

Consider the following functions.

1.  $f(x) = \sin(x)$
2.  $f(x) = (1/x)$
3.  $f(x) = \sqrt{x}$

There is a pattern in the derivatives of these function. We will discover and establish it.

- (a) Compute the first, second, and third derivative of each of these functions.
- (b) Propose a general formula for  $f^{(n)}(x)$ . This will be in terms of  $n$ .
- (c) Verify the formula using a computer system. Check what the 10th derivative of the function is from your formula, then plug that into a computer system (Wolfram Alpha, for example) to see what it gives for the 10th derivative. They should match!
- (d) **Bonus. Do not turn in.** Verify the proposed formula in two steps. Step 1 - verify the formula when  $n = 1$ . Step 2- assuming formula is true for a certain value of  $n$ , let us say for  $n = k$ , show that formula is valid for the next value of  $n = k + 1$ .

This last part is an elegant way of establishing the truth of statements like “for all  $n$ , something statement involving  $n$  is valid ”. Convince yourself it works! This method is called ‘mathematical induction’.