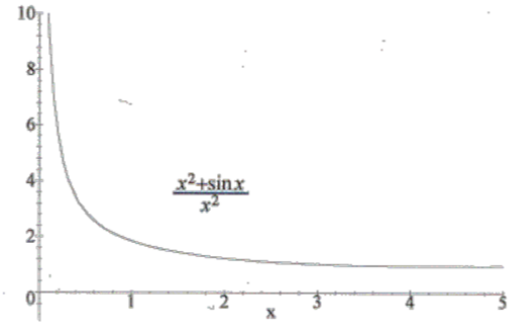
Your friend was asked to evaluate

$$\lim_{x\to\infty} \frac{x^2+\sin x}{x^2}.$$

They thought it seems like a clear case of L'Hôpital's rule, and so here is the work:

$$\begin{aligned} &\lim_{x\to\infty} \frac{x^2+\sin x}{x^2} && \left(\frac{\infty}{\infty}\right) \\ =&\lim_{x\to\infty} \frac{2x+\cos x}{2x} && \left(\frac{\infty}{\infty}\right) \\ =&\lim_{x\to\infty} \frac{2-\sin x}{2} \end{aligned}$$

and since $\lim_{x\to\infty} \frac{2-\sin x}{2}$ does not exist, neither does the original limit. However, when you plot $\frac{x^2+\sin x}{x^2}$, it sure looks like the limit exists.



1. Find the error in your friend's reasoning.
2. Provide a correct solution. What is the actual limit?