

In Section 4.2, we have discussed Rolle's Theorem, which says that if f is continuous on $[a, b]$ and differentiable on (a, b) with $f(a) = f(b)$, then there exists a point c between a and b so that $f'(c) = 0$. The point of this workshop problem is to use this to prove something potentially more useful: the Mean Value Theorem. Let g be any function that is continuous on $[a, b]$ and differentiable on (a, b) .

1. The only part we are missing is the equality of the endpoints. Let $h_1(x) = g(x) - g(a)$. What is $h_1(a)$?
2. Now, let $h(x) = g(x) - g(a) - \frac{(x-a)}{(b-a)}g(b)$. What is $h(a)$? What is $h(b)$?
3. Based on what Rolle's Theorem says, we now know that it applies to h . What does this mean in terms of h and some point c between a and b ?
4. Use the definition of h , take the derivative, and use the result from Rolle's Theorem to get the following result: There exists a c between a and b so that

$$g'(c) = \frac{g(b) - g(a)}{b - a}$$