

The assumptions on the Mean Value Theorem are important, and whenever you apply the Mean Value Theorem, you need to make sure that the assumptions are verified. For each of the following criteria, come up with a function that has those properties and does not satisfy the conclusion of the Mean Value Theorem. That is, find a function f so that $f(0) = 1$, $f(2) = 3$, and there is no point c between 0 and 2 with $f'(c) = 1$.

1. f has a jump discontinuity somewhere between 0 and 2.
2. f has an infinite discontinuity between 0 and 2.
3. f is continuous on $[0, 2]$ but still does not satisfy the hypotheses of the mean value theorem. **Hint:** What is the other hypothesis? How can we come up with continuous functions that do not have this property?