Consider the functions $f(x) = |x| \sin\left(\dfrac{\pi}{x}\right)$ and $g(x) = \sin\left(\dfrac{1}{x}\right)$. This is a strange function. Let's investigate how it behaves near 0.

1. What are $f\left(\frac{1}{10}\right)$, $f\left(\frac{-1}{100}\right)$, $f\left(\frac{1}{500}\right)$, and $f\left(\frac{-1}{634{,}298}\right)$? In general, for any nonzero integer $n$, what is $f\left(\frac{1}{n}\right)$? Does this numerical evidence prove that $\lim_{x \to 0} f(x) = 0$? Why or why not? What if you were to do the same for the function $g(x)$?

2. Using a calculator, graph the curve $y = f(x)$. Adjust the viewing window so that $x$ goes from -5 to 5 and $y$ goes from -3 to 3. Zoom in several times, observing the behavior of $f$ near 0 each time. Describe what you see. Does this graphical evidence prove that $\lim_{x \to 0} f(x) = 0$? Why or why not?

3. Recall the precise mathematical definition of the statement "$\lim_{x \to 0} f(x) = 0$". We say that $\lim_{x \to 0} f(x) = 0$ if for any arbitrarily small positive number $\epsilon$, *no matter how small* (think: $\epsilon < .01$), it is possible to find some positive number $\delta$, perhaps even smaller than $\epsilon$, such that whenever $x$ is within $\delta$ of 0, $f(x)$ is within $\epsilon$ of 0. Using this formal definition, is it true that $\lim_{x \to 0} f(x) = 0$? Justify your answer.