

Sketch the graph of the following function:

$$f(x) = \frac{x^2 - 16}{x^2 - 9}$$

To do so, carry out the following steps.

1. Factor the numerator and denominator. Where is $f(x) = 0$? Where is $f(x)$ not defined?
2. Take the limit of f as $x \rightarrow \infty$ and $x \rightarrow -\infty$. Are there any horizontal asymptotes? If so, what are they?
3. Are there any vertical asymptotes? If so, where are they? Determine what the graph of f looks like near the asymptotes by taking the one-sided limits of f as x approaches the asymptote.
4. Take the derivative of f . Where is f increasing and decreasing? Where does f have critical points?
5. Take the second derivative of f . Where is f concave up and down? Where does f have inflection points?

Collect all of your answers in the following table. Use interval notation for all of the answers for which it is relevant.

Zeros	
Vertical Asymptotes	
Horizontal Asymptotes	
Increasing	
Decreasing	
Critical Points	
Concave Up	
Concave Down	
Inflection Points	

Now, use this information to sketch the graph of the function $y = f(x)$.