

Consider the function  $f(x) = x^2 + x$ . We want to find the area under the graph of  $f$  over the interval  $[0, 1]$ .

1. Set up a sum to approximate this area using 4 rectangles, where the right endpoint is used for the test point on each interval. Evaluate this sum (using a calculator). What is the set of points that you plugged into  $f$  to evaluate this sum?
2. What if you wanted to do this with 10 rectangles? What would your collection of points look like here? Write this in terms of some number  $\alpha$  times  $k$ , where  $k$  ranges from 1 to 10.
3. What would this look like for 20 rectangles? Write this using summation notation. You should also have the points written in the form  $\alpha k$  where  $k$  ranges from 1 to 20.
4. What do I get for  $N$  rectangles, where  $N$  is some large number? Use the pattern you've seen from the previous two parts to figure out what this sum would look like in summation notation.
5. Expand out the terms in this sum, and use the power sum formulas from class/the textbook to convert this to an expression depending on  $N$ .
6. Take the limit as  $N$  goes to  $\infty$  to get the area.
7. Carry out the steps (at least (d), (e), and (f)) for using the left endpoints as the test points. When writing your sum, you are allowed to take your index  $k$  going from 0 to  $N - 1$  instead of 1 to  $N$ . Check to see that you get the same answer as the previous part.