Suppose $f(x) = x^3 + x - 1$.

1. Explain why f has a root in the interval $[0, 1]$.
2. Suppose A is a constant and $g(x) = x^3 + x - 1 + Ax(x-1)(2x-1)$. Show that g has at least one root in the interval $[0, 1]$.
3. Calculate $g\left(\frac{1}{3}\right)$ and $g\left(\frac{2}{3}\right)$. Show that if A is large enough, g must have three roots in the interval $[0, 1]$. For what values of A do we know for sure that g has three roots in $[0, 1]$?