

Workshop 2 Solution/Rubric

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We start by proving

The first identity. The rhs is the sum of the product

$$\sinh x \cosh y = \frac{1}{2}(e^x - e^{-x})\frac{1}{2}(e^y + e^{-y}) \quad (1)$$

$$= \frac{1}{2} \cdot \frac{1}{2}(e^x - e^{-x})(e^y + e^{-y}) \quad (2)$$

$$= \frac{1}{4}(e^{x+y} - e^{-x+y} + e^{x-y} - e^{-x-y}) \quad (3)$$

and the product

$$\cosh x \sinh y = \frac{1}{2}(e^x + e^{-x})\frac{1}{2}(e^y - e^{-y}) \quad (4)$$

$$= \frac{1}{2} \cdot \frac{1}{2}(e^x + e^{-x})(e^y - e^{-y}) \quad (5)$$

$$= \frac{1}{4}(e^{x+y} + e^{-x+y} - e^{x-y} - e^{-x-y}) \quad (6)$$

which comes to

$$= \frac{1}{4}(e^{x+y} - e^{-x+y} + e^{x-y} - e^{-x-y}) + \frac{1}{4}(e^{x+y} + e^{-x+y} - e^{x-y} - e^{-x-y}) \quad (7)$$

$$= \frac{1}{4}(2 \cdot e^{x+y} - 2 \cdot e^{-x-y}) \quad (8)$$

$$= \frac{e^{x+y} - e^{-x-y}}{2} = \frac{e^{x+y} - e^{-(x+y)}}{2} = \sinh(x+y) \quad (9)$$

The second identity is proved similarly; its rhs is the sum of the products

$$\cosh x \cosh y = \frac{1}{2}(e^x + e^{-x})\frac{1}{2}(e^y + e^{-y}) \quad (10)$$

$$= \frac{1}{4}(e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y}) \quad (11)$$

and

$$\sinh x \sinh y = \frac{1}{2}(e^x - e^{-x})\frac{1}{2}(e^y - e^{-y}) \quad (12)$$

$$= \frac{1}{4}(e^{x+y} - e^{x-y} - e^{-x+y} + e^{-x-y}) \quad (13)$$

which comes to

$$= \frac{1}{4}(e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y}) + \frac{1}{4}(e^{x+y} - e^{x-y} - e^{-x+y} + e^{-x-y}) \quad (14)$$

$$= \frac{1}{4}(2 \cdot e^{x+y} + 2 \cdot e^{-(x+y)}) = \cosh(x+y) \quad (15)$$

Mathematical rubric 13 (resp., 12) pts per identity:

- **2 pts each** (4 pts total): Correctly writes out each product in terms of exponential function
- **2 pts each** (4 pts total): Distributes the results of the previous bullet to give correct products (*e.g.*, in line 3)
- **3 pts**: Correctly sums the results of the previous bullet
- **2 pts (resp., 1 pt for second identity)** Explicit comparison to lhs function.

NOTE. As much as reasonable, I will score each bullet point independently of each other bullet; *e.g.*, it will be possible to score points for correctly adding two products in terms of exponential function even if you do not receive points for those products. This policy will prevent a “cascading effect” in which you could potentially lose many points for a minor error.

Writing quality rubric

Remember that I am looking for a

- succinct,
- fluent,
- well-integrated commentary that
- uses complete sentences and
- demonstrates that you have a firm and confident command of the material needed to solve the problem.

In particular, you should avoid any errors pointed out in the “Workshop 1 Observations” announcement.

- I will give any write-up meeting these requirements an **A** for writing quality.
- Write-ups not meeting the format requirement will receive an **F** for writing quality.
- Completely incorrect answers (receiving zero points for mathematical correctness) will receive an **F** for writing quality.
- Whether I can easily follow a written argument from beginning to end will determine if it receives (1) a **B** or higher or (2) a **C** or lower.