Here we will explore some properties of inverse functions.

1. The function $f(x) = 4x - 3$ is invertible. Find the inverse function $f^{-1}(x)$ and plot a graph of both f and f^{-1}.

2. The function $f(x) = x^2$ is not invertible. Try to solve for the inverse anyway, and sketch graphs of f and everything you get by trying to solve for the inverse.

3. What do we notice about these graphs? In particular, what is strange about the graph(s) for f^{-1} when f was not invertible? Think about a test we have for determining if a graph is represented by a function.

4. Look at the graphs you have above and see if you can find a relation between the graph of f and the graph of f^{-1}. From a more algebraic perspective, if the point (a, b) is on the graph of f, what point has to be on the graph of f^{-1}? What does this mean in terms of the geometry of the graphs?

5. With this relation you described in the previous part, how does the function test for f^{-1} relate to a different type of test on the graph of f? That is, is there a test we could perform on the graph of f to see if it will be invertible?

6. We can fix the problem of x^2 being non-invertible by changing the domain. Draw a graph of $f(x) = x^2$ on $x \geq 0$. Does it pass the new test? If so, what is the inverse function to this restricted f?