Students were given the following problem: Find all points where the graph of the function
$$f(x) = (x^2 - 1)(x^3 - 9x^2 + 24x + 92)$$
has a horizontal tangent line.

First, solve this problem. **Note:** You will need a calculator to find the values where the tangent line is horizontal, but you should be able to get to an equation for those points by hand. Then, grade each of the following student responses out of 10 points. You should provide feedback to explain what was done wrong and why the student received the number of points you gave them.

1. The places where the graph of f has a horizontal tangent line are the places where $f(x) = 0$, which we can find by

$$(x^2 - 1)(x^3 - 9x^2 + 24x + 92) = 0$$

which has solutions $x = -2, -1, 1$.

2. The places where the graph of f has a horizontal tangent line are the places where $f'(x) = 0$. We compute the derivative

$$f'(x) = 2x(3x^2 - 18x + 24) = 6x(x^2 - 6x + 8) = 6x(x - 2)(x - 4)$$

Setting this to zero, we find the places with a horizontal tangent line are $x = 0, 2, 4$.

3. First, we need to take the derivative

$$\begin{aligned} f'(x) &= (x^2 - 1)(3x^2 - 18x + 24) + 2x(x^3 - 9x^2 + 24x + 92) \\ &= 3x^4 - 18x^3 + 24x^2 - 3x^2 + 18x - 24 + 2x^4 - 18x^3 + 48x^2 + 92x \\ &= 5x^4 - 36x^3 + 69x^2 + 110x - 24 \end{aligned}$$

We then plug in $x = 0$ to get horizontal tangent lines, giving us -24 as the point with a horizontal tangent.