

Each of the following “derivatives” has been done *incorrectly*. In each case below, find a counterexample that shows why the reasoning is incorrect. Then correct the mistakes by finding the actual derivative of the left hand side.

For example: The statement “ $\frac{d}{dx}f(2x) \stackrel{?}{=} f'(2x)$ ” is false because if $f(x) = x^2$, then we have $\frac{d}{dx}f(2x) = \frac{d}{dx}[4x^2] = 8x$, but on the other hand, $f'(x) = 2x$, so $f'(2x) = 4x$. In general, the correct derivative would be “ $\frac{d}{dx}f(2x) = f'(2x) \cdot 2$ ” by the chain rule.

(a) $\frac{d}{dx}f(x+5) \stackrel{?}{=} f'(x)$

(b) $\frac{d}{dt}g(t/3) \stackrel{?}{=} g'(t)/3$

(c) $\frac{d}{ds}h(s^2) \stackrel{?}{=} h'(2s)$

(d) $\frac{d}{dt}k(x) \stackrel{?}{=} k'(x)$

(f) $\frac{d}{dx}l(x \cos(x)) \stackrel{?}{=} l(\cos(x) - x \sin(x)) \cdot x \cos(x)$

(g) $\frac{d}{du}[F(u) \sin(2u)] \stackrel{?}{=} 2F'(u) \cos(2u)$

(h) $\frac{d}{dt}G(\sin(t)) \stackrel{?}{=} G'(\sin(t)) + \cos(t)$