1. (a) Most identities about regular trig functions are related to the fundamental identity $\sin^2(x) + \cos^2(x) = 1$. What is the fundamental identity for hyperbolic trig functions? Show that this identity holds by using the definitions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \text{and} \quad \cosh(x) = \frac{e^x + e^{-x}}{2}.$$

(b) Divide the identity from (a) by $\sinh^2$ or $\cosh^2$ to get two new identities.

(c) Use Implicit Differentiation to show that the derivative of $\tanh^{-1}(x)$ is $\frac{1}{1-x^2}$. You will need to make use of the identity in (a) or one of the identities in (b). [Hint: If you're not sure how to do this problem, you may use part (d) as a roadmap.]

(d) Below is an attempt to compute the derivative of the inverse hyperbolic cosecant function $\operatorname{csch}^{-1}(x)$. There is a subtle difference between the formula obtained below and the one given to you in class / in the textbook. Locate the error in this argument and fix it.

Let $y = \operatorname{csch}^{-1}(x)$. Then $\operatorname{csch}(y) = x$, i.e. $1 = x\sinh(y)$. Hence

$$0 = x\cosh(y)\frac{dy}{dx} + \sinh(y).$$

Therefore $\frac{dy}{dx} = -\frac{\sinh(y)}{x\cosh(y)}$. Since $\sinh(y) = x^{-1}$,

$$\cosh(y) = \sqrt{1 + x^{-2}} = x^{-1}\sqrt{x^2+1}.$$

So finally we get

$$\frac{dy}{dx} = -\frac{x^{-1}}{x \cdot x^{-1}\sqrt{x^2+1}} = -\frac{1}{x\sqrt{x^2+1}}.$$