Consider the equation

$$ax^2 + bxy + cy^2 = 1$$

for three parameters a, b, and c.

1. This is the equation corresponding to graphs of conic sections. What kind of a shape is drawn if:

   (a) $b^2 - 4ac < 0$?

   (b) $b^2 - 4ac = 0$?

   (c) $b^2 - 4ac > 0$?

   You can use a calculator here to draw graphs and identify the curves from that.

2. Find $\frac{dy}{dx}$ for the first equation in this problem. Your answer will depend on the parameters a, b, and c. (You should solve for $\frac{dy}{dx}$ here. It'll make your life a lot easier.)

3. Set $\frac{dy}{dx} = 0$ and see what this means in terms of the values of x and y. Plug this into the initial equation to see what that means in terms of where horizontal tangent lines exist.

4. Is it possible to choose values for a, b, and c so that the graph both is a hyperbola *and* there is a point where the graph has a horizontal tangent line? If so, come up with values for a, b, and c that do this, as well as the point (x, y) that has the horizontal tangent line. If not, explain why not.

5. Is it possible to choose values for a, b, and c so that the graph both is a hyperbola *and* there are no points where the graph has a horizontal tangent line? If so, come up with values for a, b, and c that do this. If not, explain why not.

6. Is it possible to choose values for a, b, and c so that the graph both is an ellipse *and* there is a point where the graph has a horizontal tangent line? If so, come up with values for a, b, and c that do this, as well as the point (x, y) that has the horizontal tangent line. If not, explain why not.

7. Is it possible to choose values for a, b, and c so that the graph both is an ellipse *and* there are no points where the graph has a horizontal tangent line? If so, come up with values for a, b, and c that do this. If not, explain why not.