Problems esergel52, esergel53, and esergel54 are intended to be used together.

1. (Bonus problem, not to be turned in) Recall the formula

$$\sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}$$

Our goal is to compute the N-th left- and right-endpoint approximations L_N and R_N for the area under the curve $f(x) = x^2$ between $x = 0$ and $x = b$ when $b > 0$.

(a) What is Δx? What is x_j? These should depend on b and N.

(b) Set up a formula for R_N as a sum. Then rearrange the terms so that you can use the formula above to get an expression for R_N which has no $\sum$ or $\ldots$ in it.

(c) Write a formula for $\displaystyle\sum_{j=0}^{N-1} j^2$. Check that your formula works when $N = 4$.

(d) Set up a formula for L_N as a sum. Then use the formula from part (c) to get an expression for L_N which has no $\sum$ or $\ldots$ in it.

(e) Now take the limit of the expressions you got in parts (b) and (d) as $N \to \infty$. Check that you get $\frac{b^3}{3}$ for both of these limits.