

Earlier in this course, we learned the Chain Rule and how it can be applied to find the derivative of a composite function. In particular, if u is a differentiable function of x and f is a differentiable function of $u(x)$, then $\frac{d}{dx}[f(u(x))] = f'(u(x)) \cdot u'(x)$.

In other words, we say that the derivative of a composition function $c(x) = f(u(x))$, where f is considered the “outer” function and u the “inner” function, is “the derivative of the outer function, evaluated at the inner function, times the derivative of the inner function”.

1. For each of the following functions, use the Chain Rule to find the function’s derivative. Be sure to label each derivative by name (e.g., the derivative of $g(x)$ should be labeled $g'(x)$).

- $g(x) = e^{3x}$
- $h(x) = \sin(5x + 1)$
- $p(x) = \arctan(2x)$
- $q(x) = (2 - 7x)^4$
- $r(x) = 3^{4-11x}$

2. For each of the following functions, use your work in (a) to help you determine the general antiderivative of the function. Label each antiderivative by name (e.g., the antiderivative of m should be called M). In addition, check your work by computing the derivative of each proposed antiderivative.

- $m(x) = e^{3x}$
- $n(x) = \cos(5x + 1)$
- $s(x) = \frac{1}{1+4x^2}$
- $v(x) = (2 - 7x)^3$
- $w(x) = 3^{4-11x}$

3. Based on your experience in parts (a) and (b), conjecture an antiderivative for each of the following functions. Test your conjectures by computing the derivative of each proposed antiderivative.

- $a(x) = \cos(\pi x)$
- $b(x) = (4x + 7)^{11}$
- $c(x) = xe^{x^2}$