Here we will explore some calculations of limits.

1. A fellow student claims to be able to prove that $1 = 0$ by writing

$$\begin{aligned} 1 &= \lim_{t\to 0} 1 = \lim_{t\to 0} \tan t \cot t \\ &= \left(\lim_{t\to 0} \tan t\right)\left(\lim_{t\to 0} \cot t\right) \\ &= 0\left(\lim_{t\to 0} \cot t\right) = 0 \end{aligned}$$

   Something is definitely wrong here. Explain what this student did wrong and why this does not prove that $1 = 0$.

2. Another student claims to have calculated the following limit:

$$\lim_{x\to 0} x \sin\frac{1}{x} = \left(\lim_{x\to 0} x\right)\left(\lim_{x\to 0} \sin\frac{1}{x}\right) = 0\left(\lim_{x\to 0} \sin\frac{1}{x}\right) = 0$$

   Do you agree with the student's logic here? If not, what is wrong with it? (Hint: Look what you said for the previous part.)

3. A third student claims that because

$$\lim_{x\to 0} \frac{x^3 - 4x^2 + 4x}{x^3 + 5x^2 + 4x}$$

   is of the form $\frac{0}{0}$, then the limit must be 1. Show that the student did get to the correct answer, but explain how their reasoning is wrong by providing an example of an indeterminate form where the limit does not evaluate to 1.

4. The student's answer in the second part was correct, even though they got there by the wrong method. Use the squeeze theorem to find a correct proof of this statement.