Students in a calculus class were asked to solve the following problem:

> A company is manufacturing widgets and selling them. Based on their current setups, they can only produce between 0 and 10,000 widgets per day. If they produce x thousand widgets, the price they can sell them at is $p(x) = 20x - x^2 + 30$. However, the cost to produce x widgets is $C(x) = 5x^2 + 105x + 10$. Find the production level that maximizes profit.

A student's work on this problem is as follows.

> We know that the formula for profit is
>
> $$P(x) = xp(x) - C(x) = 20x^2 - x^3 + 30x - (5x^2 - 105x + 10) = -x^3 + 15x^2 - 75x - 10$$
>
> We then take the derivative and set it to zero:
>
> $$0 = P'(x) = -3x^2 + 30x - 75 = -3(x^2 + 10x - 25) = -3(x - 5)^2$$
>
> Thus our only critical point is $x = 5$. This must be the point that maximizes profit, so the desired production level is 5,000 widgets.

1. There is a significant step at the end of the process that is missing. What is it?
2. Sketch a graph of the profit function over the range 0 to 10. What happens at the critical point $x = 5$?
3. Work out a full solution to the problem. What should the company do?