

Sketch the graph of the following function:

$$f(x) = \frac{x^2 - 16}{x^2 - 9}$$

To do so, carry out the following steps.

1. Factor the numerator and denominator. Where is  $f(x) = 0$ ? Where is  $f(x)$  not defined?
2. Take the limit of  $f$  as  $x \rightarrow \infty$  and  $x \rightarrow -\infty$ . Are there any horizontal asymptotes? If so, what are they?
3. Are there any vertical asymptotes? If so, where are they? Determine what the graph of  $f$  looks like near the asymptotes by taking the one-sided limits of  $f$  as  $x$  approaches the asymptote.
4. Take the derivative of  $f$ . Where is  $f$  increasing and decreasing? Where does  $f$  have critical points?
5. Take the second derivative of  $f$ . Where is  $f$  concave up and down? Where does  $f$  have inflection points?

Collect all of your answers in the following table. Use interval notation for all of the answers for which it is relevant.

<b>Zeros</b>	
<b>Vertical Asymptotes</b>	
<b>Horizontal Asymptotes</b>	
<b>Increasing</b>	
<b>Decreasing</b>	
<b>Critical Points</b>	
<b>Concave Up</b>	
<b>Concave Down</b>	
<b>Inflection Points</b>	

Now, use this information to sketch the graph of the function  $y = f(x)$ .