

1. Suppose $f(x) = \frac{\sqrt{2-\sqrt{4-x^2}}}{x}$.

(a) Sketch the graph of $y = f(x)$ in the viewing window $[-2, 2] \times [-1, 1]$.

(b) Based on the graph, what do you think the values of $\lim_{x \rightarrow 0^-} f(x)$, $\lim_{x \rightarrow 0^+} f(x)$, and $\lim_{x \rightarrow 0} f(x)$ should be?

(c) Consider the following argument: We see algebraically that

$$f(x) = \frac{\sqrt{2-\sqrt{4-x^2}}}{x} \cdot \frac{\sqrt{2+\sqrt{4-x^2}}}{\sqrt{2+\sqrt{4-x^2}}} = \frac{1}{\sqrt{2+\sqrt{4-x^2}}}.$$

Therefore

$$\lim_{x \rightarrow 0} f(x) = f(0) = \frac{1}{2}.$$

Does this agree with your prediction? Which do you think is wrong, the graph or the argument? Either way, find the reason for the discrepancy.