NOTE: To avoid "cascading," parts after are usually scored based on correctness given answers to previous parts.

Question 1 [20 pts] A spring-mass system vibrates in a medium with damping constant zero; the spring has a constant of $k = 27$ N / m and mass $m = 9$ kg.

3pts (a) Write the homogeneous second-order differential equation that this spring-mass system satisfies.

10pts (b) Write an expression $y(t)$ for the position of the mass at time t if, at time $t = 0$, the mass is pulled down 1 m and then pushed *upward* at 3 m/s. (Use the convention that "down" is oriented positively and pay close attention to your signs.)

7pts (c) Express the above function as a single function in the form

$$y(t) = R\cos(\omega_0 t - \delta)$$

where $R > 0$ and $-\pi < \delta \leq \pi$.

(a) 3pts $9y'' + 27y = 0$

(b) 3pts: G.S w/ coeffs

2pts: soln using $y(0) = 1$

2pts: soln using $y'(0) = -3$

3pts: final answer.

(c) 2pts: R

2pts: δ

3pts: final ans

Question 2 [25 pts] Let $L[y]$ indicate the third order linear differential operator

$$y^{(3)}(t) + 3y''(t) - y'(t) - 3y(t)$$

(*i.e.*, the operator that takes a function y and returns its third derivative w/r to time plus three times its second derivative w/r to time plus ...

7 **(a)** Find the general solution to the homogeneous equation

$$L[y] = 0$$

13 **(b)** Find the general solution of the differential equation

$$L[y] = e^{-t} \tag{1}$$

5 **(c)** Circle the bullet point of each choice that represents a possible end behavior as $t \to \infty$ for solutions of Equation 1:

- Increase without bound
- Decrease without bound

} 1pt for either

- ~~Oscillate~~
- Approach a limit of zero — 3pts
- ~~Approach a finite limit other than zero~~

$\boxed{\text{− 1 or 2 pts for incorrect add'l selections}}$

NOTE. *Partial credit is possible for part (c) if work or justification is shown.*

(a) 3pts C. Pol.

3pts Correct roots

1pts general equation

(b) 5pts Guess of ate^{-t}

8pts Rest of solution

(gen. −2pts for minor course-rel. mistakes

−4pts for major-

Question 3 [20 pts] Let

$$A = \begin{pmatrix} -2 & 2 \\ 0 & 3 \end{pmatrix}$$

and consider the matrix equation

$$\mathbf{u}' = A\mathbf{u} \tag{2}$$

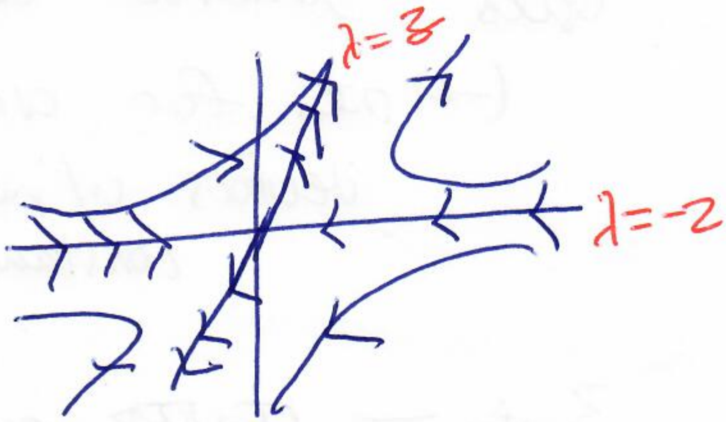
2pts (a) Find the eigenvalues of A and their corresponding eigenvectors.

2pts (b) Calculate the general solution of Equation 2.

16pts (c) Draw a reasonably accurate phase portrait (phase plane) of the solution of Equation 2; show the straight-line solutions and a few representative curves, and draw arrows oriented towards increasing time.

(c) The G.S. is

$$y(t) = c_1 e^{-2t}\begin{pmatrix}1\\0\end{pmatrix} + c_2 e^{3t}\begin{pmatrix}2\\5\end{pmatrix}$$



16 pts

= 8 × 2pts

The 2pts are for:

- Four s.l. solutions, WITH correctly directed arrows
- Four curves in each "quadrant" " WITH "

Question 4 [20 pts] The matrix

$$A = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$$

has complex (*i.e.*, with imaginary part non-zero) eigenvalues. Find the general solution to

$$\mathbf{u}' = A\mathbf{u}$$

in terms of *real-valued* functions and constants. Also describe whether its phase portrait is a spiral sink, source, or center, and what direction its spiral turns in with passing time.

2pts char polyn

2pts roots of c.p.

2pts eigenvectors

2pts one particular solution

6pts general solution

(-4pts for correct vectors w/ no constants)

3pts — CENTER and why

3pts — clockwise and why

Question 5 [15 pts] The matrix

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$$

has only one eigenvector up to scalar multiples. Find the general solution to

$$\mathbf{u}' = A\mathbf{u}$$

5 pts - Eigenvector & eigenvalue V of A

5 pts - solve for w in $(A-\lambda I)w = V$

5 pts - give g.s

(all or nothing unless derivation shown)