

1. A form is indeterminate if it can take on more than one value. In parts (a)-(c) below, show that the given form is indeterminate. In other words, find two limits which have this form but evaluate to different values.

- (a) Show that the form $\frac{\infty}{\infty}$ is indeterminate.
- (b) Show that the form $0 \cdot \infty$ is indeterminate.
- (c) Show that the form 0^0 is indeterminate.
- (d) Explain why the forms $\frac{0}{\infty}$ and 0^∞ are not indeterminate. Your answer should consider what happens when the parts of these forms are not precisely 0 or ∞ , but very close.
- (e) We have seen in class that the form 1^∞ is indeterminate. For instance, in class we showed that $\lim_{x \rightarrow \infty} 1^\infty = 1$ and $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$. However, a friend makes the following argument that 1^∞ is always ∞ . What is wrong with this argument?

The form 1^∞ means we are taking a limit of $f(x)^{g(x)}$ where $f(x)$ is approaching 1 and $g(x)$ is approaching ∞ . But when you raise a number to a bigger and bigger power, it grows to infinity. For instance, 1.01 is small, but $1.01^{1,000}$ is over 20,000 and $1.01^{10,000}$ is way over a billion. So $1^\infty = \infty$.