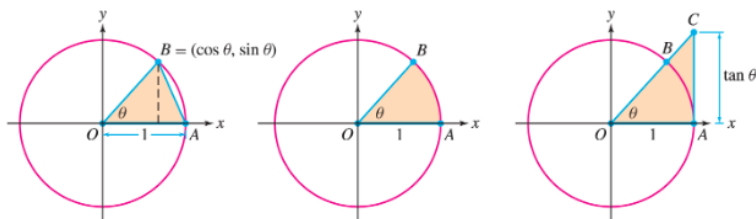


1. (a) Suppose  $0 < \theta < \frac{\pi}{2}$ . Recall the formulas for the area of a triangle and a sector (pie slice) of a circle. Find the area of each shaded region below. Explain why the height of the triangle on the right is  $\tan(\theta)$ .



- (b) Order the areas you computed in part (a) from smallest to largest. Use this to argue that  $\cos(\theta) \leq \frac{\sin(\theta)}{\theta} \leq 1$  for  $0 < \theta < \frac{\pi}{2}$ . [Actually, it is true for  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ .]
- (c) Use the Squeeze Theorem to argue that  $\lim_{x \rightarrow 0} \frac{\sin(\theta)}{\theta} = 1$ . State specifically what  $l(x)$ ,  $f(x)$ ,  $u(x)$ ,  $c$  and  $L$  are.
- (d) Consider the graph of  $\sin(\theta)$ . What is the slope of the secant line between the points  $(0, 0)$  and  $(\theta, \sin(\theta))$ ? What is the slope of the tangent line through  $(0, 0)$ ?
- (e) Consider the graph of  $\cos(\theta)$ . What do you think the slope of the tangent line through  $(0, 1)$  is? How does this relate to the fact that  $\lim_{x \rightarrow 0} \frac{1 - \cos(\theta)}{\theta} = 0$ ?