Teaching Portfolio - Teaching Experience Supplement

Rutgers University

Matthew Charnley

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Contents

Chapter 1

Introduction

This document provides an in-depth summary of my experiences of the classes I have taught on my own during my time at Rutgers. It includes my summary of each of these classes, a sample of the materials that I created for these classes, and the student feedback from these classes.

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EDUCATION

Doctor of Mathematics,
Rutgers University, Piscataway, NJ
May 2019 (expected)
Concentration: Partial Differential Equations
Advisor: Michael Vogelius

Bachelor of Science, Mathematics and Chemical Engineering
University of Notre Dame, Notre Dame, IN
May 2013

HONORS AND AWARDS

Spring 2016 - Invitation to RASTL
Spring 2018 - TA Teaching Excellence Award, Rutgers University Math Department
Fall 2018 - TAPCast, Invited Guest, Season 1 Episode 4: Active Learning, with Sandra Medina (Spanish). Released October 15, 2018. `http://go.rutgers.edu/y87h66ik`

TEACHING EXPERIENCE

Fall 2018 - Introduction to Probability
Summer 2018 - Differential Equations
Summer 2017 - Ordinary Differential Equations
Summer 2016 - Introduction to Probability
Summer 2015 - Calculus 3

TEACHING ASSISTANT EXPERIENCE

Spring 2018 - Numerical Methods in PDEs/Numerical Analysis 2
Fall 2017 - Advanced Calculus for Engineers
Fall 2016 - Graduate Real and Complex Analysis
Spring 2016 - Numerical Methods in PDEs
Spring and Fall 2015 - Calculus 3
Fall 2014 - Calculus 1

PROFESSIONAL DEVELOPMENT

Rutgers Academy for the Scholarship of Teaching and Learning (RASTL) — Fall 2016 - Spring 2019
Rutgers University, New Brunswick, NJ

- Met with graduate students from various departments at Rutgers to discuss pedagogy and other issues with teaching.
- Led workshops for the group on ‘Engaging Students and Managing Discussions’ and ‘Classroom Expectations’.

Rutgers TA Project — Fall 2016 - Spring 2019
Rutgers University, New Brunswick, NJ

- Ran and assisted in workshops run by the TA Project for graduate students at Rutgers interested in learning more about teaching.
- Discussed topics such as ‘Teaching a Summer Course’, ‘Teaching Non-Majors’, and ‘Providing Feedback that Matters’.
- Presented to new graduate students at TA Orientation, informing them of how the TA program in the math department works and where they can go to get extra teaching resources.

Rutgers Math Teaching Group Fall 2016 - Spring 2019
Rutgers University, Piscataway, NJ

- Planned a weekly seminar where graduate students from the math and math education departments could discuss teaching issues.
- Invited speakers from other departments at Rutgers to discuss programs that exist to support teaching.
- Participated in a semester-long discussion of the Calculus sequence at Rutgers and ways that it could be improved.

Math TA-At-Large Program Organizer Fall 2017 - Spring 2018
Rutgers University, Piscataway, NJ

- Organized the online office hours for the TA-At-Large program.
- Gathered class information from the TAs to provide to the technology staff to get the classes set up in the online system.
- Scheduled the 12 sets of office hours in the single technologically-capable office each of two semesters.
- Provided an outline to the department on how to run this program in future semesters.

MENTORING EXPERIENCE

Directed Readings Program Fall 2014 - Spring 2018
Rutgers University, Piscataway, NJ

- Mentored advanced undergraduates in independent study projects, showing them a little bit about what advanced math looks like.
- Advised students on giving a 15 minute presentation about the semester's work at the end of the project.
- Supervised 6 projects related to the Hydrogen Atom, Fourier Analysis, Functional Analysis and the δ-function, and Markov Chains.

Summer Session Head TA Summer 2016 - Summer 2018
Rutgers University, Piscataway, NJ

- Served as a peer reviewer for other graduate students teaching classes over the summer.
- Provided a formative assessment to each graduate student near the start of their summer class.
- Advised peers on how to improve their teaching before the summative review later in the summer by a full-time faculty member.

PUBLICATIONS

1. Charnley, M and M. Vogelius. In preparation.
2. Charnley, M and A. Wood. In preparation.
3. Charnley, M. “The average of a polygon is an ellipse.” 2018. MAA Mathematics Magazine. (submitted)
4. Charnley, M. and A. Wood. “A linear sampling method for through-the-wall radar detection.” 2017. DOI: `10.1016/j.jcp.2017.06.035`.
5. Charnley, M. and A. Wood. “Through-the-wall radar detection analysis via numerical modeling of Maxwell's equations.” 2016. DOI: `10.1016/j.jcp.2016.01.039`.

EXTERNAL TALKS

1. "A Modern Approach to Gårding's Asymptotics Result." Rutgers Camden Several Complex Variables Learning Seminar. Rutgers University - Camden Campus. March 30, 2018.
2. "An energy lemma and an application to thin inhomogeneities." NYS Regional Graduate Mathematics Conference. Syracuse University. March 24, 2018.
3. "Flipped Classrooms for Higher Level Mathematics." NE RUME Conference. Montclair State University. November 11, 2017.
4. "A Linear Sampling Method for Through-the-Wall Radar Detection." Colloquium at the Air Force Institute of Technology. March 2, 2017.
5. "Numerical Simulation of Maxwell's Equations for Radar Detection Analysis." Brown Bag Seminar at the Air Force Institute of Technology. August 27, 2015.

OTHER RESEARCH

ORISE Summer Researcher Summer 2015 - Present
Air Force Institute of Technology, Dayton, OH

- Continued work from a Master's Thesis on "Through-the-Wall" imaging.
- Developed techniques for generating data and analyzing the results numerically, implementing both existing research and new ideas.
- Presented findings at AFIT at several of their seminars.

Chapter 2

Teaching Course Summaries

2.1 Summer 2015 - Math 251

2.1.1 Class Overview

Math 251 is Rutgers' version of Multivariable Calculus. The difference for this instance is that it was run over the summer. During the summer, Rutgers runs compressed classes for students who are either trying to get ahead or catch up on classes. This class in particular met 4 days a week for 8 weeks, with 2 hour sessions every day. The format of these courses makes running them very interesting, because in addition to covering material very quickly, the lecture sessions are also longer than classes during the semester, and keeping the students' focus is also an issue. Running classes over the summer also involves planning out a syllabus for the course and deciding how you are running the class, because it is not identical to classes that are run during the semester. The material is the same, but the structure is up to the instructor to decide. As this was my first class, I kept things fairly simple, but it was still a good exercise in planning and organizing a class.

2.1.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix A.1. Included in this section are

1. The course syllabus
2. A sample practice problem with its solution
3. A sample quiz with its solution
4. A review sheet handed out before the final exam

As this was my first summer class, I stuck fairly close to the standard syllabus and class format. The only main thing I introduced were the Practice Problems that were done in the middle of class.

2.1.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix B, at the end of this document. A few selected comments from these surveys are

- "I love to see teachers who are passionate about what they are teaching it makes me wish I could enjoy the subject as much as they do."
- "He is always showing us how even the hardest of topics can be tackled in the simplest of ways."
- "He was always encouraging and made sure everyone had a fair chance at asking their questions as well as doing a great job answering them."

- "Mr. Charnley was excellent in explaining all the concepts, presenting examples and applications, and answering any questions I had."

2.1.4 Class Reflection

This was my first time teaching a class for myself, and for a first experience, I think it went really well. I had an 8 AM class time, which I know can be difficult for students, but I tried to make sure to bring as much energy to the class as I could in the morning. The students seemed very responsive to the early class, much more than I had anticipated, and I think that fed into the atmosphere of the classroom. Even though I was doing a lot of lecturing during the class time, I tried to encourage discussion and questions from the students to help break up the class period. Twice a week, the second half of class was devoted to a recitation-type period, where students were encouraged to ask homework questions, and I would work out solutions on the board. I think this also helped to break up the aggressive schedule of meeting four days a week for two hours at a time.

As the summer courses are supposed to be equivalent to those taught during the semester, the topics on the syllabus were pretty much set for me from the start. In addition, since this was an 8 week class, there were actually more class meetings than a lecture would get during the semester, so it was pretty easy to figure out how to schedule everything for the summer. This also meant that, because the semester courses have recitations, there was also supposed to be some time for that built into the schedule. My general plan for the course just came from putting all of that information together. Thankfully, it worked out nicely, but it was good practice for the more complicated courses I needed to plan out in future semesters.

A component that I added to this class that is not usually seen during the semester were my 'Practice Problems.' These were generally given out around the middle of class for two purposes. They were meant to give the students a chance to practice what I had just been talking about, and they were also a way to break up the 2 hour lectures. I would tend to lecture for around an hour, then give the students a practice problem to work on, followed by a break. Depending on the class, after the break would consist of more lecture, recitation time, or a quiz. I felt like these worked out well, but I should have worked them more into the lecture. That is, I should have given them time to work on the problem, and then discussed it afterwards. I did this a few times, with some of the harder sections, but it probably should have been done more often. These practice problems continued to be present in all of my later classes, and I tried to make them more involved in the structure of the class as a whole.

Overall, this class was a lot of work, but a great experience. I had been a TA for the same class the previous semester, so I knew the material very well, but being in front of the class the entire time, and being the main lecturer, was new to me. I very much enjoyed the experience and felt like I better understood the dynamics of a classroom environment after this class. I think I did a good job promoting the type of environment that I wanted to see in the classroom, but it could have been better. This is something that I have striven to improve on in all of my future classes.

2.2 Summer 2016 - Math 477

2.2.1 Class Overview

Math 477 is a class on the Theory of Probability. This is the senior-level version of probability, requiring Calculus 3 as a prerequisite, which goes into both discrete and continuous probability. It covers the general axioms of probability, a basic introduction to combinatorics (in terms of counting outcomes for probabilities), independence, conditional probability, expectation, and the same concepts for continuous random variables. It is also a prerequisite for the classes on stochastic processes and mathematical statistics.

During the summer, this class was run as a 6-week course, meeting 4 days a week for 2 hours each meeting. With only 6 weeks, the class moved very quickly, and it was a struggle to keep up with everything. With this, there was room for only one midterm along with the final exam, and only 4 quizzes. There would generally be a lot more of these during the semester, but there really isn't time for it over the summer. This was also the second class I had taught over the summer, so I had some experience with this format, but the class itself was very different.

2.2.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix A.2. Included in this section are

1. The course syllabus
2. A Calculus 3 review that was given to the students early on in the class to make sure they were ready for the second half of the course
3. A sample homework solution that was posted for the students
4. A sample quiz with its solution
5. A topic list for the final exam that was given to the students
6. A set of review problems for the final exam

2.2.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix B, at the end of this document. A few selected comments from these surveys are

- "in class practice problems that reinforced what the class learned that day"
- "go over the harder homework problems assigned. give more examples that were not in the textbook"

- "This class was tough and had dry material but the professor definitely made it more bearable"
- "I found that the last three chapters seemed rushed. A longer focus on this material may be more beneficial."
- "I would assign less homework; maybe one assignment every other day. Even though you should be studying (i.e. doing problems) every day in a summer course, some days it was hard to complete homework on time if you have other obligations in addition to the course."

2.2.4 Class Reflection

This class was one of the most difficult experiences I had teaching a class. The main reason for this was that I have never taking a college level probability class, and so had to learn the material right before teaching the class. This meant that not only was I not as confident in my knowledge of the material as I should have been, but I also didn't know any of the common pitfalls or issues that students would run into with the material over the course of the summer. There were many instances over the summer where immediately after class, I knew that it didn't go as well as I would have liked, and I knew what I needed to change to make it better. It was a frustrating experience, but by the time the class started, there wasn't really an opportunity to make up ground and fix the problem because of how fast the course progresses. The midterm is three weeks in, and by that point, the class is already done with discrete random variables and moving through continuous probability. I felt like I was behind where I wanted to be the entire class and didn't really have the ability to make it up, which can be seen in the fact that at least one student felt that the last three chapters were rushed. On top of the issues with the material, I felt like this course had a fairly distinct end point (getting to the law of large numbers), which meant that even though I felt like I was behind, I wanted to get to that ending, causing me to rush through the last part of class.

I also had difficulty with structuring this class properly because of the schedule. Since this is only a three credit class, it only runs for 6 weeks and there is no built-in time for recitation sessions. Thus, for a class like this, it is supposed to be basically all lecture over the summer, which is impossible and doesn't work very well in a class that meets 4 times a week for 2 hours at a time. It was very difficult to cover the necessary material within that time, with also trying to allow for breaks and the practice problems that I used the previous year. My intention at the start of the summer was to lecture for about an hour, give a practice problem, and then lecture for 40 minutes or so after that. In reality, since I was not as strong with the material, the first part of lecture regularly took 70 to 80 minutes, meaning that after the practice problem (which also took longer than expected because I didn't know how to write good and easy problems on the material) left like 20 minutes to wrap up the class. I never really got much done in this time, but felt like I needed to keep pushing on, otherwise I was not going to make it through all of the material that I wanted to for the class.

This class really taught me how important it is to be prepared for a class; not just in terms of knowing the material, but knowing how to teach it. I felt like I knew a decent amount of probability going into this class (since I had worked through the book on my own to learn it) but I didn't really know how to teach it. I didn't know where the common mistakes were going to be and what problems to assign to flush them out. I didn't have an array of extra examples in my head that I could bring out to use at any time, so I mostly stuck with using the examples in the book because I didn't know any better. The class still went ok, and I had plenty of students do well, but I knew that I could have done better with it.

2.3 Summer 2017 - Math 244

2.3.1 Class Overview

Math 244 is one of Rutgers' two classes on Differential Equations. This is a four-credit version of the class that mostly consists of students from the School of Engineering. For some of them, it is the last math class they need to take, while the rest of the students will go on to take Math 421. The class covers the basics of differential equations: Solving first-order equations, second and higher order equations, linear systems, and analysis of non-linear systems. As the course is meant for engineers, it tends to focus on applications, showing the engineers how the material in this class will be used in their future courses and jobs. It also includes an introduction to linear algebra, since that is not a prerequisite for this course and is required in order to analyze the linear systems that they will see. Since they will see more linear algebra in 421, this is a very brief introduction, only covering the material needed to do the problems from this class.

As a four credit class, the course lasted for 8 weeks, meeting 4 days a week for 2 hours. My version of the class covered the same material as during the semester, but my course looked substantially different because I flipped my class, recording video lectures and posting them for the students to watch before they came to class. In class, they would then work on problems in groups, discussing the material with their classmates and me, and then class would end with either in-class presentations or a quiz. While watching the videos and reading the sections in the book, students would have a worksheet to fill out that they would bring to class before starting on the group problems, but these were eventually replaced with quizzes at the start of class due to poor attendance at the start of class. The idea with this class format was to focus the in-class time on doing problems, which, in my opinion, is the most important part of Differential Equations, and how a student will best learn the material. I also developed two new projects for this class since I felt that projects (and more involved applications of the material in general) were important for engineers to see while they were in my class. All of this went into how I decided to run my class over the summer.

2.3.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix A.3. Included in this section are

1. The course syllabus
2. The Fluid Flow project that I created
3. The Bifurcation project that I made for the class
4. A problem set that was done in the library to give them practice with numerical methods

5. A fairly standard problem set to be worked on in class
6. A sample worksheet that the students would do while watching the videos
7. The worksheet quiz that corresponds to the previous worksheet that would be completed when the students arrived to class

The first three documents here after the syllabus are the things I am most proud of from this course. These are three project type activities that I designed in addition to the content needed to flip the course. The first two of these were designed to be completed by the students individually over the course of a week. The first, on Fluid Flow, was meant to introduce another application of differential equations, modeling of fluid flow, to the class, and give them an opportunity to play with it more. The second, the Bifurcation project, was meant to let them explore the idea of bifurcations, which are not generally covered in this class, through numerical experiments. The last of these assignments was supposed to let them practice numerical methods in a computer lab, where they could actually code the methods, but didn't work nearly as well as I would have hoped.

The last two documents show how the videos were integrated into the class, with the worksheet that the students would need to complete while watching the videos and the corresponding quiz that was given at the start of the next class. In addition, all of the vidoes that I created for this course are also on YouTube. The channel can be found here:
`https://www.youtube.com/channel/UCWmjNk4wyUW98C-SrctULQQ`.

2.3.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix B, at the end of this document. A few selected comments from these surveys are

- "The teaching style of this class was great for actually learning the material and understanding it."
- "Very helpful in class, encouraged questions, and was every enthusiastic about this course in class."
- "He has reinvigorated my passion for the subject"
- "I like the structure of the class. It is very different than the other calc classes I have taken at Rutgers. The way this class is set up encourages learning in a great new way. All math classes at Rutgers should be taught in this way. Active collaborative learning is the best!"
- "I believe that although there are online videos available, a brief lecture in class is still necessary."
- "Do an example before class to make sure students understand the videos"

2.3.4 Class Reflection

This class was a fantastic experience. It was a lot of work, even more than I had expected, but as my first foray into the world of Active Learning, I think it went very well. On they whole, the class seemed to buy in to what I was trying to do and participated in the activities and group work in class. I got several comments, both on the end of semester surveys and during the class itself that students enjoyed the videos and like the way the class was run. I feel like the students learned a lot in this class, and I gained a lot more experience in running an active classroom. I also learned what needs to go into preparing and recording video lectures, which, as the way things are heading with technology in the classroom, will likely be very useful in the future. Recording lectures caused me to think a lot more about what I was saying or writing and why I was doing it. Trying to keep videos under 10 minutes when recording lectures for a differential equations class, which has long computations involved, is difficult, and it made me realize what was really important in the lecture and what I could cut out. It also encouraged extra thought into the structure of the lecture, because even though the lectures ended up being 3 or so videos per section, they still needed to be distinct videos, containing some form of complete information on a topic. I know this practice of planning lectures carefully has already come into play with more recent classes I have taught, and it will continue to do so.

The semester was not without its issues, however. At the beginning of the semester, there was a student who contacted the Summer Section coordinator to tell him that I was 'not teaching' the class, because I was having the students watch videos outside of class and in class had them working on problems. The coordinator responded and took care of the situation and I never heard anything else after that. I also had an issue of students not arriving to class on time in the morning. My assumption to this end was that, since I was not lecturing in class, the students who were showing up late didn't feel like they were missing out on anything by showing up late. It got to the point where I needed to change something, because students were showing up late enough to throw off the class. This motivated me to change the worksheets that students would do outside of class while watching the videos to worksheet quizzes that the groups would have to do at the start of class. These quizzes were graded as a group (resulting in less grading for me) and needed to be turned in within the first 15 minutes of class, forcing students to be on time. I could have also moved some of the actual quizzes to the start of class, which is something I decided would happen in my future classes, not only for making sure students arrive on time, but also give them time to practice the material outside of class before the quiz. I had a few students make comments to that end during the class, and I feel like that's something that could be implemented fairly easily. I did not want to do this initially because my class started at 8 AM, but I could (and probably should) have done it anyway.

In addition with getting practice with a more active classroom, this summer also afforded me the opportunity to start working towards designing my own assignments and classes that were not quite identical to the ones that are run during the semester. The Fluid Flow project was a little bit of a struggle for the students to get through, but I think it went well on the whole. I'm fairly certain the students did not enjoy having to present problems in front of the class, but I think most of them gained something out of the experience. Overall, the combination of designing my own assignments as well as running a flipped classroom is an experience that will pay dividends as I continue to improve as an instructor in years to come.

2.4 Summer 2018 - Math 252

2.4.1 Class Overview

Math 252 is Rutgers' other version of Differential Equations. This one is more directly aimed at math majors (or non-Engineering majors) and requires Linear Algebra as a prerequisite. This allows the class to cover the same amount of material as Math 244, while only being a 3 credit class. This works out fairly well during the school year, but as a summer class, it means everything is even more compressed than it was in Math 244. In addition, this class ran 3 times a week, for 2 hours and 45 minutes a session, which is a significantly different schedule than what I had taught for summer courses in the past. In addition to the accelerated rate of covering material, this assignment also brought with it the challenges of how to handle a class period that was twice as long as a normal lecture during the semester, while still covering the material that would be discussed over a full week of classes.

In order to do this, I ran this course as a very active classroom. The goal was to minimize the amount of time I was lecturing to the students and maximize the amount of time they would spend working on problems during class. They were free to work in groups and discuss these problems, and I hoped that this format would allow them to get more comfortable talking about math, as well as help them understand the material better. To facilitate spending time on problems in class, I needed to move some of the introduction of material to outside of class. This was done via assigning sections of the textbook to read, which was tested via open-book Readiness Assessments at the start of class. In addition, to cut down on the amount of grading I had to do for the class (and prevent the academic integrity issues I saw the previous summer), no homework was collected. Instead, problems were recommended, and these were assessed via Mini-Quizzes that happened at the start of every class that didn't contain a larger assessment. Examples of these will be presented on the following pages.

2.4.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix A.4. Included in this section are

1. The course syllabus
2. A sample Mini-Quiz with its solution
3. A sample practice problem with its solution
4. A sample Readiness Assessment with its solution
5. A fairly standard example of a problem set that would be worked on in class
6. Another problem set that had more of an activity component to it

7. The set of review problems that made up the Gallery Walk activity on the first day of class
8. Assignment sheet for the bifurcation jigsaw activity
9. Assignment sheet for the oscillators workshop activity
10. Assignment packet for the SIR modeling activity
11. Assignment sheet for the end-of-semester student generated test questions

After the syllabus, the next four documents outline the general procedure of each class: a mini-quiz, followed by a readiness assessment, the problem set, and then a practice problem. The rest of the materials beyond that are different projects and activities that I developed to add more to the active components of the course. The next problem set was given out as a standard problem set, but the second half of it has more of an activity component, where the students worked on a problem in groups and wrote their final answer on the chalkboards around the room so that everyone could see it. The next set of problems was put around the room on large poster paper on the first day of class, and everyone did one of the problems to help review for the prerequisite midterm that they had on the second day of class.

The next three documents are three of the new activities that I designed for this class. The first is a jigsaw activity based around bifurcations, where each group of students would work one a different bifurcation problem, and then the groups were mixed so that each new group had someone from each old group, and they all discussed the problems they had previously solved. The next activity was an exploration of harmonic oscillators, with the goal of them getting to dive deeper into these calculations on their own to get a feel for how it works. The last one, on SIR models, gave them a bunch of examples of models similar to SIR, and this was provided with MATLAB code that ran these models, so the students could play with the models and see how they work. The final document included in this section is an assignment for student generated test questions, which is something new that I tried for this class, and then gave these questions back to the class as a review sheet.

2.4.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Chapter B, at the end of this document. A few selected comments from these surveys are

- "I really enjoyed the active learning activities that we did. Each of the practice problems and mini quizzes helped reinforce what we learned and helped me have a better sense of what problems I understood and which I needed to review again."

- "I like how the professor would engage the students and would always be receptive to questions. I also liked how I never was afraid to ask questions or felt stupid for asking."
- "Maybe shift the class towards being slightly more lecture based but I think the course worked well as is."
- "I was not really pleased with the readiness assignments he gave us because they were unnecessary."
- "So many ways. I want to be a high school math teacher so every class I take I'm always learning classroom management and teaching techniques from the teacher even if it isn't an education class. This class deepened my love of calculus and my want to be a teacher. The professor is inspired, engaging and implemented many different learning techniques."

2.4.4 Class Reflection

Overall, this class was a success and a great conclusion to my run of teaching summer classes at Rutgers. The structure of the course caused me to think a lot about how I planned my lessons and what was really important to discuss in class. I also had the pleasure of meeting with an instructional designer to talk about how to plan out a class that meets for almost three hours at a time, and how to structure it in a way that allows for active learning in the classroom. This led me to figure out what the main components of the course were and design my lessons and weekly plans around getting these topics across to the students. I feel like this could have been done a little bit better, but it was a good start to the process, and a good exercise that I will continue to do in future classes. I felt that by the end of the class, I could have used some extra time, and a little extra foresight would have helped me speed things up at the start of the course to get this extra time at the end. I also noticed that, when structuring the class to involve short lectures and in-class problem solving, the material covered in the lectures gets reduced to only the essential elements, which means there isn't time to talk about more of the side material or personal experiences. There were several moments in the course where I felt like I could have spent quite a bit of time talking about related ideas, but just did not have the time with this format. It was an interesting dilemma, and something that I think requires more consideration in the future.

This was also the first class where I attempted the Readiness Assessments and Mini-Quiz approach. I think, on the whole, the Mini-Quizzes were successful. They did eat up a little bit of class time, but I feel like they did the job of making sure the students had, at least to some degree, understood the material from the previous class before we moved on to new material. It made for extra preparatory work on my part to get these problems ready, but slightly reduced the grading, as I only needed to look at one problem for each student instead of a full homework set. The Readiness Assessments, on the other hand, were less successful. The goal of these was to make sure that students had read the sections in the textbook before coming to class, so I only needed to give a brief lecture on the material before they would be ready to work on problems. After the first one didn't go so well, I decided to make the rest of the open book, open note, but timed, so that if you hadn't read the sections in advance, you likely would not be able to complete the assessment in time. Grades on these stayed relatively low throughout the semester, and I have a few ideas as to why. The first is the possibility that the class was just not reading the textbook before class, and besides making these worth more, there's not too much to be done about that. Another possibility is that they didn't understand what I was expecting them to get out of the reading. I had hoped that this would sort itself out as the class went on, but I'm not sure if it did. This could be remedied by providing notes on the section(s) in addition to the textbook, or switching the in-class assignments to worksheets like I had the previous summer. The last option is that I was writing the Assessments too hard or too complicated that the students could not complete them before class. I feel like this may have been the case for a few of the assignments, so making them simpler may have been more beneficial to the students. The worksheets might be the way to go here; while they would give me less control over the environment in which the students were completing the assignment, it would make it more likely that they could find the answers and get things sorted out before coming to class. In addition, the student feedback showed that they were also not thrilled about the readiness assessments, which I think was a combination of the fact that they did not go well overall and that students were unsure how to do well on them. Both of these could be improved in the future by making my expectations more clear and writing better assignments.

Another addition that I made to this class was a prerequisite midterm on the second day of class. Since this course requires linear algebra as a prerequisite, I wanted to make sure the students were aware of this and that I was expecting them to be able to do linear algebra when we came to that point of the course. I think the prospect of having an exam on the second day of class scared the students a bit, but it did make sure they were ready for the class. Scores on this exam were high (as they should have been) and I believe the exam got my expectations across to them more clearly than anything I could have said on the first day of class. I also introduced Exam Rewrites to this course, mostly copied from another colleague at Rutgers. With this, students were able to rewrite any problem from either of the midterms (the prerequisite exam or the actual midterm) that they did not get full credit on for a chance to earn back half of the points they missed. In order to do so, they needed to write up a completely correct solution to the problem, but could use their textbook, classmates, or me to make sure that this happened. For the prerequisite midterm, this went very well, but there were a few problems on the second midterm that several students did not manage to write correctly. While these were difficult to grade, I feel like giving the students the opportunity to review their mistakes (and encouraging them to do so with points) was worthwhile.

In addition to the standard 'active learning' problem solving sessions, I also created activities to take place during the Wednesday classes after quizzes.

These included more group-centered activities like jigsaws or group presentations. The goal with these was also to encourage discussion and help the students to become more confident in discussing math with each other. I feel like the activities did a fairly good job of this, but they could have been better designed to accomplish more at the same time. For instance, I did not anticipate how difficult it would be to write a jigsaw activity that would allow all of the groups to finish at approximately the same time. The other activities were similarly successful, but had their issues with bringing the class back together as a whole after working on individual problems in groups. This is something that I feel like I will get better at personally with more practice.

I feel like a lot of these experiences are things that I will take with me as I move on to teach more classes in the future. I think the active components of the class went well, and will definitely keep trying to implement them in my future classes. It was also a good experience to see that not everything works the first time I try it out. While the Readiness Assessments are something that I took from other instructors, they had not really been used in math classes before. Conceptually, they are a good idea, but the implementation may not have been the best, and this takes trial and error. The same goes for any new component I try to add to a class; the way I try to include it may not work perfectly well the first time, and that is ok. It is still worth it to try them out and see what kind of feedback I get about it, so that it can be improved the next time around. The end results of this class also taught me about sticking to the numbers in terms of assigning final grades. Even though students do well and are active participants in class doesn't necessarily mean that they deserve an A in the class. They have to put in the work for it and do well enough on the exams. I had several students this summer that were active participants and were involved throughout the entire class, but ended up with a B based on exam and other scores. I felt like I wanted to give them a higher grade, because they were good participants, but the numbers said otherwise, and so I had to stick with those grades. All in all, I think this class taught me a lot about myself as well as how stepping out of the lecture role doesn't necessarily mean the students learn less in the class. They may actually learn more this way, and I definitely plan to take that with me in the future.

2.5 Fall 2018 - Math 104

2.5.1 Class Overview

Math 104 is an introduction to probability class directed at students in the humanities and social sciences. There are a few majors, including Human Resource Management, that require this class, but for most other students, they are in this class because they are interested in probability and it meets the university's quantitative requirements. It is possibly the hardest class, outside of the calculus sequence, that meets these requirements, so it poses an interesting set of challenges to teach. The course is basically an introduction to discrete probability, counting problems, conditional probability, and the normal distribution and Z-scores.

This was a very interesting course to teach because it is at a much lower level than anything I had taught up to this point. Everything else I had taught involved calculus, and so the students involved are at a level of mathematical maturity that has gotten them to that point. For this class, however, there are many students who have not taken a math class in several years and think about problems significantly differently than those who are taking calculus. It was also a good experience teaching this class as two of the other instructors in the department who had taught this class many times before provided me with most of their in-class materials. Preparing for this class then consisted of looking over the textbook and both sets of notes from the two other instructors and trying to combine them into something that I enjoyed and that worked with me as an instructor.

This class was also the first time I was teaching a class that met on a standard two days a week for 80 minutes schedule, which I had to adjust to. After the first few weeks, I settled in to how much material I could actually cover in an 80 minute class and determined the best way to bring the practices I had developed from my summer classes into this shorter class. I decided to bring back my practice problems from before, but these would take place at the end of class instead of the middle. The beginning of class would consist of a short homework quiz on the set that had been turned in previously (and which the students already had the answers to), followed by an interactive lecture, with the class wrapping up with a practice problem. I think this format worked well for this class in terms of keeping people interested in the material and showing them how to do problems before they have to do them for homework.

2.5.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix A.5. Included in this section are

1. The course syllabus

2. A worksheet I designed about counting problems, which was built from previous work with Pascal's Triangle

3. A worksheet I designed introducing the idea of continuous distributions and the normal distribution
4. A sample practice problem that students were given at the end of class
5. A sample "small quiz" that was given to students the class after turning in a homework set
6. A sample "big quiz" that was given to cover material from about two weeks worth of lectures
7. A sample homework solution that was given to the grader

2.5.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Chapter B, at the end of this document.

2.5.4 Class Reflection

I feel like teaching this class was a great experience for me on several levels. First of all, teaching a class that is separate from the Calculus sequence was a very interesting challenge, especially near the end of the class, where the topics covered in class were things that I had learned using Calculus. I had to think about different ways to present the material in order to avoid using both the methods and terminology from Calculus when discussing things like normal distributions. For instance, the concept of "area under a curve" isn't really something that they have seen before, so discussing continuous probability distributions (and, in particular, the normal distribution) becomes a much trickier prospect. After a decent amount of planning, I believe I was able to cover the topics in a way that was accessible to the class, but also covered them in a way so that if they were to see them again later, they would be able to adapt their knowledge to the new setting. This class taught me a lot about how to present things to students and attacking problems in ways that are different from the way that I would normally approach them in order to accommodate the different prior knowledge and skills of my students.

Secondly, this is the first time that I taught a class in 80 minute blocks, with another class immediately before and after mine. In the previous classes I had taught, my room was either empty before or after my class, so I didn't have to be too worried about when I got there or exactly when I ended. In addition, my summer classes were at least 2 hours long, so there was plenty of time to do a variety of activities, and I had a feel for when the class would be getting close to done. With the 80 minute blocks, I did not have this intuition, and so had to make sure to keep an eye on the clock to make sure I did not go over time. I also had to recalibrate how much material could be covered in a class, because it was substantially less than during a two-hour class over the summer. It took some getting used to, but I got better at this during the semester, and it's something that I know I will continue to get better at as I teach more classes under these kind of time constraints.

Finally, this is the first time I taught a class during the semester, where the class runs over a 14 week period instead of 6 or 8 weeks over the summer. There is a very different type of planning that needs to happen for a semester long class instead of a summer class, particularly in how time needs to be budgeted. Over the summer, I tended to spend almost all of my time working on class preparation and grading during the short time the class was happening, and then would do my other work over the rest of the summer. That is not possible for a semester class, so I needed to learn how to ration my time and not spend all of it on teaching, because I had other things that needed to get done while the class was running. It took some getting used to, but I figured out a way to organize my time to allow me to both prepare for the class as much as I felt I needed to, as well as get the other work done that needed to happen. I know that in future jobs, I will likely be teaching multiple classes at the same time, and being able to organize my time will be a very important part of staying on top of everything that I need to get done. Outside of the experience with the actual class material, this practice with organizing my time was one of the most important skills I learned this semester.

Appendix A

Sample Course Materials

A.1 Summer 2015 - Math 251

This section contains the following document from my Summer section of Math 251:

1. The course syllabus
2. A sample practice problem with its solution
3. A sample quiz with its solution
4. A review sheet handed out before the final exam

MATH 251, Section C1: Summer 2015

Syllabus

Instructor – Matt Charnley

Email: charnley@math.rutgers.edu

Office: Hill 606, Busch Campus

Office Hours: Monday and Tuesday, 10 am – noon, or by appointment.

Note: Office Hours on June 1 will be moved to June 4.

Course Website: SAKAI and math.rutgers.edu/~mpc163/Courses/SM15_MATH251.html

Class Meetings

MTWTh, 8:00am – 10:00 am, SEC-218

Exams

First Midterm Exam – Thursday, June 11th

Second Midterm Exam – Thursday, July 2nd

Final Exam – Thursday, July 16th

Course Information

Information about this course can be found on SAKAI, as well as the course website. SAKAI will be updated very regularly, and this will be the best place to obtain course materials. My personal website may lag behind in updates.

Grade Breakdown

Grades will be assigned according to the following weights

Homework & Practice Problems	10%
Maple Assignments	5%
Quizzes	15%
Midterms	17.5% each, 35% total
Final Exam	35%

Textbook

Jon Rogawski; *Calculus: Early Transcendentals.*
Custom Edition for Rutgers University; 2011; ISBN: 1-4641-0376-3

Class Attendance

Attendance at ALL class meetings is mandatory. Attendance will be taken at each class, and will factor into final grade assignments in the form of the practice problems and quizzes. This class will move very quickly through material, even more so than in a normal Spring or Fall semester, and missing one day of class will put you significantly far behind. If you must miss class for any reason, you should let me know well in advance, and plan to stop by office hours to catch up on missed material.

Quizzes

Thirty minute quizzes will be given every Thursday at the end of class, covering the material from that week of class. These will be closed book, closed note quizzes. Calculators and other electronic devices will NOT be allowed on any of the quizzes. You will be provided with any formulas you may need. The goal of these quizzes is to give you more practice working problems in an environment similar to that of an exam. There will be 5 of these quizzes over the course of the class. There will also be smaller 'practice problems' that will need to be written up during most class sessions. These will be handed out in the middle of the lecture, and will be open-book, open-note exercises.

Homework

Homework will be collected almost every day at the beginning of class. If the homework is not turned in to me by 8:10, it is late, and will receive at most half credit. If I do not have it by 9:00, it will not be accepted. In order to get credit, you must show all your work. The problem sets due each day will be assigned in class the day before. A tentative schedule is outlined below, but the final schedule will be announced in class, since it may change due to the rate at which we cover material. The homework sets are all assigned at the back of this packet, and are taken from the normal online syllabus:

http://www.math.rutgers.edu/courses/251/syllabushw2nd.html

I would highly recommend doing all these problems, because it will give you more practice with the material and a better understanding of it. However, since it is a shorter class over the summer, not all of these problems will be assigned or graded.

Problem Sessions

Almost every week, we will have two problems sessions for the second half of the class period. This will be time for you to ask about any homework problems that have been causing difficulty or go back over topics that are confusing. I will not prepare anything specific to talk about in these sessions, but will leave the entire time for you to ask questions. This is to fill the role of recitations during the Spring and Fall semesters. If there are no questions from the class, then I will just continue lecturing on the next section.

Maple Assignments

Maple is a computing software commonly used to carry out complicated calculations or visualize problems in 3 dimensions. This course requires 3 Maple assignments in addition to an optional introductory one. The first Wednesday of class, we will have a brief introduction to Maple and how to use it. After that, there will be three labs due over the summer. Those dates are already available below, and all of the necessary files will be posted on Sakai. These labs will need to be given to me, as a hard copy, on the specified due date. Emailed labs will not be accepted.

Exams

There will be two midterm exams given on the dates above. Calculators and other electronic devices will NOT be allowed on any of the exams. These will be 80 minute exams, which will start at 8:00 each day. These will be closed book, closed notes, with a formula sheet provided by me. After the 80 minutes, we will go over the exam in class. The final exam will be **three hours** on July 16th, with the same procedure. The exact time of the final exam will be specified at a later date, but it will run either 7:00 – 10:00 or 8:00 – 11:00 am.

Make-Up Policies

Exams and quizzes can NOT be made up under any circumstances. If there is a legitimate excuse for missing an exam, i.e. a doctor's note, then we can discuss a way to make the grades work out in the end, but you will not be allowed to take the exam late. Homework and Maple assignments will not be accepted past their due date.

Academic Integrity

All students in this course are expected to be familiar with and abide by the academic integrity policy (http://academicintegrity.rutgers.edu/academic-integrity-at-rutgers). Violations of this policy are taken very seriously. In short, don't cheat, and don't plagiarize.

Disability Accommodations

I will be happy to provide appropriate accommodations for students who provide me with a letter of accommodation from the Office of Disability Services (ODS). For more information, see http://ods.rutgers.edu/.

Class Schedule

Date	Sections	Assignments Due
5/26	12.1, 12.2, 12.3	
5/27	12.4, Intro to Maple	HW 1 Due
5/28	12.5, QUIZ 1	HW 2 Due
6/1	13.1, Problem Session	HW 3 Due
6/2	13.2, 13.3	HW 4 Due
6/3	13.4, Problem Session	HW 5 Due
6/4	14.1, QUIZ 2	HW 6 Due
6/8	14.2, Problem Session	HW 7 Due
6/9	14.3, 14.4	HW 8 Due
6/10	Catch Up / Problem Session	HW 9 Due
6/11	EXAM 1, Go over Exam	Maple Lab 1 Due
6/15	14.5, 14.6	
6/16	14.7	
6/17	14.8, Problem Session	HW 10 Due
6/18	15.1, QUIZ 3	HW 11 Due
6/22	15.2, Problem Session	HW 12 Due
6/23	15.3	HW 13 Due
6/24	12.7, Problem Session	HW 14 Due
6/25	15.4, QUIZ 4	HW 15 Due, Maple Lab 2 Due
6/29	15.4, Problem Session	HW 16 Due
6/30	15.6	HW 17 Due
7/1	16.1, 16.2	HW 18 Due
7/2	16.3, 16.4	HW 19 Due
7/6	Catch Up, Problem Session	HW 20 Due
7/7	EXAM 2, Go over Exam	HW 21 Due
7/8	16.5, Problem Session	
7/9	17.1, QUIZ 5	Maple Lab 3 Due
7/13	17.2, Problem Session	HW 22 Due
7/14	17.3	HW 23 Due
7/15	Catch up, Review	HW 24 Due
7/16	FINAL EXAM	HW 25 Due

Homework Sets

Sections	Problems	Tentative HW Set Number
12.1	5, 11, 15, 21, 39, 45	1
12.2	11, 13, 25, 33, 53	1
12.3	1, 21, 29, 66, 71	2
12.4	5, 11, 22, 39, 41	2
12.5	1, 13, 17, 33, 51	3
13.1	4, 7, 15, 21	4
13.2	10, 27, 28, 31, 45	5
13.3	3, 11, 14, 18	5
13.4	3, 17, 19, 23	6
14.1	7, 19, 20, 23	7
14.2	7, 15, 22, 29	8
14.3	3, 19, 22, 33, 49, 55	9
14.4	1, 7, 15, 21, 25	9
14.5	7, 13, 38, 39, 44	10
14.6	1, 5, 13, 27, 39	10
14.7	1, 3, 10, 19, 21, 35	11
14.8	2, 7, 11, 13, 15	12
15.1	17, 27, 29, 41, 45	13
15.2	3, 11, 27, 32, 33, 49	14
15.3	5, 9, 15, 19, 35	15
12.7	1, 5, 23, 35, 42, 53, 59	16
15.4	1, 5, 13, 17	17
15.4	22, 23, 27, 33, 41, 47, 49	18
15.6	1, 5, 15, 29, 33,	19
16.1	1, 3, 10, 15, 25, 31	20
16.2	3, 15, 21, 35, 43	20
16.3	5, 9, 11, 15, 21	21
16.4	1, 5, 8, 17, 23	21
16.5	1, 9, 12, 15, 17, 23	22
17.1	1, 6, 9, 13, 23, 25	23
17.2	1, 5, 9, 11, 21, 23	24
17.3	1, 5, 7, 9, 16, 17	25

MATH 251: Practice 10

June 8, 2015

Name: Solutions

1. Compute the gradient of

$$f(x, y, z) = e^{xy} + 2xz^2 + \sin(yz)$$

$$\nabla f = \langle ye^{xy} + 2z^2,\ xe^{xy} + z\cos(yz),\ 4xz + y\cos(yz)\rangle$$

2. Find the directional derivative of f in the direction $\langle 1, 1\rangle$ at $(-1, 2)$ for the function

$$f(x, y) = x^2 + 2y^2$$

$$\nabla f = \langle 2x, 4y\rangle \qquad \nabla f\big|_{(-1,2)} = \langle -2, 8\rangle.$$

$$\nabla f \cdot \vec{v} = -2\cdot 1 + 8\cdot 1 = 6.$$

$$D_{\vec{v}} f = \frac{1}{\|\vec{v}\|} \nabla f \cdot \vec{v} = \frac{1}{\sqrt{2}} \cdot 6 = \boxed{6/\sqrt{2}}$$

$$= \boxed{3\sqrt{2}}$$

MATH 251: Quiz 4

June 25, 2015

Name: Solutions

1. Integrate $f(x, y, z) = 2x + 3yz$ over the rectangular prism $0 \le x \le 2$, $1 \le y \le 5$, $0 \le z \le 1$.

/3

$$\int_0^2 \int_1^5 \int_0^1 2x + 3yz \, dz \, dy \, dx$$

$$= \int_0^2 \int_1^5 2xz + \frac{3}{2} yz^2 \Big|_0^1 \, dy \, dx$$

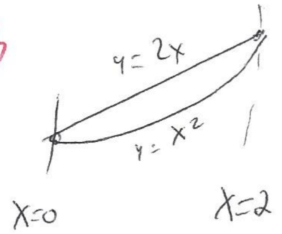
$$= \int_0^2 \int_1^5 2x + \frac{3}{2} y \, dy \, dx = \int_0^2 2xy + \frac{3}{4} y^2 \Big|_1^5 \, dx$$

$$= \int_0^2 8x + \underbrace{\frac{3}{4}(25-1)}_{18} \, dx = 4x^2 + 18x \Big|_0^2$$

$$= 16 + 36 = \boxed{52}$$

2. Integrate $f(x, y) = 2xy$ over the region between the graphs of $y = 2x$ and $y = x^2$.

/3



$$\int_0^2 \int_{x^2}^{2x} 2xy \, dy \, dx$$

$$= \int_0^2 xy^2 \Big|_{x^2}^{2x} \, dx$$

$$= \int_0^2 x(2x)^2 - x(x^2)^2 \, dx$$

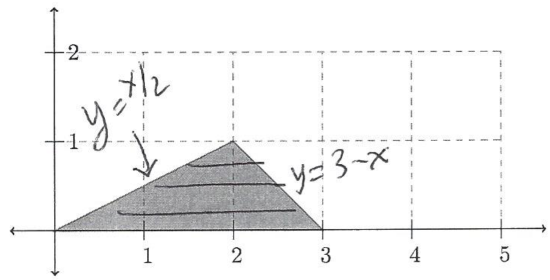
$$= \int_0^2 4x^3 - x^5 \, dx = x^4 - \frac{x^6}{6} \Big|_0^2$$

$$= 16 - \frac{64}{6} = 16 - \frac{32}{3}$$

$$= \boxed{16/3}$$

3. Integrate $f(x,y) = x + 2y$ over the triangle pictured below.

/4



$0 \le y \le 1$

$2y \le x \le 3-y$

$$\int_0^1 \int_{2y}^{3-y} x+2y \; dx \, dy$$

$$= \int_0^1 \frac{x^2}{2} + 2xy \Big|_{2y}^{3-y} dy$$

$$= \int_0^1 \frac{(3-y)^2}{2} + 2y(3-y) - \frac{(2y)^2}{2} - 2(2y)y \; dy$$

$$= \int_0^1 \frac{9}{2} - 3y + \frac{y^2}{\cancel{2}} + 6y - 2y^2 - 2y^2 - 4y^2 \, dy$$

$$= \int_0^1 \frac{9}{2} + 3y - \frac{15}{2}y^2 \, dy = \frac{9}{2}y + \frac{3}{2}y^2 - \frac{5}{2}y^3 \Big|_0^1$$

$$= \frac{9}{2} + \frac{3}{2} - \frac{5}{2} = \boxed{7/2}$$

4. Integrate $f(x, y, z) = x$ over the region in the first octant $[x \geq 0,\ y \geq 0,\ z \geq 0]$ bounded from above by the plane $x + 2y + z = 6$.

4

$0 \leq z \leq 6-x-2y$ $\qquad$ $6-x-2y=0$

$0 \leq x \leq 6-2y$ $\qquad$ $x=6-2y$

$0 \leq y \leq 3$ $\qquad$ $6-2y=0 \Rightarrow y=3$

$$\int_0^3 \int_0^{6-2y} \int_0^{6-x-2y} x \, dz\, dx\, dy$$

$18 \cdot 9 = 81 \cdot 2 = 162$

$27 \cdot 8 = 216$ $\qquad$ $36 \cdot 6 = 216$

$$= \int_0^3 \int_0^{6-2y} xz \Big|_0^{6-x-2y} dx\, dy$$

$\frac{6^4}{24} = \frac{\cancel{6} \cdot 6 \cdot 3 \cdot 3}{\cancel{4}} = 54$

$$= \int_0^3 \int_0^{6-2y} 6x - x^2 - 2xy \, dx\, dy$$

$$= \int_0^3 3x^2 - \frac{x^3}{3} - x^2 y \Big|_0^{6-2y} dy$$

$$= \int_0^3 3(6-2y)^2 - \frac{(6-2y)^3}{3} - y(6-2y)^2$$

$$= \int_0^3 3(6-2y)^2 - \frac{(6-2y)^3}{3} - y(36-24y+4y^2)\, dy$$

$$= \int_0^3 3(6-2y)^2 - \frac{(6-2y)^3}{3} - 36y + 24y^2 - 4y^3 \, dy$$

Using $u = 6-2y$, $du = -2y\, dy$

$$= \left[-\frac{(6-2y)^3}{2} + \frac{(6-2y)^4}{24} - 18y^2 + 8y^3 - y^4\right]\Big|_0^3 \qquad \boxed{27}$$

$$= 0 + 0 - 18 \cdot 9 + 8 \cdot 27 - 81 \ \overset{3}{+} \frac{6^3}{2} - \frac{6^4}{24} = -\cancel{162} + \cancel{216} - 81 + 108 - \cancel{54}$$

5. Convert $(x, y, z) = (0, 3, 4)$ to both cylindrical and spherical coordinates.

/3

Cylindrical: $(3, \pi/2, 4)$

Spherical: $(5, \pi/2, \cos^{-1}(4/5))$

6. Convert the following equations to spherical coordinates.

/3

(a) $z^2 = x^2 + y^2$.

(b) $z = x^2 + y^2$.

(c) $x^2 + y^2 + z^2 = 4$.

(a) $\cos^2\varphi = \sin^2\varphi \Rightarrow \boxed{\varphi = \pi/4.}$

(b) $\cos\varphi = \rho \sin^2\varphi \Rightarrow \boxed{\rho = \dfrac{\cos\varphi}{\sin^2\varphi}}$

(c) $\boxed{\rho = 2}$

Conversion Formulas

Cylindrical		Spherical	
$x = r\cos(\theta)$	$r = \sqrt{x^2+y^2}$	$x = \rho\cos(\theta)\sin(\phi)$	$\rho = \sqrt{x^2+y^2+z^2}$
$y = r\sin(\theta)$	$\tan(\theta) = \frac{y}{x}$	$y = \rho\sin(\theta)\sin(\phi)$	$\tan(\theta) = \frac{y}{x}$
$z = z$	$z = z$	$z = \rho\cos(\phi)$	$\cos(\phi) = \frac{z}{\rho}$

MATH 251 Final Exam Review

Matt Charnley

July 7, 2015

These are some sample problems for the final exam. Look at these problems, the ones on previous reviews, as well as ones in the book to prepare for the exam. There may be things on the exam that are not directly listed here. The problems may be more complicated than the ones here.

1. Draw the following vector fields.
   - $\langle x, y \rangle$.
   - $\langle -y, x \rangle$
   - $\langle x^2, 2y \rangle$.
2. Check if the vector field $\vec{F}$ is conservative.
   - $\vec{F} = \langle yz, xz, xy \rangle$.
   - $\vec{F} = \langle x^2, xy, 3z^3 \rangle$.
3. Compute a line integral (scalar and vector versions).

$$\int_{\mathcal{C}} \langle 2x, 3xy \rangle \cdot d\vec{s} \qquad c(t) = \langle \cos(t), \sin(t) \rangle \quad 0 \leq t \leq \pi$$

$$\int_{\mathcal{C}} x + y + z \; ds \qquad c(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle \quad 0 \leq t \leq 1$$

4. Use the Fundamental Theorem for Conservative Vector Fields to compute Line Integrals

$$\int_{\mathcal{C}} \langle 2x, 3y \rangle \cdot d\vec{s} \qquad c(t) = \langle \cos(t), \sin(t) \rangle \quad 0 \leq t \leq \pi$$

$$\int_{\mathcal{C}} \langle e^x + y, x + 2yz, y^2 + z^2 \rangle \; ds \qquad c(t) = \langle t, t^2, t^3 \rangle \quad 0 \leq t \leq 1$$

5. Find a potential function for $\vec{F} = \langle yz + 2x\sin(z), xz + 3y^2, xy + x^2\cos(z) \rangle$.
6. Find the equation for a normal vector for the parametrizations and surfaces.
   - $\mathcal{S}_1 = G(u, v) = (2u + 1, u + v, 3u + v)$, $\mathcal{D} = \{0 \leq u, v \leq 5\}$
   - $\mathcal{S}_2 = G(u, v) = (u\cos(v), u\sin(v), 1 - u^2)$, $\mathcal{D} = \{0 \leq u \leq 3, 0 \leq v \leq 2\pi\}$
   - $\mathcal{S}_3 = G(u, v) = (u, v, uv)$, $\mathcal{D} = \{0 \leq u, v \leq 5\}$.
7. Find the tangent plane to any of the above surfaces at a given value of u and v.
8. Compute the area of the above surfaces.
9. Compute scalar and vector surface integrals.
   - Integrate $x^2 + 3yz$ over $\mathcal{S}_1$.
   - Integrate $\langle xy, x^2, z \rangle$ over $\mathcal{S}_1$ with the upward normal.

- Integrate $\langle x, y, z\rangle$ over $\mathcal{S}_2$ with the upward normal.
- Integrate z over $\mathcal{S}_3$.

10. Use Green's Theorem to compute line integrals, area of curves, area integrals.

    - Integrate $\langle x^2 \sin(x) - x^2 y, ye^y + x^2 y\rangle$ over the triangle bounded by $x = 0$, $y = 2$ and $y = x$ oriented counterclockwise.
    - Find the area of the curve parametrized by $\langle \frac{3t}{1+t^3}, \frac{3t^2}{1+t^3}\rangle$, $0 \leq t \leq \infty$.
    - Find the integral of $\langle x^2(1-y), xy^2 + y\sin(y)\rangle$ over the top half of the circle $x^2 + y^2 = 1$, oriented counterclockwise.
    - Find the integral of $\langle x^2 \sin(x) + y^2, xy\rangle$ over the three sides of the square $[0, 1] \times [0, 1]$, going from $(0, 0)$ to $(1, 0)$ to $(1, 1)$ to $(0, 1)$.

11. Identify the boundary of:

    - $\mathcal{S}_1 = \{x^2 + y^2 + z^2 = 4,\ z \geq 0\}$.
    - $\mathcal{S}_2 = \{x^2 + z^2 = 9,\ 1 \leq y \leq 3\}$.
    - $\mathcal{S}_3 = \{x^2 + y^2 + z^2 = 16\}$.

12. Use Stokes' Theorem to compute line and surface integrals.

    - Integrate $curl(\langle -y + z, z^2 + 2x, xyz\rangle)$ over $\mathcal{S}_1$, upward normal.
    - Integrate $\langle x^2, y^2, z + 5\rangle$ over $\partial\mathcal{S}_2$, oriented correctly with $\mathcal{S}_2$ having the outward normal vector.

13. Identify the boundary of:

    - $\mathcal{W}_1 = \{x^2 + y^2 + z^2 \leq 4\}$
    - $\mathcal{W}_2 = \{x^2 + y^2 \leq 9,\ 1 \leq z \leq 4\}$
    - $\mathcal{W}_3 = \{x^2 + y^2 + z^2 \leq 1,\ z \geq 0\}$.

14. Use the Divergence Theorem to compute surface and volume integrals.

    - Integrate $\langle 2x, 3y, z\rangle$ over $\partial\mathcal{W}_1$ with the outward normal vector.
    - Integrate $\langle -3y, 2x^2, z^2\rangle$ over $\partial\mathcal{W}_2$ with the outward normal vector.
    - Integrate $\langle -3xy, 2x^2, z^2\rangle$ over $\partial\mathcal{W}_3$ with the outward normal vector.

Formulas

If we want to decompose $\vec{u}$ into $\vec{u} = \vec{u}_{//} + \vec{u}_{\perp}$ with respect to $\vec{v}$, then we have

$$\vec{u}_{//} = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right)\vec{v} \qquad\qquad \vec{u}_{\perp} = \vec{u} - \vec{u}_{//}.$$

Area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$: $||\vec{v} \times \vec{w}||$.

Volume of the parallelepiped spanned by $\vec{u}$, $\vec{v}$, and $\vec{w}$: $|\vec{u} \cdot (\vec{v} \times \vec{w})|$.

Curvature Formulas:

$$\kappa(t) = \left|\left|\frac{d\vec{T}}{ds}\right|\right| \qquad \kappa(t) = \frac{||\vec{r'}(t) \times \vec{r''}(t)||}{||\vec{r'}(t)||^3} \qquad \kappa(x) = \frac{|f''(x)|}{(1 + f'(x)^2)^{3/2}}$$

Linearization of a function at (a, b):

$$z = L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b).$$

Gradient of a Function $f(x, y, z)$:

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

Chain Rule for Paths: For a function $F(x, y, z)$ and a curve $\vec{c}(t)$,

$$\frac{d}{dt}F(\vec{c}(t)) = \nabla F(\vec{c}(t)) \cdot \vec{c'}(t)$$

Directional Derivative of the function f in the direction of $\vec{v}$:

$$D_{\vec{v}}f = \frac{1}{||\vec{v}||}\nabla f \cdot \vec{v}$$

General Chain Rule: For a function $f(x, y, z)$ with $x = x(s, t)$, $y = y(s, t)$ and $z = z(s, t)$, then

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$$

Implicit Differentiation: If $F(x, y, z) = 0$ and z can be written as a function of x and y, we have that

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Second Derivative Test: If (a, b) is a critical point of f, then for

$$D = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}(a, b)^2$$

- If $D > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a minimum.
- If $D > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a maximum.
- If $D < 0$, then (a, b) is a saddle point.
- If $D = 0$, the test is inconclusive.

Conversion Formulas

Cylindrical		Spherical	
$x = r\cos(\theta)$	$r = \sqrt{x^2+y^2}$	$x = \rho\cos(\theta)\sin(\phi)$	$\rho = \sqrt{x^2+y^2+z^2}$
$y = r\sin(\theta)$	$\tan(\theta) = \frac{y}{x}$	$y = \rho\sin(\theta)\sin(\phi)$	$\tan(\theta) = \frac{y}{x}$
$z = z$	$z = z$	$z = \rho\cos(\phi)$	$\cos(\phi) = \frac{z}{\rho}$
$dV = r\ dr\ d\theta\ dz$		$dV = \rho^2\sin(\phi)\ d\rho\ d\phi\ d\theta$	

Change of Variables Formula: If $G : \mathcal{D}_0 \to \mathcal{D}$ is a map given by $G(u, v) = (x(u, v), y(u, v))$, then

$$\iint_{\mathcal{D}} f(x,y)\ dx\ dy = \iint_{\mathcal{D}_0} f(x(u,v), y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right|\ du\ dv$$

where

$$\frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v}\end{pmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

Line Integrals: For a curve $\mathcal{C}$ given by $\vec{c}(t) = \langle x(t), y(t), z(t)\rangle$ for $a \le t \le b$, a scalar function $f(x, y, z)$ and a vector field $\vec{F}(x, y, z)$,

$$\int_{\mathcal{C}} f(x,y,z)\ ds = \int_a^b f(\vec{c}(t))||\vec{c}'(t)||\ dt$$

$$\int_{\mathcal{C}} \vec{F}(x,y,z)\cdot d\vec{s} = \int_a^b \vec{F}(\vec{c}(t))\cdot\vec{c}'(t)\ dt$$

Surface Integrals: For a surface $\mathcal{S}$ given by $G(u, v) = \langle x(u, v), y(u, v), z(u, v)\rangle$ for $(u, v) \in \mathcal{D}$, a scalar function $f(x, y, z)$ and a vector field $\vec{F}(x, y, z)$,

$$\iint_{\mathcal{S}} f(x,y,z)\ dS = \iint_{\mathcal{D}} f(G(u,v))||\vec{n}(u,v)||\ du\ dv$$

$$\iint_{\mathcal{S}} \vec{F}(x,y,z)\cdot d\vec{S} = \iint_{\mathcal{D}} \vec{F}(G(u,v))\cdot\vec{n}(u,v)\ du\ dv$$

Green's Theorem: For a domain $\mathcal{D}$ with $\partial\mathcal{D}$ positively oriented,

$$\oint_{\partial\mathcal{D}} \langle F_1, F_2\rangle\cdot d\vec{s} = \oint_{\partial\mathcal{D}} F_1\ dx + F_2\ dy = \iint_{\mathcal{D}} \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\ dA$$

$$\text{Area}(\mathcal{D}) = \frac{1}{2}\oint_{\partial\mathcal{D}} \langle -y, x\rangle\cdot d\vec{s} = \frac{1}{2}\oint_{\partial\mathcal{D}} x\ dy - y\ dx$$

Stokes' Theorem: For a surface $\mathcal{S}$ with $\partial\mathcal{S}$ positively oriented with respect to the normal $\vec{n}$ of $\mathcal{S}$,

$$\oint_{\partial\mathcal{S}} \vec{F}\cdot d\vec{s} = \iint_{\mathcal{S}} \text{curl}(\vec{F})\cdot d\vec{S}$$

$$\text{curl}(\vec{F}) = \nabla\times\vec{F} = \left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial y} - \frac{\partial F_1}{\partial x}\right\rangle$$

Divergence Theorem: For a volume $\mathcal{W}$ with boundary surface $\partial\mathcal{W}$ given the outward normal,

$$\iint_{\partial \mathcal{W}} \vec{F} \cdot d\vec{S} = \iiint_{\mathcal{W}} \operatorname{div}(\vec{F})\ dV$$

$$\operatorname{div}(\vec{F}) = \nabla \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Integral Formulas

$$\int \sin^2(x)\ dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C$$

$$\int \cos^2(x)\ dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C$$

A.2 Summer 2016 - Math 477

This section contains the following documents from my Summer section of Math 477:

1. The course syllabus
2. A Calculus 3 review that was given to the students early on in the class to make sure they were ready for the second half of the course
3. A sample homework solution that was posted for the students
4. A sample quiz with its solution
5. A topic list for the final exam that was given to the students
6. A set of review problems for the final exam

MATH 477, Section B1: Summer 2016

Syllabus

Instructor – Matt Charnley

Email: charnley@math.rutgers.edu

Office: Hill 606, Busch Campus

Office Hours: Monday and Wednesday, 10 am – noon in LSH 102B, or by appointment.

By appointment office hours will be held in Hill 606, Busch Campus.

Course Website: Sakai. Make sure you can access this as soon as possible.

Class Meetings

MTWTh, 8:00am – 9:55 am, TIL-103A

Exams

Midterm Exam – Thursday, June 16th

Final Exam – Thursday, July 7th

Course Information

Information about this course can be found on Sakai. Sakai will be updated very regularly, and this will be the best place to obtain course materials. Calculus 3, or multivariable calculus, (Math 251 at Rutgers) is a mandatory prerequisite for this class. You will need to know how to deal with multiple integrals to get through the course. Please see me with any concerns about this.

Grade Breakdown

Grades will be assigned according to the following weights

Homework / Practice Problems	15%
Quizzes	20%
Midterm	25%
Final Exam / Project	40%

Textbook

Sheldon Ross: *A First Course in Probability*, **9th edition (2012)**,
ISBN # 978-0321794772, Prentice-Hall.

Class Attendance

Attendance at ALL class meetings is mandatory. Attendance will be taken at each class, and will factor into final grade assignments in the form of the practice problems and quizzes. This class will move very quickly through material, even more so than in a normal Spring or Fall semester, and missing one day of class will put you significantly far behind. If you must miss class for any reason, you should let me know well in advance, and plan to stop by office hours to catch up on missed material.

Quizzes

Thirty minute quizzes will be given every Thursday at the end of class, covering the material from that week of class. These will be closed book, closed note quizzes. Calculators and other electronic devices will NOT be allowed on any of the quizzes. You will be provided with any formulas you may need. The goal of these quizzes is to give you more practice working problems in an environment similar to that of an exam. There will be 4 of these quizzes over the course of the class. There will also be smaller 'practice problems' that will need to be written up during most class sessions. These will be handed out at some point during the lecture, and will be open-book, open-note exercises.

Homework

Homework will be collected every day at the beginning of class. If the homework is not turned in to me by 8:10, it is late, and will receive at most half credit. If I do not have it by 9:00, it will not be accepted. In order to get credit, you must show all your work. The problem sets due each day will be assigned in class two days in advance, that is, homework assigned on Tuesday will be collected on Thursday, homework assigned on Wednesday will be collected on Monday, etc. Problem sets will be posted on Sakai, and any significant changes will be announced in class. A tentative schedule as to the sections that will be covered each day is outlined below, and the homework assignments will follow the same schedule, but are always subject to change based on the rate at which we cover material. This schedule is also fairly flexible, so pay attention in class or to Sakai for updates. For each assignment, there will be recommended homework problems and a smaller subset of them that will be collected and graded.

Homework assignments will consist of two types of problems: "Problems" and "Theoretical Exercises". The Problems are more computational, direct applications of the material in the sections, while the Theoretical Exercises require a little more thought to solve. When turning in the problems, these should be turned in separately; one packet with the Problems, and a second one with the Theoretical Exercises. There are also Self-Test exercises in the book, which are another set of practice exercises. The solutions to the Self-Text exercises are in the back on the book, so you can do these on your own to make sure you know how to solve the problems.

I would highly recommend doing the suggested problems in addition to the assigned ones, because it will give you more practice with the material and a better understanding of it. However, since it is a shorter class over the summer, not all of these problems will be assigned or graded. All of the homework sets are subject to change, and I will announce any changes at the end of each class and post them on Sakai.

Exams

There will be one midterm exam, currently scheduled for June 16th. Calculators and other electronic devices will NOT be allowed on either the midterm or the final. The midterm will be a 100 minute exam, starting at 8:10, and will cover chapters 1-4. It will be closed book, closed notes, with a formula sheet provided. The final exam will be **three hours** on July 7th, with the same procedure. The exact time of the final exam will be specified at a later date, but it will run either 7:00 – 10:00 or 8:00 – 11:00 am.

Make-Up Policies

Exams and quizzes can NOT be made up under any circumstances. If there is a legitimate excuse for missing an exam, i.e. a doctor's note, then we can discuss a way to make the grades work out in the end, but you will not be allowed to take the exam late. Homework assignments will not be accepted past their due date.

Academic Integrity

All students in this course are expected to be familiar with and abide by the academic integrity policy (http://academicintegrity.rutgers.edu/academic-integrity-at-rutgers). Violations of this policy are taken very seriously. In short, don't cheat, and don't plagiarize.

Disability Accommodations

I will be happy to provide appropriate accommodations for students who provide me with a letter of accommodation from the Office of Disability Services (ODS). For more information, see http://ods.rutgers.edu/.

Tentative Class Schedule

Date	Sections	Assignments Due
5/31	2.1 – 2.5	
6/1	1.1 – 1.6	
6/2	2.5, QUIZ 1	HW 1 Due
6/6	3.1 – 3.3	HW 2 Due
6/7	3.4 – 3.5	HW 3 Due
6/8	4.1 – 4.4	HW 4 Due
6/9	4.5 – 4.7, QUIZ 2	HW 5 Due
6/13	4.8 – 4.10	HW 6 Due
6/14	5.1 – 5.3	HW 7 Due
6/15	5.4, REVIEW	HW 8 Due
6/16	EXAM 1	
6/20	5.5 – 5.6	HW 9 Due
6/21	5.7, 6.1 – 6.2	
6/22	6.3	HW 10 Due
6/23	7.1 – 7.2, QUIZ 4	HW 11 Due
6/27	7.2 – 7.4	HW 12 Due
6/28	6.4 – 6.5	HW 13 Due
6/29	7.5, 7.7	HW 14 Due
6/30	8.1 – 8.2, QUIZ 5	HW 15 Due
7/5	8.3 – 8.4	HW 16 Due
7/6	REVIEW	
7/7	FINAL EXAM	HW 17 Due

Calculus Review

Matt Charnley

June 20, 2016

(a) Compute the following integrals.

(i)

$$\int_0^5 x^3 - 5x^2 + 3 \, dx$$

(ii)

$$\int_0^{10} e^{-3x} \, dx$$

(iii)

$$\int_0^{\infty} xe^{-10x} \, dx$$

(iv)

$$\int_0^{2\pi} x\sin(x) \, dx$$

(v)

$$\int_0^{\infty} xe^{-x^2} \, dx$$

(b) Change the order of integration in the following integrals.

(i)

$$\int_0^2 \int_0^x f(x,y) \, dy \, dx$$

(ii)

$$\int_0^4 \int_{x^2}^{16} f(x,y) \, dy \, dx$$

(iii)

$$\int_0^{10} \int_{-y}^{y} f(x,y) \, dx \, dy$$

(c) Set up the integral of a function $f(x,y)$ over the regions sketched out on the final page.

(d) Compute the following double integrals.

(i)

$$\int_0^2 \int_0^x e^{x^2} \; dy \; dx$$

(ii)

$$\int_0^4 \int_0^3 x^2 + 3xy^3 \; dx \; dy$$

(iii)

$$\int_0^\pi \int_0^3 r^2 \cos(\theta) \; rdr \; d\theta$$

(iv)

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x^2 + y^2 \; dy \; dx$$

MATH 477: Homework 10 Solutions

Matt Charnley

June 22, 2016

1.

(a) We compute the integral

$$c\int_{-1}^{1}(1-x^2)\ dx = c\left[x - \frac{x^3}{2}\right]_{-1}^{1} = c\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right) = \frac{4}{3}c$$

Since this must equal 1, we need to have $c = \frac{3}{4}$.

(b) For the cumulative distribution function, we know that it must be 0 if $x < -1$ and 1 if $x > 1$. For everything in between, we have

$$F(x) = \int_{-1}^{x}\frac{3}{4}(1-x^2)\ dx = \frac{3}{4}\left[(x-\frac{x^3}{3})+\frac{2}{3}\right]$$

Therefore, the cumulative distribution function is

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{3}{4}\left(x-\frac{x^3}{3}\right)+\frac{1}{2} & -1 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

.

2. First, we need to compute the constant C that makes this a probability density function. Thus, we need to compute integrals.

$$\begin{aligned} \frac{1}{C} &= \int_{0}^{\infty} xe^{-x/2}\ dx = -2xe^{-x/2}\ |_0^\infty + 2\int_0^\infty e^{-x/2} \\ &= 0 - 4e^{-x/2}\ |_0^\infty = 4 \end{aligned}$$

Therefore, we have $C = \frac{1}{4}$. Then, we can compute that

$$\begin{aligned} P(X \ge 5) &= \frac{1}{4}\int_5^\infty xe^{-x/2} = \frac{1}{4}\left[-2xe^{-x/2}\ |_5^\infty + 2\int_5^\infty e^{-x/2}\ dx\right] \\ &= \frac{1}{4}\left[10e^{-5/2} + 4e^{-5/2}\right] = \frac{14}{4}e^{-5/2} \end{aligned}$$

.

4.

(a) We compute this probability by integrating

$$P(X > 20) = \int_{20}^{\infty} \frac{10}{x^2}\,dx = -\frac{10}{x}\,|_{20}^{\infty} = 0 + \frac{10}{20} = \frac{1}{2}$$

(b) Changing the x to other numbers, we see that the cumulative distribution function is

$$F(x) = \begin{cases} 1 - \frac{10}{x} & x \geq 10 \\ 0 & x < 10 \end{cases}$$

(c) Using this, we see that $P(X \geq 15) = \frac{10}{15} = \frac{2}{3}$. Thus, if we assume that each of the 6 devices will last a certain amount of time independently, we have that, the probability of at least 3 of them functioning for 15 hours is

$$\binom{6}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 + \binom{6}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + \binom{6}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + \binom{6}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0$$

which simplifies to $\frac{650}{729}$.

.

6.

(a)

$$\begin{aligned} E[X] &= \frac{1}{4}\int_0^{\infty} x^2 e^{-x/2} = \frac{1}{4}\left[-2x^2 e^{-x/2}\,|_0^{\infty} + 2\int_0^{\infty} x e^{-x/2}\,dx\right] \\ &= \frac{1}{2}\int_0^{\infty} x e^{-x/2} = -x e^{-x/2}\,|_0^{\infty} + \int_0^{\infty} e^{-x/2}\,dx \\ &= 0 + -2e^{-x/2}\,|_0^{\infty} = 2 \end{aligned}$$

(b) From problem 1, we know that $c = \frac{3}{4}$. Then, we can calculate that

$$E[X] = \frac{3}{4}\int_{-1}^{1} x(1 - x^2)\,dx = \frac{3}{4}\int_{-1}^{1} x - x^3\,dx = \frac{3}{4}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{1} = 0$$

(c)

$$E[X] = \int_5^{\infty} \frac{5}{x}\,dx = 5\ln(x)\,|_5^{\infty} = \infty$$

.

7. From the information in the problem (the fact that we have a probability distribution and we know the expected value), we know that

$$1 = \int_0^1 a + bx^2 = ax + \frac{bx^3}{3}\,|_0^1 = a + \frac{b}{3}$$

$$\frac{3}{5} = \int_0^1 x(a+bx^2) = a\frac{x^2}{2} + b\frac{x^4}{4} \mid_0^1 = \frac{a}{2} + \frac{b}{4}$$

This becomes a system of equations of the form

$$3 = 3a + b \qquad\qquad \frac{12}{5}2a + b$$

Solving this, we see that $a = \frac{3}{5}$ and $b = \frac{6}{5}$.

.

8. The expected value is

$$E[X] = \int_0^\infty x^2 e^{-x}\ dx = -x^2 e^{-x} \mid_0^\infty + \int_0^\infty 2xe^{-x}\ dx = -2xe^{-x} \mid_0^\infty + 2\int_0^\infty e^{-x}\ dx = 2$$

MATH 477: Quiz 3

June 23, 2016

Name: Solutions

1. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} c\,(8 - x^3) & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the constant c that makes this a probability density function.

(b) Calculate $P(X \geq 1)$.

(c) Find the expected value and variance of this random variable.

(a) $$1 = \int_0^2 c(8-x^3)\,dx = c\left[8x - \frac{x^4}{4}\right]\Big|_0^2 = c[16-4] = 12c$$

So $\underline{c = \frac{1}{12}}$

(b) $$P(X \geq 1) = \frac{1}{12}\int_{1}^{2} (8-x^3)\,dx = \frac{1}{12}\left[8x - \frac{x^4}{4}\right]\Big|_1^2$$

$$= \frac{1}{12}\left[16-4-\left(8-\frac{1}{4}\right)\right] = \cancel{\frac{1}{12}\left[8+\frac{1}{4}\right]}$$

$$= \frac{1}{12}\left[4+\frac{1}{4}\right] = \underline{\frac{17}{48}}$$

(c)
$$E[X] = \frac{1}{12}\int_0^2 x(8-x^3)\,dx = \frac{1}{12}\left[4x^2 - \frac{x^5}{5}\right]\Big|_0^2 = \frac{1}{12}\left[16 - \frac{32}{5}\right]$$

$$= \frac{1}{12}\left[\frac{80-32}{5}\right] = \frac{1}{12}\left[\frac{48}{5}\right] = \underline{\frac{4}{5}}$$

$$E[X^2] = \frac{1}{12}\int_0^2 x^2(8-x^3)\,dx = \frac{1}{12}\left[\frac{8}{3}x^3 - \frac{x^6}{6}\right]\Big|_0^2$$

$$= \frac{1}{12}\left[\frac{64}{3} - \frac{64}{6}\right] = \frac{1}{12}\cdot\frac{64}{6} = \frac{16}{18} = \underline{\frac{8}{9}}$$

So $\boxed{\cancel{Var(X) = \left(\frac{8}{9}\right) - (\ldots)^2}}$ $\boxed{Var(X) = \frac{8}{9} - \left(\frac{4}{5}\right)^2}$

2. Assume that a person's height H is normally distributed with mean 68 and variance 9. Using the table at the back of the quiz, calculate

(a) $P(H > 72)$.

(b) $P(66 < H < 71)$.

(a) $P(H > 72) = P\left(\frac{H-68}{3} > \frac{72-68}{3}\right)$

$= P(Z > 4/3)$

$= 1 - P(Z \le 1.33)$

$= 1 - .9082 = \underline{.0918}$

(b) $P(66 < H < 71) = P\left(\frac{66-68}{3} < Z < \frac{71-68}{3}\right)$

$= P(-2/3 < Z < 1)$

$= P(Z < 1) - P(Z < -2/3)$

$= .8413 - .2514 = \underline{0.5889}$

3. Assume that the amount of time an individual spends taking a psychology survey is uniformly distributed between 10 and 30 minutes.

(a) What is the probability that I take more than 25 minutes to complete the survey?

(b) If 5 people go to take the survey, what is the probability that exactly 4 of them finish the survey *before* 25 minutes have passed?

Uniform on $(10, 30)$ $\Rightarrow f(x) = \frac{1}{20}$

(a) $P(X > 25) = 5/20 = 1/4$

(b) p (less than 25) $= 3/4$

$$P = \binom{5}{4}(3/4)^4(1/4)^1$$

4. I have a biased coin that shows up heads with probability .8. Let X count the number of heads when I flip the coin 625 times. Use the normal approximation to the binomial distribution to calculate the probability that I see strictly between 485 and 510 heads; i.e. $P(485 < X < 510)$.

$$np = 625 \cdot \frac{8}{10} = 625 \cdot \frac{4}{5} = 125 \cdot 4 = \underline{500}$$

$$np(1-p) = 500\,(.2) = \underline{100}$$

$$P(485 < X < 510) \sim P(485.5 < X < 509.5)$$

$$= P\left(\frac{485.5 - 500}{10} < Z < \frac{509.5 - 500}{10}\right)$$

$$= P(-1.45 < Z < .95)$$

$$= P(Z < .95) - P(Z < -1.45) = .8289 - .0735 = \boxed{.7554}$$

5. Assume that the length of a card game is exponentially distributed and has an average length of 20 minutes.

(a) Describe this distribution.

(b) What is the probability of a game lasting longer than 30 minutes?

(c) If the game has already lasted for 15 minutes, what is the probability that it lasts for another 40?

(a) This is an exponential random variable with

$\underline{\lambda = 1/20}$

(b) $$P(X>30) = \int_{30}^{\infty} \frac{1}{20} e^{-x/20}\, dx$$

$$= -e^{-x/20} \Big|_{30}^{\infty} = \boxed{e^{-3/2}}$$

(c) By the fact that the exponential distribution is memoryless, this is the same as

$$P(X>40) = -e^{-x/20} \Big|_{40}^{\infty} = \boxed{e^{-2}}$$

MATH 477: Final Exam Topic Review

Matt Charnley

June 29, 2016

Chapter 5: Continuous Random Variables

(a) Continuous Random Variables: What are they? How do we compute probabilities using these density functions? How do we check if something is a density function?

(b) Expectation and Variance of Continuous Random Variables. Expected Value of Functions of a random variable. Calculating these using integrals.

(c) Examples of Random Variables: Uniform, Normal, Exponential. Calculating probabilities and setting up word problems using these random variables.

(d) Normal approximation to the binomial distribution. How to set this up, using the continuity correction.

(e) Functions of a random variable. Using the cumulative distribution function to start the problem, then getting to the density function.

Chapter 6: Jointly Distributed Random Variables

(a) How do we talk about two random variables at the same time? What are joint distribution functions? How do we calculate probabilities when we have two random variables?

(b) What does it mean for two random variables to be independent? How does this help us compute probabilities?

(c) How do we add two random variables together? If we add two normal random variables, what is the result? What if we add two exponential random variables? What about Poisson or Binomial?

(d) How do we calculate discrete conditional distributions? What is the formula, and what does it mean?

(e) For the continuous case, what do conditional distributions look like? How can we calculate probabilities using them?

Chapter 7: Properties of Expectation

(a) Linearity of Expectation. What is it? How do we use it? How can we set up problems to use this to make things easier?

(b) Expectation of functions of two random variables. How do we do it, and what can we do with it?

(c) Covariance and Correlation. What is covariance? What does it mean, and how can we calculate it? There are two formulas here. How do we relate covariance to correlation? What are the properties of covariance that we can use to make it easier to calculate things? Some of the linearity stuff from earlier might show up here as well.

(d) Variance of a sum of random variables. Covariance shows up in the formula.

(e) Conditional Expectation. What is it? What does it mean? How can we use this to help with things?

(f) Computing Expectations using conditioning. What types of problems apply here? When is this easier than just computing the expected value outright? How can we just this to also calculate probabilities by conditioning, with Indicator random variables? When is this useful/what does this remind you of?

Chapter 8: Limit Theorems

(a) Markov and Chebyshev inequalities: When do they apply? How do you use them? Know the specific types of problems they show up in.

(b) Central Limit Theorem: Same Idea. Know how to identify a problem as a CLT problem, and then know how to get to the answer.

(c) Strong Law: It's cool, but not really any way to test it.

Overall Key Points

(a) Know the definitions of all of the terms and random variables so that you can use them in problems.

(b) Know how to break down a word problem to figure out what it is asking for, and what different techniques you need to apply from the course to solve the problem.

(c) Know how to set up the double integrals for evaluating probability appropriately, because once it is set up, the rest of the problem is just calculus.

(d) Think through problems carefully before you start to work on them. Sketch out a picture if you need to. Once you figure out what is going on and how it all works, putting together the proper probabilities becomes a lot easier.

(e) There are a lot of different ways and techniques to approach problems. Know how to figure out which technique you need, and which would be the best way to approach the problem.

MATH 477: Final Exam Review Problems

Matt Charnley

July 1, 2016

1. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} C(8 - x^2) & -2 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the constant C to make this a probability density function.

   (b) Calculate $P(X < 1)$, $P(|X| > 1)$.

   (c) Find $E[X]$ and $E[X^2]$.

2. Assume that the lifetime of a system component (in hours) is a random variable given by the density function

$$f(x) = \begin{cases} \frac{50}{x^2} & x \geq 50 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is the probability that a component lasts longer than 150 hours?

   (b) If we have 5 of these components, what is the probability that exactly 3 of them will need to be replaced within the first 100 hours?

3. Let X be the continuous random variable defined by the density function

$$f(x) = \begin{cases} \frac{x}{2} & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find $E[X]$.

   (b) Find $E[X^3]$.

   (c) Find $Var(X)$.

   (d) Find $E[e^X]$

4. There is a single cop car patrolling a stretch of road of length L, however, to be more efficient, he only actually moves back and forth between $L/4$ and $3L/4$. Assume that an incident happens at a point uniformly distributed along the full length of the road, and the cop's position is uniformly distributed in the range from $L/4$ to $3L/4$.

   (a) What is the probability density function of the position of the incident?

   (b) What is the average distance between the cop and the incident?

5. Assume that the length of a card game is approximately normally distributed with mean 30 minutes, and variance 9 minutes.

   (a) What is the probability of a game lasting longer than 35 minutes?

(b) What is the probability that a game lasts between 26 and 31 minutes?

6. Use the normal approximation to the binomial distribution to approximate the probability that, if I roll a 6 sided die 1000 times, that I see more than 200 5's.

7. Approximate the probability that, if I survey 400 people, I get at least 125 votes for a proposition, where 25% of the population approves of the proposition.

8. Assume that the length of a computer repair is an exponential random variable with parameter $\lambda = \frac{1}{10}$ minutes.

   (a) What is the average length of a repair?

   (b) What is the variance in the time a repair will last?

   (c) What is the probability that a repair lasts longer that 25 minutes?

   (d) If a repair has already lasted for 10 minutes, what is the probability that it will, in total, last for at least 25 minutes?

9. Let X be a continuous random variable with density function f and cumulative distribution function F. What is the density function of the random variable $Y = X^3$? What about $Y = e^X$?

10. Let X_1, X_2, Y_1, Y_2 be four random variables so that X_1 and Y_1 are jointly continuous with density function f_1, and X_2 and Y_2 are jointly continuous with density function f_2, as shown below

$$f_1(x,y) = \begin{cases} \frac{1}{64}xy & 0 < x < 4,\ 0 < y < 4 \\ 0 & \text{otherwise} \end{cases} \qquad f_2(x,y) = \begin{cases} \frac{1}{144}[x+3y] & 0 < x < 4,\ 0 < y < 4 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Looking at X_2 and Y_2, calculate $P(X > 2, Y < 1)$ and $P(Y > 2)$.

   (b) One of these pairs is independent. Which one is it and why?

   (c) Using the independent density function you chose in part a), compute the marginal density functions of X and Y, $f_X(x)$ and $f_Y(y)$.

   (d) Calculate $P(X < 1|Y = 2)$ and $P(Y > 3|X = 2)$.

11. I have a bucket containing 6 red, 5 white, and 3 blue balls, from which I am going to draw 3. Let X be the number of red balls drawn, and Y the number of blue.

   (a) Find the joint probability mass function of X and Y.

   (b) Find the marginal distribution of X and Y.

   (c) Find the conditional distribution of $X|Y$ for any y.

12. Let X and Y be two independent normal random variables so that

$$E[X] = 10 \qquad Var(X) = 9 \qquad\qquad E[Y] = 15 \qquad Var(Y) = 16$$

   (a) What is the probability that $X \leq 12$?

   (b) What is $P(X > 14, Y < 13)$?

(c) What is the distribution of $X + Y$?

(d) What is the probability that $X \geq Y$?

13. Let X and Y be jointly continuous random variables with joint probability density function

$$f(x,y) = \begin{cases} 6e^{-2x}e^{-3y} & x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal distributions $f_X(x)$, $f_Y(y)$.

(b) Calculate $E[XY]$ and $E[X + Y]$

14. Consider a standard 52 card deck of cards, which has 26 red cards and 26 black. Assume I shuffle the deck and lay out all the cards. What is the expected number color changes in the deck, i.e., the number of times we have a red card followed by a black, or a black card followed by a red. Hint: Linearity of expectation.

15. A building has $N + 1$ floors, and anyone who enters the elevator is equally likely to go to any of the N floors above the first, independently of all other people in the elevator. Assume that k people enter the elevator on the first floor, and let X denote the number of floors that the elevator will stop at on the way up. Find $E[X]$. Hint: Linearity of Expectation. Let X_i be 1 if the ith floor is selected, and 0 otherwise. It may be easier to think about the probability of a floor not being selected.

16. 5 people are playing a game. Each player has their own hand of 10 distinct cards, and each player has the exact same hand. The game is played by each player picking a card from their hand, and revealing them all at once. A player wins if no other cards on the table match the one that he picked. There can be multiple winners in a given game. What is the expected number of winners each time this game is played? Hint: Linearity of Expectation.

17. I have a bag of balloons, containing approximately 1/3 red, 1/2 blue, and 1/6 green balloons. I am going to take out ten balloons, and there are enough balloons in the bag that each draw can be assumed to be independent of all others. Let R denote the number of red balloons I get, and G the number of green.

(a) What is the covariance of R and G?

(b) What is the correlation between R and G? Does this make sense?

18. I am going to roll a pair of 6 sided dice 30 times. Let X denote the number of times I see a pair on the dice, and Y the number of times I see a sum of 2, 5, 7, or 12. What is the covariance of X and Y? Can you say anything else about the events X and Y?

19. Let X and Y be jointly continuous random variables defined by the density function

$$f(x,y) = \begin{cases} \frac{1}{2}\left[2x + 3y^2\right] & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal densities f_X and f_Y.

(b) Find $E[X]$, $E[Y]$ and $E[XY]$.

(c) Calculate $Cov(X,Y)$.

(d) Find the conditional density function $f_{X|Y}(x|y)$.

(e) Find $E_X[X|Y=y]$.

(f) Calculate $E_Y[E[X|Y=y]]$ and see that this is the same as $E[X]$.

20. You are playing a game with a pair of two 6-sided dice. You roll the dice. If a sum of 7 shows up, you win nothing and the game is over. If anything other than 7 shows up, you can either win that amount of money (the sum of the two dice), or you can roll again, following the exact same procedure. Your strategy for this game is to pick a number k, and stop as soon as you see a sum of at least k. What is your expected winnings, which will depend on k? Hint: You will want to condition on the initial roll of the dice. The result should look like the dumb miner problem.

21. Let X and Y be jointly continuous with density function

$$f(x,y) = \begin{cases} \frac{1}{y}e^{-y} & 0 < x < y,\ 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate $E[X]$, $E[Y]$ and $E[XY]$.

(b) Find $Cov(X,Y)$.

(c) Find $E[X|Y=y]$ and show that $E[X] = E[E[X|Y]]$.

(d) Calculate $E[X^3|Y=y]$.

22. A coin having probability p of landing on heads is flipped until both a heads and a tails has appeared.

(a) What is the expected number of flips?

(b) What is the probability that the last flip is heads?

23. Let X be a random variable that can only take on positive values and has expected value 10.

(a) What can be said about the probability that $X \geq 20$?

(b) What can be said about the probability that $X \geq 50$?

(c) What can be said about the probability that $X \geq 5$?

24. Let X be a random variable with mean 25 and variance 9.

(a) What can we say about the probability that $19 < X < 31$?

(b) What is the likelihood that I see a value for X less than 15 or bigger than 35?

25. Let X be a random variable with mean 50 and variance 16. I am going to sample X 100 times. Use the Central Limit Theorem to approximate the probability that the sum of all of these samples, $\sum_{i=1}^{100} X_i$ is between 490 and 520.

26. Let X be a normally distributed random variable with mean 10 and variance 4. I am going to sample from X 20 times, so that I get a sequence X_i from $i = 1$ to 25 of identical normally distributed random variables. I am interested in the probability that the sum is between 225 and 260.

  (a) Use Chebyshev's Inequality to approximate this probability.
  (b) Use the central limit theorem to approximate this probability.
  (c) Calculate this probability using the fact that the sum of normally distributed random variables is normal. What do you notice about these last two results?

A.3 Summer 2017 - Math 244

This section contains the following documents from my Summer section of Math 244:

1. The course syllabus
2. The Fluid Flow project that I created
3. The Bifurcation project that I made for the class
4. A problem set that was done in the library to give them practice with numerical methods
5. A fairly standard problem set to be worked on in class
6. A sample worksheet that the students would do while watching the videos
7. The worksheet quiz that corresponds to the previous worksheet that would be completed when the students arrived to class

MATH 244, Section C1 – Summer 2017
Syllabus

Instructor – Matt Charnley

Office: Hill 606, Busch Campus
Course Website - Canvas: canvas.rutgers.edu
Personal Website: math.rutgers.edu/~mpc163/Courses/SM17_MATH244.html

Class Meetings

MTWTh, 8:00 AM – 10:00 AM, Tillett 204, Livingston Campus

Office Hours

Monday and Wednesday – 10:00 AM – 12:00 noon, LSH 102C
By Appointment - Hill 606, Busch Campus

Exam Schedule

Midterm 1: Thursday, June 15 – In Class
Midterm 2: Thursday, July 6 – In Class
Final Exam: Thursday, July 20 – 8:00 AM – 11:00 AM

Course Information

The information for this course can be found on Canvas. Canvas is a newer Learning Management System that Rutgers is looking to implement in their classes. While I haven't used it for an actual class before, I feel like it's a lot better than Sakai, and I hope you'll like it too. The website is canvas.rutgers.edu. For the first week or so, I will also be posting links to everything on my personal website math.rutgers.edu/~mpc163/Courses/SM17_MATH244.html to allow everyone to keep up with the class if there are any issues with the Canvas site. After that, we will be exclusively using Canvas. If there are any issues, let me know as soon as possible.

Textbook

The textbook for this course is *Elementary Differential Equations*, **10th edition**. William Boyce and Richard DiPrima. ISBN: 978-0-470-45832-7.

Class Summary

This class is an introduction to Ordinary Differential Equations. In particular, this class is directed at engineering and physics students who will need knowledge of ordinary differential equations for future classes. To best do this, the class will focus on both the qualitative and quantitative aspects of differential equations, showing how both can be useful in different situations. There will be some instances where it may seem like the math has no connection to engineering or physics, but there will always be an attempt to emphasize the applications at every step. The topics covered in this class include:

- First Order Differential Equations
- Second and Higher Order Linear Differential Equations
- Systems of First Order Differential Equations
- Numerical Methods for Solving Differential Equations
- Non-Linear Differential Equations

Grade Breakdown

The final grades for this course will be calculated using the following distribution

Worksheets / Participation / Presentations	10%
Homework Writeups	5%
Quizzes (Including Syllabus Quiz)	10%
Projects and Maple Labs	15%
Midterms	28% (14% each)
Final Exam	32%

Note: No student will receive a final grade more than one mark higher than their average grade on the 3 exams, weighted in an appropriate manner. For instance, if your average exam grade is a C, you can receive no higher than a B for a final grade.

Class Structure

This class will be run as a flipped classroom. What this means is that the process of learning basic concepts and content (normally done via lecture) will be done on your own outside of class, and the practice of problem solving (normally done as homework) will be done in class when I am present to help you. The learning process outside of class will be facilitated by videos that I will be making over the course of the summer and worksheets that I will give you corresponding to both sections in the textbook and these videos. The worksheets will be due at the start of each class, at which point is expected that you will have read the sections in the textbook and completed the worksheet. These worksheets will contain questions about the important parts of the textbook sections as well as a few simple problems to get you started. Class will consist of a short lecture discussing the material and answering any questions from the previous night's reading, followed by group problem solving work. This will be similar to the workshop process in Calculus 1 and 2. The end of class will either consist of presentations or quizzes. The last page in this syllabus outlines the general process for a given class period.

Academic Integrity

All students in this course are expected to be familiar with and abide by the academic integrity policy (http://academicintegrity.rutgers.edu/academic-integrity-at-rutgers). Violations of this policy are taken very seriously. In short, don't cheat, and don't plagiarize. In terms of exams, it's fairly easy to understand what cheating/plagiarism is. However, this class is going to be heavily based on group work and projects. Everyone is expected to submit their own work, which means copying or borrowing answers from someone else in the class is plagiarism. Since you are expected to work together for some of the problems, this can be tricky. The general method that you should use in this class is that during the group work, you should only write notes about the problems, but don't work on the actual write-up. Then, outside of class, you can do the write-up using your notes, which will result in your write up still coming from the work you did in class, but will not be identical to your classmates. See the Canvas page for more information.

Attendance

Attendance at every class meeting is mandatory. Attendance will be taken in the form of the worksheets at the beginning of class and participation points for the problems solved in class. You are also expected to watch all of the videos that I link on the Canvas page. With the speed of all summer classes, missing any class will result in you falling significantly behind. If you must miss a class for any reason, come talk to me as soon as possible.

Problem Sets

The in-class problem sets will consist of three parts. The first, 'Warm-ups,' consists of problems that give a basic idea of the topics. All groups in the class should do all of these problems, or at least verify that they know how to do all of them. These are all fair game for quiz and exam questions. The second section, 'Exercises,' are a little more complicated and involved. These are the problems that will be presented at the end of class (see the Presentations section), and problems similar to these are around the level of exam questions. The final section, 'Problems,' are more involved and multi-step problems. These will be turned in as a part of the homework write-ups (see the Homework section).

Presentations

Whenever there is not a quiz at the end of class, there will be in-class presentations of the problems that were worked on that day. Each group will get a different problem to work through (assigned around the middle of class), and one person from the group will need to present it to the class in the last 30 minutes of class. The person presenting at the board will need to rotate every time a presentation is done, but the entire group can help in presenting the problem or giving guidance from their seats. The entire group can also work together to write the solution on the board before the presentation, and then only one person will talk through it to the class. The goal here is to build confidence in talking about the course material, as well as give everyone practice talking about math.

Quizzes

Every Tuesday and Thursday class (except exam days) will end with a quiz. This quiz will cover material from the previous two days of class, but can also be more cumulative depending on the situation. The problems on the quizzes will be on a comparable level to the in-class exercises. They will be closed book, closed note, individual quizzes. The first quiz is already posted on Canvas. It is a quiz about this syllabus and how the class is structured. It is worth triple of all of the other quizzes, and can not be dropped. It is due at the end of the day on Tuesday, June 6. You have unlimited attempts to get it right, and can use this syllabus and the Canvas website while you are taking the quiz.

Homework

Homework for this class will not be assigned in the typical manner. Before each class, you will be expected to complete a worksheet summarizing sections in the textbook. There will also be problems on here to be completed from the videos that you need to watch. This will be due at the start of class, graded during class, and returned to you the same day. At the end of each class, approximately one problem from each homework set covered that day will be assigned. These will need to be written up individually, although you will be doing the problems in groups, so you are definitely welcome to talk about the problems as you do them. The write-ups should be fairly complete, somewhere between workshops and normal homework. The write-ups should also be your own work, completed without collaborating with other students. These will be graded and returned to you.

Maple Assignments

There are 3 Maple labs that will need to be completed and turned in over the course of the summer. The dates are included in the tentative schedule below, and all of the necessary materials will be posted to Canvas. There will also be some introductory materials posted there if you need more information. If you have any issues with the Maple labs, come talk to me in office hours or send me an email. I will briefly mention each of them as they are assigned, but you will be overall responsible for completing them on your own, and coming to me with any questions. I will be spending minimal in-class time discussing the Maple labs.

Projects

There will be two projects assigned over the course of the summer. The due dates for these projects are in the schedule at the end of the syllabus. These projects will be somewhat similar to lab reports, in that math and writing will be incorporated together into a single document. You will need to both present solutions to the given math problems and discuss the implications of the results in an actual situation. The project description will make it clear what you are expected to talk about and how to use the mathematical results to do so. The idea with the projects is to show you how the math you do in this class is applicable to physical situations and understand yourself how to do these applications. The projects will be done individually, although you are allowed to discuss it with both me and your fellow students.

Exams

There will be two midterm exams and a final exam. The dates for these exams are posted above as well as on the tentative schedule below, but are subject to change. These will be exams in the standard sense, 80 minutes for the midterms and 180 minutes for the final, and will be taken individually. Calculators and electronic devices will not be permitted on the exams, and they will be closed book and closed note.

Make-Up Policies

There will be no make-ups for any of the in-class activities or homework assignments. In order to compensate for this, the lowest quiz, lowest 2 worksheets, and lowest 2 homework grades will be dropped at the end of the summer. Under no circumstances can an exam be made up after the fact. If there is a legitimate reason for missing an exam, i.e., doctor’s note, then we can discuss possibilities moving forward, but you will not be able to take the exam later. If there is an excessive need to miss class, talk to me about it sooner rather than later.

Disability Accommodations

I will be happy to provide appropriate accommodations for students who provide me with a letter of accommodation from the Office of Disability Services (ODS). For more information, see http://ods.rutgers.edu/.

Changes

This syllabus is subject to change at any point. Any changes will be announced in class and posted on the Canvas site.

Tentative Class Schedule

DATE	SECTIONS	DUE DATES
5/30	1.1, 1.3	
5/31	1.2, 2.2	
6/1	2.1	Quiz
6/5	2.3	
6/6	2.4, 2.8	Quiz
6/7	2.5	
6/8	2.6	Quiz
6/12	2.9	Project 1 Due
6/13	3.1, 3.2	Quiz
6/14	5.4, Review	
6/15	MIDTERM 1	
6/19	2.7, 8.1-8.3	Maple 1 Due
6/20	3.3, 3.4	Quiz
6/21	3.7	
6/22	3.5, 3.6	Quiz
6/26	3.8	Project 2 Due
6/27	Chapter 7, Day 1	Quiz
6/28	Chapter 7, Day 2	
6/29	7.5, 7.6	Quiz
7/3	7.8, 9.1	
7/4	NO CLASS	NO CLASS
7/5	C7, Day 3, Review	
7/6	MIDTERM 2	
7/10	7.7	Maple 2 Due
7/11	7.9	Quiz
7/12	9.2, 9.3	
7/13	9.4, 9.5	Quiz
7/17	9.7, 9.8	Maple 3 Due
7/18	Chapter 4 and 5 Summary	Quiz
7/19	Review	
7/20	FINAL EXAM	

Class Structure

Outside of Class:

1. Read assigned sections in the textbook.
2. Watch the corresponding videos through Canvas.
3. Fill out the worksheet and complete the problems from the videos.
4. Complete in-class problem write-up.

In Class:

1. Turn in worksheet, previous class's homework, and ask any questions from the previous sections.
2. Listen to brief lecture about the material.
3. Work on book problems or other assigned problems in groups.
4. Ask questions about the problems as needed.
5. Midway through class, end of class and homework assignment will be discussed.
6. Class will end with either presentations or a quiz.

Worksheets will be returned the same day they are collected. Homework assignments will be returned a day later.

Project 1: Fluid Flow

Matt Charnley

April 15, 2017

All fluids have a material property called viscosity, which basically measures how much the fluid likes to move or flow. For instance, honey is much more viscous (has a higher viscosity) than water. In this project, you will investigate how using ODE's to model fluid flow can help us to calculate the viscosity of a fluid in two types of viscometers.

The main equations we will be using is the Navier-Stokes equations, which are, in their original form, the following partial differential equation:

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho(\vec{u} \cdot \nabla)\vec{u} = -\nabla P + \mu \Delta \vec{u} + \rho \vec{g} \tag{1}$$

where this is actually 3 equations, one for each coordinate direction, i.e., one of them is

$$\rho \frac{\partial u_x}{\partial t} + \rho(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z}) = -\frac{\partial P}{\partial x} + \mu \Delta u_x + \rho g_x \tag{2}$$

and the Laplacian Δ is

$$\Delta u_x = \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}$$

μ is the viscosity parameter we want to find, and ρ is the density of the fluid.

In each of the cases below, we'll be able to simplify the equation in order to reduce it down to a simple ODE that you can solve. This assignment will guide you through the simplifications needed, and allow you to do some analysis on this system. Note, the ODEs in this problem are really simple. Don't overthink it.

1 Torque Viscometer

For this first version of the calculation, we will consider fluid flowing between two parallel plates, the bottom plate being fixed in place, and the top moving at a velocity V.

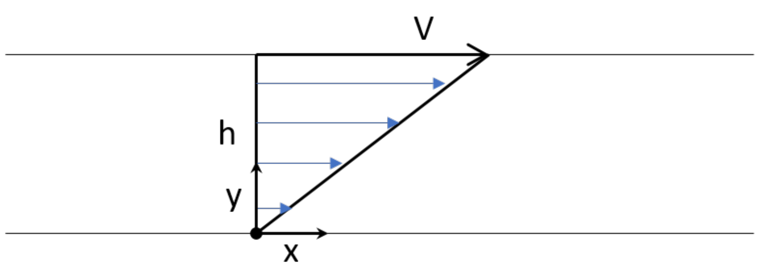


Figure 1: Sketch of the flow between parallel plates

We will assume that coordinates are aligned so that the origin is as marked in the image, the x direction is to the right, the y direction points up, and the z direction is back into the page. The assumptions we are allowed to make here are that the fluid only moves in the x direction, that is, $u_y = u_z = 0$ everywhere. We also assume that nothing depends on the z coordinate because the problem is infinite in that direction, that is, every derivative in z is zero. Finally, a different property, the continuity equation, lets us conclude that $\frac{\partial u_x}{\partial x} = 0$.

(a) Those assumptions tell us that we only need to care about the u_x equation from Navier-Stokes, i.e., the equation in 2. Using the above assumptions, plus the fact that the system is at steady state (time derivatives are zero) and gravity points in the y direction ($g_x = 0$), simplify 2 to get a simple ODE for u_x.

(b) What is the general solution of this ODE? Again, don't overthink it.

(c) Assuming the channel is of height h, the "no-slip" condition tells us that we must have $u_x(0) = 0$ and $u_x(h) = v$. Using these conditions, solve for the constants to get a specific solution.

(d) For this type of situation, the *stress* on the lower plane (something like the force the fluid imparts on the plane) is given by $\tau = \mu \frac{\partial u_x}{\partial y} \mid_{y=0}$. Compute this in terms of the given situation above.

(e) Now, we want to use this in a specific situation to compute viscosity. A torque viscometer is a pair of nested cylinders, where the fluid lies between them.

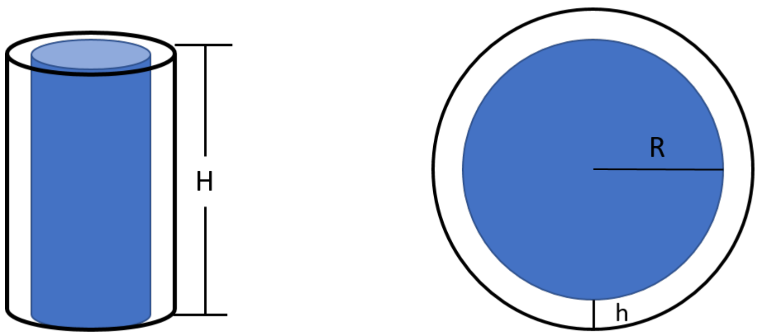


Figure 2: Sketch of a Torque Viscometer, both in profile and in cross-section

If the radius of the outer cylinder is really large compared to the gap between the cylinders, we can ignore the fact that the setup is curved, and pretend it is flat, which is what we just solved above. Assume we have a setup like this of height H in the z-direction. The torque T on the inner cylinder is given by the stress factor τ multiplied by the surface area of the cylinder (giving force) and then multiplied by the radius. Find a formula for the torque in terms of v, R, H, h, μ and other constants.

(f) Describe a procedure to use this system to calculate the viscosity of a fluid. You can leave all of the parameters of the problem in terms of variables or pick values for them. Determine which things are physical parameters of the system, which things you can measure, and how to calculate the viscosity from that.

We are a little bit off in the calculations here, because we assumed the round cylinder was flat, but it's only around a 1% error in most situations. So, a simple setup like this will allow us to approximate the viscosity of a fluid.

2 Capillary Viscometer

In this problem, we will consider flow in a cylindrical pipe.

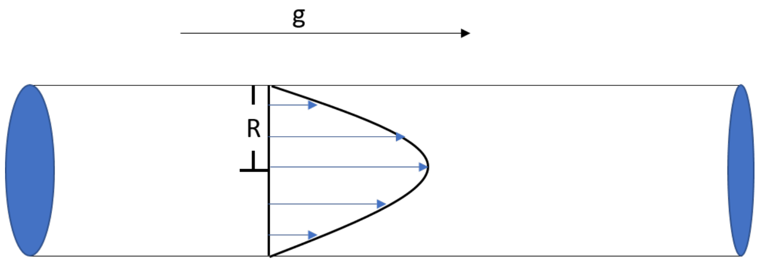


Figure 3: Sketch of flow in a cylindrical pipe

In order to find the equations here, we will want to use cylindrical coordinates, because that best fits the geometry of our system. We will assume that z points along the axis of the pipe, r is in the direction of the radius of the pipe, and θ points around the pipe. We will again assume that the fluid only moves in the z direction, so that $u_r = u_\theta = 0$. Thus, we only care about the u_z equation, which, in these coordinates is

$$\rho\frac{\partial u_z}{\partial t} + \rho(u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}) = -\frac{\partial P}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u_z}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z \tag{3}$$

(a) Making the assumptions above, along with the steady state assumption from part 1, and the additional fact that $\frac{\partial u_z}{\partial z} = 0$, we can simplify this equation. For this part of the problem, we will assume that $\frac{\partial P}{\partial z} = 0$, but $g_z = g$ is the normal acceleration due to gravity. Make the cancellations above to get an ODE for u_z.

(b) Integrate both sides twice in r, clearing the factors of r each time, to get a general solution for u_z.

(c) The boundary conditions we want here are that $u_z(R) = 0$, another "no-slip" condition, and that $u_z(0)$ is finite, because fluid moving infinitely fast is generally not a good thing. Using these conditions, find a specific solution for u_z.

(d) In order to calculate the flow rate Q, we need to integrate this velocity profile over the circular cross-section of the cylinder. Integrate u_z over the circle of radius R to find a formula for Q.

(e) Describe a procedure where you use this information to calculate the viscosity of a fluid. Hint: The fact that the fluid flow is being driven by gravity means you need to have the pipe be vertical. You will want to have the fluid flow through the pipe and measure something, just like in part 1 you measured the torque.

These calculations are the basis of the Capillary viscometer, which can also be used to find the viscosity of a fluid.

Project 2: Harvesting from a Population and Bifurcation Analysis

Matt Charnley

April 15, 2017

We recently investigated the logistic model for population growth. In this project you will look at how this is affected by harvesting from the population, whether this is constant-yield or constant-effort harvesting. We will see that there is a specific value of the harvesting parameter for which the type of solution changes. The name for this point (where the existence and stability of critical points changes) is a **bifurcation point**. Here, you will look at the bifurcation point in the case of a harvesting model, and then extend this analysis to a general first order ODE.

All of your answers for this project should be done two ways, both explicitly/by hand and using Maple to sketch graphs. The equations are simple enough that everything can be calculated by hand, but the pictures will help to illustrate your point. For instance, if a critical point is stable, you should show some solutions that converge towards it. If a point is unstable, you should show solutions that start near the solution, and then move away from it. You should also have multiple graphs for each parameter that is varying.

1 Single Species with Harvesting

(a) Consider a single species whose population grows under the logistic model, namely

$$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y$$

where K is the carrying capacity of the system and r is the normal growth rate of the population. Find and analyze the critical points of this system.

(b) Now, we want to implement harvesting. First we will use constant-effort harvesting. This means that there will be a constant effort put forth to harvesting, namely, the extra amount that the population is reduced is proportional to how many creatures there are. Let E be this proportionality constant. Write a new differential equation for the rate of change of the population. Analyze the critical points of this equation, which will now depend on E. What happens as E changes? Are there any important values of E where the equation changes significantly?

(c) Finally, we will look at constant yield harvesting. This means that the population will be depleted by a specific constant amount, call it H, each year. Write a new differential equation for this system. Analyze the critical points here as well, which will depend on H. What happens as H changes? Are there any specific values of H for which the equation changes significantly?

(d) What are your recommendations for what the values of H or E should be? This is much more of an opinion question. Interpret the results of the previous parts to discuss harvesting habits. Your answer should discuss some/all of the following.

1. If we have estimates for r and K (you can make up values if you would like), then what would you recommend for values of E and H?
2. How would you be able to check if the values for E or H are too high or low? How does the population over time tell you this?
3. If environmental factors change, which produces an effective change in r or K, how could/should E and H change to compensate? Make up a few example of this to fuel the discussion.
4. How sensitive is the situation to changes in H? For instance, if one person goes against fishing limits, and H jumps by 10 for a given time step, what could happen to the population? Why do limits on the number of fishing or hunting licenses exist?

2 General Bifurcation Analysis

In this section, you will analyze a different ODE that also exhibits bifurcation behavior. Between your in-class group, you should each choose a different one of the following four ODEs to analyze. If you have a group of three people, you should choose between the first three ODEs, if there are four of you, you should add in the fourth ODE as well. In each case, the capital letter is the parameter to be varied.

(a) $\frac{dy}{dt} = y^3 - Ay$

(b) $\frac{dy}{dt} = y^2 - By + y$

(c) $\frac{dy}{dt} = Cy - y^2$

(d) $\frac{dy}{dt} = Dy - y^3$

For your given ODE, you should perform the following analysis:

1. Analyze your problem (by hand) to find the critical points and their stability. Look at where specific values of the parameter cause changes in the stability of critical points.
2. Show Maple graphs in each of the situations that you described in the first part. They should clearly state the value of the parameter and the stability of each critical point.
3. Draw a bifurcation plot. This is a plot of the phase line (critical points) against the value of the parameter. For instance, if at $A = 1$, the ODE has a critical point at $y = 2$ and $y = 4$, the graph should have curves at $y = 2$ and $y = 4$ over the point $A = 1$ (your axes should be y vs. A).
4. Talk about what you see in these plots and what it means for the ODE or the physical system governed by it.

Problem Set 11: Sections 2.7 and 8.1-8.3

Matt Charnley

June 20, 2017

This worksheet is a little different because of what this section is. We're talking about numerical methods, and we will be spending most of the class in the library so that you all can practice this. Follow the problem set from start to finish, and you should hopefully be able to complete the coding part of the write-up before you leave class. What you turn in will be partially hand-written and partially computer code. I am looking to see your code for this assignment. There is a sample file on Canvas with some basic loops that you can use as a starting point.

1 Problem 1

Look at the ODE $y' = 3 - t + y$

(a) Solve the ODE explicitly (first order linear).

(b) Assume we want to start with the initial condition $y(0) = 2$. Write out the first 2 steps of Euler's method with $h = 0.5$. How much error is there at $t = 1$?

(c) Write out the general form of Euler's Method for this equation (how you get from y_n to y_{n+1}) with a step size h.

(d) Do the same for the backwards Euler's method, and solve it explicitly for y_{n+1}.

(e) Do this for the centered difference or Heun's method and solve out for y_{n+1}.

Now, we want to move all of this into the computer.

(a) Write a Maple code (or really, whatever language you want to use, I'm just providing maple) to do the normal Euler's Method for the above ODE. Get to the value at $t = 1$ with a step size of $h = 0.05$, $h = 0.025$ and $h = 0.0125$. Keep track of your errors in each case. You should see approximately the proper decrease in error as you decrease the step size.

(b) Do the same for the backwards Euler, Heun's method, and Runge-Kutta method. You should be able to reuse most of your code for each of the steps. Record the error here as well.

The rest of this problem set will be due tomorrow at the start of class.

2 Problem 2

Look at the ODE $y' = 4 + 2t - 5y$

(a) Solve the ODE explicitly (first order linear).

(b) Assume we want to start with the initial condition $y(0) = 2$. Write out the first 2 steps of Euler's method with $h = 0.5$. How much error is there at $t = 1$?

(c) Write out the general form of Euler's Method for this equation (how you get from y_n to y_{n+1}) with a step size h.

(d) Do the same for the backwards Euler's method, and solve it explicitly for y_{n+1}.

(e) Do this for the centered difference or Heun's method and solve out for y_{n+1}.

Now, we want to move all of this into the computer.

(a) Write a Maple code (or really, whatever language you want to use, I'm just providing maple) to do the normal Euler's Method for the above ODE. Get to the value at $t = 1$ with a step size of $h = 0.05$, $h = 0.025$ and $h = 0.0125$. Keep track of your errors in each case. You should see approximately the proper decrease in error as you decrease the step size.

(b) Do the same for the backwards Euler, Heun's method, and Runge-Kutta method. You should be able to reuse most of your code for each of the steps. Record the error here as well.

3 Problem 3

In this problem, we are looking at a non-linear ODE, so solving for the backwards Euler and Heun's method is more difficult. So you don't have to do that.
Consider the ODE

$$y' = (2x + 4)y^2 \qquad\qquad y(0) = 10$$

(a) Solve this separable equation explicitly.

(b) Use the Euler's method code you wrote earlier to approximate the solution at $x = 1$ for using $h = 0.05$, $h = 0.025$ and $h = 0.0125$. Write down the error in each case.

(c) Use the Runge-Kutta code you wrote earlier to do the same with the same values of h. Again, keep track of the error in each case.

4 Discussion

(a) For Problem 2, what do you notice about the errors as you decrease the size of h for all 4 of the methods you used?

(b) For Problem 3, do the same analysis for the two methods you used for the non-linear problem.

Problem Set 17: Sections 7.5 and 7.6

Matt Charnley

June 29, 2017

These problems are to be worked on in class. All groups should work on the first set of problems, then move on to the second and third sets. The second set will be done as presentations, and the third set will have a problem assigned as homework. This problem is due at the start of next class.

1 Warm-ups

(a) Problem 3 on page 405

(b) Problem 15 on page 405

(c) Problems 24-27 on page 406 (only need to do part (a) and take a guess at part (b))

(d) Problem 3 on page 417

(e) Problem 9 on page 417

(f) Problem 13 on pages 417-418

2 Exercises/Presentations

The first two problems here I think are really nice and tie these new calculations back to the second order systems. The other ones are more computation.

(a) Problem 29 on page 406

(b) Problem 28 on page 419

(c) Problem 1 on page 405

(d) Problem 6 on page 405

(e) Problems 1 and 6 on page 417

(f) Problems 15 and 18 on pages 417-418

3 Problems

Write up the solution to the following problems. In both cases, solve the given initial value problem and describe the behavior of the solution as $t \to \infty$.

1. Problem 16 on page 405:
$$\vec{x}\,' = \begin{bmatrix} -2 & 1 \\ -5 & 4 \end{bmatrix} \vec{x} \qquad\qquad \vec{x}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

2. Problem 10 on page 417:
$$\vec{x}\,' = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} \vec{x} \qquad\qquad \vec{x}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Worksheet 18: Sections 7.5 and 7.6

June 29, 2017

Name: ____

Assignment: Read Sections 7.5 and 7.6 and watch the 'Day 18 Videos' posted to the Canvas site. The last few questions for each section will come from the ends of the videos.

1 Section 7.5

1. What name do we give to the drawings of a sample of trajectories for a system of the form $\vec{x}' = A\vec{x}$, particularly when there are two functions in $\vec{x}$?

2. If we know that ξ is an eigenvector of A with eigenvalue r, then what function do I know solves $\vec{x}' = A\vec{x}$?

3. What are the three different options for the type of equilibrium point at 0 for a system $\vec{x}' = A\vec{x}$ where A has two distinct real eigenvalues?

4. What are the other two options for the eigenvalues of a 2×2 matrix (other than real and distinct roots)?

2 Section 7.6

1. If the eigenvalues of A are complex conjugates, then what do we know about the corresponding eigenvectors?

2. If $r = \lambda + i\mu$, what does the **complex valued** solution look like?

3. How do we get from the above solution to two independent solutions? How is this more complicated than the similar process for the second order ODE case?

4. What kind of equilibrium points can we get for complex eigenvalues?

3 Video Questions

1. Section 7.5 Video 1 Question

2. Section 7.5 Video 2 Question

Worksheet 18 Quiz

June 29, 2017

Name: ____

Find the general solution of the following ODE system

$$\vec{x}\,' = \begin{bmatrix} 2 & 2 \\ 0 & 3 \end{bmatrix} \vec{x}$$

A.4 Summer 2018 - Math 252

This section contains the following documents from my Summer section of Math 252:

1. The course syllabus
2. A sample Mini-Quiz with its solution
3. A sample Readiness Assessment with its solution
4. A fairly standard example of a problem set that would be worked on in class
5. A sample practice problem with its solution
6. Another problem set that had more of an activity component to it
7. The set of review problems that made up the Gallery Walk activity on the first day of class
8. Assignment sheet for the bifurcation jigsaw activity
9. Assignment sheet for the oscillators workshop activity
10. Assignment packet for the SIR modeling activity
11. Assignment sheet for the end-of-semester student generated test questions

MATH 252 - Section B6 - Summer 2018

Contact Information

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Email: charnley@math.rutgers.edu
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Office Hours

Mondays, 3:30 - 5:00 PM
Tuesdays, 5:00 - 7:00 PM
or by appointment.

Class Meetings

MWF 6:00-8:45 PM, SEC 220

Exam Schedule

- Midterm 1: Friday, June 1 - First half of class
- Midterm 2: Wednesday, June 13 - First half of class
- Final Exam: Friday, July 6 - 6:00 - 9:00 PM

Textbook

This course will use *Differential Equations*, **4th edition**, by Paul Blanchard, Robert Devaney, and Glen Hall. ISBN-13: 978-1-133-10903-7.

Course Information

The information for this course can be found on Sakai. All announcements and assignments will be posted to this site. If you have any issues accessing the Sakai site, let me know as soon as possible. My personal website will also likely have some of these materials posted to it, but the most up-to-date resource will be Sakai.

Learning Goals

During this course, students will

1. Gain a familiarity with differential equations, which will show up in a variety of places after this class.
2. Understand how qualitative, quantitative, and numerical techniques can be applied to a problem, and when each one should be used.
3. Improve skills and confidence in talking about and presenting mathematics to their peers.
4. Become exposed to some of the ideas from higher-level mathematics, which will be expanded upon in future classes.

Class Overview

MATH 252 is an introduction to differential equations, generally directed at math majors. The course takes a three-pronged approach to studying differential equations: Qualitative methods (general behavior of solutions), Quantitative methods (analytical solutions), and Numerical methods (approximating solutions on a computer). We will take a look at all three of these over the course of the summer. The topics this course will cover are:

- First Order Differential Equations
- Systems of Differential Equations
- Linear Systems and Higher-order Linear Differential Equations
- Numerical Methods for ODEs
- Non-linear Systems of ODEs

NOTE: Linear Algebra (Math 250) is a prerequisite for this class. You will be expected to know the basics of linear algebra and matrix manipulation for this class. See the Midterm 1 topic outline for more information.

Grade Breakdown

Final grades for the class will be decided according to the following breakdown:

In-Class Assignments	10%
Quizzes	10%
MATLAB	10%
Midterm Exams	10% + 25%
Final Exam	35%

Class Structure

This class will be run in a mix between lecture and workshop formats. A lot of research has been done fairly recently on the implementation of Active Learning practices in math classrooms, and an article to this end has been posted to Sakai. I personally feel like these types of activities are very helpful in learning math, as you learn math best by doing problems, not by sitting around listening to lectures. Therefore, my plan is to implement several different activities in the classroom to move it more towards Active Learning. These may be things that you haven't seen in a math class before, but both I and the current research in the field believe that they are helpful in developing a better understanding of the course material. It only really works if you buy into it though, so I'm hoping you can give it a shot with me. If you have any questions, let me know and I'd be happy to talk about it.

The general plan for the class is as follows. Monday and Friday classes will generally start with a mini quiz and a Readiness Assessment, the first of which will test your knowledge of the homework problems assigned in the previous class, and the second will cover the readings assigned for the current class. You will be expected to have read the appropriate sections of the book before coming to class, and the Readiness Assessment will test your basic knowledge of these sections. These will be done individually, but there may be a group component to them afterwards. The rest of the class will consist of a mixture of workshop time, various activities, and lecture on the topics of the day. Over the course of each day, there will be two Practice Problems which will be completed, allowing you to practice the topics we are going over in class and show me that you know what is going on in class. Wednesday classes will be slightly different in that they will start with a quiz, testing material from the previous week of class. The rest of class will more or less be the same, consisting of lectures, activities, and workshops to help deepen or expand your knowledge of the material being covered.

Academic Integrity Policy

All students in this course are expected to be familiar with and abide by the academic integrity policy (`http://academicintegrity.rutgers.edu/academic-integrity-at-rutgers`). Violations of this policy are taken very seriously. In short, dont cheat, and dont plagiarize. In terms of exams, its fairly easy to understand what cheating/plagiarism is. For homework, you are definitely allowed to work with other students, but everything you turn in should be your own work. In particular, this means that you should NOT just write down and turn in a solution that you got from a friend, classmate, or the Internet. You should also be able to explain every step of what you turn in to me if asked. I would much prefer that you turn in a half-finished assignment than one that you looked up online or took from a classmate. In the first case at least both you and I know what you need to improve on and can work towards it. If you have any questions about this policy, please let me know. I am more than happy to talk about it.

Attendance Policy

Attendance is mandatory at every class. Each day in the classroom during the summer corresponds to an entire week of class during the semester. Therefore, missing a single class can be very detrimental to your learning and development in this course. Attendance will be taken in the form of practice problems and Readiness Assessments given out every day in class. If you need to miss a class, come talk to me about it as soon as possible.

Homework Assignments

There will be no traditional homework assignments for this course. There will be homework assigned each night, but the assignments will not be collected or graded. It is up to you to determine how many of the problems you want to do and how completely you want to work them out. You will get credit for this homework in terms of the quizzes and Readiness Assessments that will take place at the start of each class. These will be taken from problems very similar to those on the homework, so doing these problems will directly help you to do well on the mini-quizzes, as well as on the exams. I am also planning to post answers (not solutions) to the homework if the answer is not already in the book, so that you can check your work after doing the problems.

You will also be expected to read sections of the textbook and/or watch videos online before class. These will be announced at the end of the preceding class, and your knowledge of these sections will be tested with the Readiness Assessments at the start of each class. These assessments will cover basic knowledge of these sections. With the active learning component of the class, there will be less time for lecturing, so you all having a base level understanding of the material before you show up to class will help things to run smoothly. I also taught MATH 244 as a flipped classroom last summer, which means I made video lectures for the entire course. I may, at points throughout the summer, send you links to the videos for the appropriate sections for you to watch before class.

Projects / MATLAB Assignments

There will be 5 MATLAB assignments over the course of the summer. Each of these will involve taking pre-written code and modifying it to run some experiments that will illustrate concepts from class. These are assignments that have been given for the last several years, and I think they are well-made assignments. The assignments and sample code will be posted to Sakai, and you can download MATLAB by following the link here: `https://software.rutgers.edu/product/3437`.

In-Class Assignments

In-class assignments will take 3 forms. Monday and Friday classes will start with a mini-quiz about the homework from the previous class. This will be very similar to the homework problems assigned from that class. These mini-quizzes will be followed by a Readiness Assessment, covering basic knowledge on the reading assignments for the current class. Finally, there will be Practice Problems assigned each day in class, giving samples of the types of problems that could be seen on quizzes or tests related to the material being discussed in class.

Quizzes

On each Wednesday class that does not have a midterm, the class will start with a quiz. This quiz will cover the material that has been discussed over the last week. The quiz will last approximately 40 minutes and will be of similar difficulty to problems from the homework. These will be done individually and will be closed-book, closed-note.

Exams

This course will consist of two midterm exams and a final exam. The first midterm will happen this Friday, June 1. This exam is a prerequisite exam, covering material that you should know coming into this class, and is worth 10% of the final grade. There is an file on Sakai that contains an outline of what will be covered on this exam. This exam will take an hour. The second midterm will happen near the middle of the course and the tentative date is listed at the top of the syllabus. This exam will last 80 minutes. Due to the length of each class period, this will only take up half of the class, and we will continue with lecture/activities after the exam. These will be closed-book, closed-note exams, which will be taken individually. Calculators and electronic devices will not be permitted on exams.

Exam Rewrites

For each of the midterm exams, you will be allowed to rewrite the problems that you do not get full points on. You will receive exams with scores on the problems, but no marks on the pages, and can rewrite problems to get back half of the points that you missed. However, in order to get any points back for the rewrite, the rewritten version of the problem needs to be completely correct. More details about this process will be provided when the first midterm is returned.

Final Exam

The final exam will take place on Friday, July 6, from 6:00 - 9:00 pm in the normal classroom, and will be cumulative. Calculators and electronic devices will not be permitted on the final exam.

Make-Up Policies

There will be no make-ups for quizzes, practices problems, or exams. There is a decent chance that I will drop some number of practices problems or Readiness Assessments at the end of the course, but that will depend on how things go throughout the summer. If you have a legitimate reason for missing a midterm exam or quiz, then we can discuss potential options for your grade at that point, but try not to miss them. If you will not be in class on a day when an assignment is due, you need to send me a scanned version of the document before the end of class on that day. Pictures of the assignment will not be accepted, and anything received after the end of class will not be graded.

Disability Accommodations

Rutgers University welcomes students with disabilities into all of the University's educational programs. In order to receive consideration for reasonable accommodations, a student with a disability must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: `https://ods.rutgers.edu/students/documentation-guidelines`. If the documentation supports your request for reasonable accommodations, your campuss disability services office will provide you with a Letter of Accommodations. Please share this letter with your instructors and discuss the accommodations with them as early in your courses as possible. To begin this process, please complete the Registration form on the ODS web site at: `https://ods.rutgers.edu/students/registration-form`.

Adjustments

All information in this syllabus is subject to change at any time. Any changes will be announced on Sakai, changed on this document, and announced in class.

Tentative Course Schedule

Date	Section(s)	Topics	Due Dates
W 5/30	1.1, 1.2, Notes	Introduction, Solutions, Modeling	
F 6/1	1.2, 1.5	MIDTERM 1, Separation of Variables	
M 6/4	1.3, 1.4, 1.6	Geometric and Numerical Methods	
W 6/6	1.7	Bifurcations	MATLAB 1 Due, Quiz 1
F 6/8	1.8, 1.9	Theoretic and Analytic Methods	
M 6/11	2.1, 2.2, 2.5	Introduction to Systems	
W 6/13	2.7	MIDTERM 2, SIR Model	MATLAB 2 Due
F 6/15	3.1, 3.2, Notes	Linear Systems	
M 6/18	3.3, 3.4, 3.5	Phase Plane Analysis	
W 6/20	3.7	Trace-Determinant Plane	MATLAB 3 Due, Quiz 2
F 6/22	1.8, 2.3, Notes	Analytic Solution Methods	
M 6/25	3.6, 4.1, 4.2	Second Order Equations	
W 6/27	4.3, 4.4	Resonance, Steady State	MATLAB 4 Due, Quiz 3
F 6/29	5.1, 5.2	Non-Linear Systems	
M 7/2	5.3, 3.8, 2.8	Other Topics, Review	MATLAB 5 Due
F 7/6		FINAL EXAM	

MATH 252 - Mini-Quiz 4

June 18, 2018

Name: Key

Find the general solution to the system

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} \vec{x}$$

and determine the particular solution with initial condition $\vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Find eigenvalues: $\det \begin{bmatrix} 3-\lambda & -1 \\ -2 & 4-\lambda \end{bmatrix} = (3-\lambda)(4-\lambda) - 2$

$$\lambda^2 - 7\lambda + 12 - 2$$

$$\lambda^2 - 7\lambda + 10 = \underline{(\lambda-5)}\underline{(\lambda-2)}$$

$\lambda = 5$

$$\begin{bmatrix} 3-5 & -1 \\ -2 & 4-5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \vec{0}$$

$$-2v_1 - v_2 = 0$$
$$-2v_1 - v_2 = 0 \quad \Rightarrow \quad v_1 = 1, \; v_2 = -2 \qquad \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$\lambda = 2$

$$\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \vec{0}$$

$$v_1 - v_2 = 0 \quad \Rightarrow \quad v_1 = 1, \; v_2 = 1 \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

So, the general solution is

$$\vec{x}(t) = k_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{5t} + k_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$$

To match the initial condition, we need

$$k_1 + k_2 = 1$$
$$\underline{-2k_1 + k_2 = 2}$$
$$3k_1 = -1 \qquad k_1 = -1/3 \quad k_2 = 4/3$$

$$\Rightarrow \quad \vec{x}(t) = \begin{bmatrix} -1/3\, e^{5t} + 4/3\, e^{2t} \\ 2/3\, e^{5t} + 4/3\, e^{2t} \end{bmatrix}$$

MATH 252 - Readiness Assessment 7

June 25, 2018

Name: Key

1. Consider mass on a spring system whose model is given by the second order equation

$$\alpha y'' + \gamma y' + \beta y = 0$$

In terms of the physical problem, what is the meaning of:

(a) α

Mass

(b) γ

Drag Coefficient

(c) β

Spring Constant

(d) y

Position of the object

(e) $\frac{dy}{dt}$

Velocity of the object

2. Assume that a mass on a spring system has general solution given by

$$y(t) = c_1 e^{-t}\sin(2t) + c_2 e^{-t}\cos(2t) + \frac{1}{4}\sin(3t) - \frac{3}{4}\cos(3t)$$

(a) What is the natural response of the system? What is the natural frequency?

Natural response: $c_1 e^{-t} \sin(2t) + c_2 e^{-t} \cos(2t)$

Natural frequency is $\frac{1}{\pi}$

(b) What is the forced response of the system?

Forced response: $1/4 \sin(3t) - \frac{3}{4} \cos(3t)$

(c) What is the long-term behavior (as $t \to \infty$)?

As $t \to \infty$, the solution goes to the forced response

$1/4 \sin(3t) - 3/4 \cos(3t)$

MATH 252 - Problem Set 10

Matt Charnley

June 25, 2018

Part 1

For this set, we will be looking at section 4.1. In addition to the problems presented, you should also identify what kind of an equation it is. That is, is the system undamped, underdamped, critically damped, or overdamped, or does it not model a mass on a spring (this is when any of the coefficients are negative).

1. Section 4.1, Problems 1-11 odd.
2. Section 4.1, Problems 13, 15, 17.
3. Section 4.1, Problems 20, 21, 23.
4. Section 4.1, Problems 25, 27, 29.

Part 2

For this set, the same additions hold as for part 1. The only thing different here is going to be we have periodic forcing instead of exponentials or constants. You should begin to connect the amount of dampening in the system with what the solution formula and graphs look like. For all of these, I'm expecting you to solve them by Undetermined Coefficients, not the complex method they describe in the book. You should also be able to get the amplitude of the answer and know how to get the phase angle. I obviously won't expect you to be able to find the phase angle by hand, because that is generally impossible without a calculator.

1. Section 4.2, Problems 1-13 odd.
2. Section 4.2, Problem 17. This is a very good problem for checking your understanding of how these equations work.

MATH 252 - Practice 16

June 25, 2018

Name: Key

For the following second order equation, determine if it is undamped, underdamped, critically damped, or overdamped, find the general solution, and compute the particular solution if, at $t = 0$, the mass starts at the equilibrium point and is given an initial velocity of 2.

$$y'' + 5y' + 6y = \sin 2t$$

$r^2 + 5r + 6 = 0 = (r+2)(r+3) \longrightarrow$ Overdamped

Homogeneous Solution: $y_h(t) = C_1 e^{-2t} + C_2 e^{-3t}$

Undetermined Coefficients: Guess $y_p(t) = A \sin 2t + B \cos 2t$

$$y' = 2A \cos 2t - 2B \sin 2t$$

$$y'' = -4A \sin 2t - 4B \cos 2t$$

$$y'' + 5y' + 6y = -4A \sin 2t - 4B \cos 2t + 10A \cos 2t - 10B \sin 2t + 6A \sin 2t + 6B \cos 2t$$

$$= (2A - 10B) \sin 2t + (2B + 10A) \cos 2t$$

So $2A - 10B = 1$

$5(2B + 10A = 0)$

$52A = 1 \qquad A = 1/52$

$B = -5/52$

General Solution: $y(t) = C_1 e^{-2t} + C_2 e^{-3t} + \frac{1}{52} \sin 2t - \frac{5}{52} \cos 2t$

$$y'(t) = -2c_1 e^{-2t} - 3c_2 e^{-3t} + \frac{1}{26}\cos 2t + \frac{5}{26}\sin 2t$$

$$y(0) = c_1 + c_2 - 5/52 = 0$$

$$y'(0) = -2c_1 - 3c_2 + \frac{1}{26} = 2$$

$$c_1 + c_2 = 5/52 \rightarrow 2c_1 + 2c_2 = 5/26$$

$$2c_1 + 3c_2 = \frac{-51}{26}$$

$$-c_2 = \frac{56}{26} \qquad c_2 = -\frac{28}{13}$$

$$c_1 = \frac{5}{52} - c_2 = \frac{5}{52} + \frac{112}{52} = \frac{117}{52}$$

Particular Solution

$$\boxed{y(t) = \frac{117}{52} e^{-2t} \cancel{+} - \frac{28}{13} e^{-3t} + \frac{1}{52}\sin 2t - \frac{5}{52}\cos 2t.}$$

MATH 252 - Problem Set 8

Matt Charnley

June 20, 2018

Part 1

For each of the following matrices, use the Trace-Determinant Plane to determine what kind of equilibrium solution the origin is if this matrix was used as the coefficient matrix in

$$\frac{d\vec{x}}{dt} = A\vec{x}$$

After you get through them, find a few general solutions and draw phase portraits to confirm your answer.

1. $\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$ **2.** $\begin{bmatrix} -3 & 2 \\ 1 & -3 \end{bmatrix}$ **3.** $\begin{bmatrix} 4 & 0 \\ -3 & -1 \end{bmatrix}$ **4.** $\begin{bmatrix} 0 & -2 \\ 7 & 1 \end{bmatrix}$ **5.** $\begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$ **7.** $\begin{bmatrix} -2 & 4 \\ 9 & -1 \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix}$ **9.** $\begin{bmatrix} 2 & -5 \\ 1 & -1 \end{bmatrix}$ **10.** $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

Part 2

In this section, you will analyze one-parameter families of systems in groups. Each group will take one of the following problems, work out the details, and then present it to the class. The components of the answer that I want to see are as follows:

1. Calculation of the trace and determinant of the family.
2. A sketch of the curve traced out by this one-parameter system in the Trace-Determinant Plane and how you determined this curve.
3. An identification of where the type of critical point at the origin changes, as well as what it changes from and to at that point.
4. (Optional) A few phase portraits of the system at specific values of the parameter μ.

The problems are:

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 3-2\mu \\ -1 & \mu \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 0 & -1 \\ e^{\mu} & \mu \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & \mu \\ \mu+6 & 2 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & -1 \\ 2 & 1 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & -1 \\ 2 & -1 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 3-\mu^2 & \mu+1 \\ 2-\mu & 1 \end{bmatrix}$$

MATH 252 - Midterm 1 Review

Matt Charnley

May 24, 2018

1. Find the roots of
   (a) $x^2 - 4x - 32$
   (b) $x^2 + 3x + 18$
2. Find all roots of $x^3 + 3x^2 - 2x - 6$.
3. Find all roots of $x^3 + 3x^2 - 4$.
4. Logarithms:
   (a) Simplify $\ln(40) - 3\ln(6)\ln(10)$
   (b) Solve $\ln(x^2) = 10$
5. Exponentials:
   (a) Simplify $\left(e^{2x} + e^{x^3}\right)^2$
   (b) Solve $e^{2x-5} = 4$
6. Trig Identities: Simplify $(\sin(2t) + 2\cos(2t))^2 - \cos(4t)$
7. Find the derivative of $x^4 + 3x^2 - 2x + 1$
8. Find the derivative of $x^3 \sin(2x)$
9. Find the derivative of $\frac{x^2+3}{2x+e^x}$
10. Find the derivative of $\tan(\ln(x))$.
11. Find the derivative of $xe^{\cos(x)}$
12. Compute $\int x^3 - 4x + 5 \; dx$
13. Compute $\int 3x\cos(x^2) \; dx$
14. Compute $\int 2x(1 + 4x^2)^{10} \; dx$
15. Compute $\int x^2 \sin x \; dx$
16. Compute $\int \ln(x) \; dx$
17. Compute $\int_0^3 f'(t) \; dx$ if $f(0) = 3$ and $f(3) = 10$.
18. Compute $\frac{d}{dt}\left[t^2 \int_0^t f(s) \; ds\right]$ for a function $f(s)$.
19. Compute $A + B$ for

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 2 & 0 & -3 \\ 3 & 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 6 \\ 1 & 5 & 2 \\ -1 & -2 & 4 \end{bmatrix}$$

20. Compute AB for

$$A = \begin{bmatrix} 6 & 1 \\ 4 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 4 & 7 \end{bmatrix}$$

21. Write out

$$\begin{bmatrix} 1 & 0 & 4 \\ 2 & 4 & 1 \\ -3 & 1 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

as a linear system of equations.

22. Determine if

$$\left\{ \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

is a linearly independent set of vectors.

23. Determine if

$$A = \begin{bmatrix} 6 & 1 \\ 4 & -3 \end{bmatrix}$$

is invertible.

24. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

25. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

26. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$

MATH 252 - Bifurcation Jigsaw

Matt Charnley

June 6, 2018

Purpose

The purpose of this activity is to allow you, as groups, to explore bifurcations and the different types that exist by analyzing a problem on your own. It will also give you an opportunity to improve your skills at talking about math concepts with your peers, as you will be presenting your discoveries to the other students in the class.

Structure

This activity will have three parts. During the first part, you and your group will work on analyzing two different one-parameter families of ODEs to search for bifurcation points. Each group will be working on the same problems here, and after some time to figure it out, we will discuss the results, so you know what the answer to a problem like this looks like. During the second part of this activity, each group will analyze another bifurcation problem, but each group will have a different problem. As a group, you will be responsible for finding and analyzing the bifurcation points of your problem following the format below. Once I see that everyone has the proper results, we will move on to the last part of the activity. Finally, in the 'Jigsaw' part of the activity, the groups will be shuffled so that every new group has at least one person in it who did each of the problems. Then, everyone in the new group will take a turn sharing what they discovered about the problem they analyzed. That way, at the end of class, everyone will have heard about all of the problems.

Format of Answer

Given a one-parameter family of ODEs, your analysis of any bifurcation points should contain the following.

1. A determination of any/all bifurcation points of the family of equations. This may include sketches of the graph of the function $f_\mu(y)$.
2. For each of the bifurcation points, a phase line (with solution sketches) for a value slightly larger and slightly smaller than the bifurcation point.
3. A description of what happens at the bifurcation points. (How many equilibrium points are there before and after? What type are they?)
4. A sketch of the bifurcation diagram for this family.

In terms of turning in this assignment, each group will need to hand in one copy of their bifurcation analysis for the second part of this activity. It can be handed in today or at the start of class on Friday.

Problems

For the initial part of this activity, each group will analyze the families

$$\frac{dy}{dt} = y^2 - \mu y$$

and

$$\frac{dy}{dt} = \mu - y^3$$

For the second phase, the possible familes of ODEs are

1. $\frac{dy}{dt} = \mu y - y^3$
2. $\frac{dy}{dt} = 2y\left(1 - \frac{y}{10}\right) - \mu$
3. $\frac{dy}{dt} = y^2 - \mu y + 1$
4. $\frac{dy}{dt} = (y^2 - \mu)(y^2 - 9)$
5. $\frac{dy}{dt} = y^3 + \mu y^2$
6. $\frac{dy}{dt} = y^4 + \mu y^2$

where, in each case, μ is the parameter to be varied.

MATH 252 - Harmonic Oscillators

Matt Charnley

June 27, 2018

1 Beats and Resonance

Beats and resonance happen when an undamped oscillator is forced at specific frequencies. This worksheet here will help you explore how and why this happens. For this set here, we will be considering the mass on a spring system given by

$$y'' + 16y = f(t)$$

(a) What is the general solution of this equation if $f(t) = 0$?

(b) What is the particular solution with $y(0) = 2$, $y'(0) = 0$ in this case?

(c) Now, we want to add in a forcing term. Let $f(t) = \cos(8t)$. Find the general solution to this ODE, and the particular solution with $y(0) = y'(0) = 0$.

(d) Using the identity

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

write this solution as a product of sine functions.

(e) Sketch the graph of this solution. The way you should think about this is that one of the two sine functions oscillates slower than the other. Draw the graph of this function and the negative of this function, and then you're going to draw the actual solution between those two curves. The ratio of the two frequencies should tell you how many times the curve cycles in each of the larger bumps of the slowly oscillating function.

(f) Do the same analysis for $f(t) = \cos(5t)$, $f(t) = \cos(4.5t)$, $f(t) = \cos(4.1t)$.

(g) Consider the forcing function $f(t) = \cos(\omega t)$ for $\omega \neq 4$. What does the result look like here? What is the amplitude of the resulting solution, and how does it depend on ω?

(h) What is the beating frequency of this solution (the frequency of the slower oscillation)? How does this change with ω?

(i) What happens when $\omega = 4$? What is the general solution here? What is the particular solution with $y(0) = y'(0) = 0$? Sketch the graph of this solution.

2 Amplitude of Steady State

Here, we are looking at damped oscillators of the form

$$y'' + py' + 4y = f(t)$$

(a) If $p = 5$, what is the general solution to this equation with $f(t) = 0$?

(b) If $f(t) = \cos(3t)$, what is the solution to the non-homogeneous part of the equation? You may still assume $p = 5$ here. What is the amplitude of this solution?

(c) Try the same for $f(t) = \cos(1.5t)$. Try it for $f(t) = \cos(2t)$ and $p = 1$.

(d) Perform the same analysis as part b, but leave p unspecified. The main thing I care about here is the amplitude. How does this change as p varies?

(e) Now, consider a forcing term of the form $f(t) = \cos(\omega t)$. Do the same analysis with both p and ω unspecified. You should end up with the amplitude as a function of ω and p. For what value of ω does it achieve it's maximum? Why is this not surprising? Look at the above section.

(f) Our input force for this process was of amplitude 1, since it was just a cosine function. What kinds of amplitudes can we get for the output steady state? Is this surprising?

3 Wrap-Up

In some sense, part 2 of this worksheet is more physically relevant, because there aren't really physical situations with no damping, since air resistance is a thing. However, we see from analysis of the ODE that we can still get some interesting results, and even some drastic behavior if the situation is just right. In particular, both the damping coefficient and the forcing frequency play a role in how the system responds to the force. One could use this knowledge to make sure that, for a given range of frequencies, the amplitude is never larger than 1, which might cause a system to break. This type of problems extends more into the engineering field that relates to the system being studied. The mass (which we fixed at 4) also plays a role, but more so in where the maximum amplitude occurs instead of what it is.

The main things to take out of the first part of this exercise are the ideas of **beats** and **resonance**. Both of these can only occur in an undamped system, beats occuring when the forcing frequency is close to that of the natural frequency of the system, and resonance occurs when these frequencies are equal. Resonance can only really happen for so long, until the system grows so fast that the model is no longer reasonable, but beats occur all the time, particularly when tuning an instrument. The beats indicate that the frequency is off just a little bit, and so it needs to be tuned. You should be able to recognize when beats and resonance can occur, know how to choose a forcing function to make it occur, and sketch the graphs that correspond to each of these situations.

MATH 252 - SIR Model and Diseases

Matt Charnley

June 13, 2018

Purpose

The purpose of this activity is to give you a chance to experience the SIR model for the spread of diseases. Over the course of this activity, you will see how the different parameters in this model affect how the disease behaves, and how making changes to the model can help it to emulate different types of diseases. You'll also get more time playing around with MATLAB code and seeing how you can use this to approximate the solution to these systems of ODEs.

Introduction

The basic idea of a disease model is that there are three main groups of people with respect to a disease: people who are susceptible to the disease, people who are currently infected (and contagious), and people who have recovered from the disease and are immune to it. These populations are denoted S (susceptible), I (infected) and R (recovered), hence the SIR model for diseases. For our particular case, we will let S, I, and R be the proportion of the population that falls into each of these categories, so that all of them are always between 0 and 1. In general, we will have initial conditions where $R(0) = 0$, meaning that no one has yet recovered from the disease, and $I(0)$ is a very small fraction of the population. A disease can be characterized by its infection parameter (how often a contact between an infected and susceptible individual results in a spread of the infection) and its recovery parameter (how quickly an infected individual 'recovers' from the disease, which can mean either no longer being infected or through death, because then you can't spread the disease anymore). While these models aren't perfect, they can give us a feel for how diseases spread and what can be done to prevent it. The main assumption in this model is that people are 'well-mixed' in the sense that any two people could interact at any time. These models work better for small, isolated communities, but we can still gain information from these types of results.

The Basic Model section below will give an introduction to the simplest of these models and kind of show you how they work. In addition to this, there is a folder full of MATLAB files on Sakai that will run all of these models. These should run fairly similar to the MATLAB files that you have been seeing for the MATLAB projects so far. The basic plan will be that the code does its thing, you'll just need to change the parameters and run the code to see what it does.

Basic Model: SIS

The first model we will discuss is the SIS model. This is an ODE system that could to model the behavior of the common cold. The idea of this model is that once someone recovers from being infected, they are immediately susceptible to the disease again. Thus, the model looks like

$$\frac{dS}{dt} = -aIS + bI$$
$$\frac{dI}{dt} = aIS - bI$$

where a is a parameter that describes how likely an infected person is to spread the infection to a susceptible individual, and b represents the rate at which infected individuals recover.

1. Find (by hand) the equilibrium points of the system. Looking at the equations, determine whether the solution will go towards or away from the different equilibrium points as t increases. Hint: Maybe there's a bifurcation in here somewhere.
2. Run the code for:
   (a) $a = 1$, $b = .5$
   (b) $a = 10$, $b = 1$
   (c) $a = 10$, $b = 5$
   (d) $a = 1$, $b = 4$

   or any other set of parameters you want to see how it behaves. You should hopefully get the result that you expected from the first part. You should also look at different initial conditions. The main one to start with is $S = 0.99$, $I = 0.01$
3. Explain what the 'equilibrium solution' means here and what this means for something like the cold.

Other Models

SIR

The SIR model applies to something more like measles, where once a person gets the disease, they are immune to it from that point forward. In this case, we get a system that looks like

$$\frac{dS}{dt} = -aIS$$
$$\frac{dI}{dt} = aIS - bI$$
$$\frac{dR}{dt} = bI$$

where a and b represent the same type of quantities as before. For this system, we can ignore R, and just focus on the relation between S and I.

1. Find all equilibrium points of this system.
2. When is $\frac{dI}{dt}$ positive and when is it negative? (For what values of the parameters)
3. When does an epidemic happen? An epidemic is defined as when the proportion of infected people is increasing.

SIRS

A modification of the SIR model can be used to model something like the flu. The idea with modeling the flu is that, after getting the disease, individuals are only immune to the disease for a period of time, until the disease mutates and everyone is susceptible again. In order to model this, we pick a parameter T to be the mutation time for the disease, and people are restored to the susceptible pool based on the number of recovered individuals T time units in the past. This gives rise to the *delay differential equation*

$$\frac{dS}{dt} = -aIS(t) + cR(t - T)$$
$$\frac{dI}{dt} = aIS(t) - bI(t)$$
$$\frac{dR}{dt} = bI(t) - cR(t - T)$$

where a and b are the same type of parameter as before, and c represents how fast recovered individuals are returned to the susceptible pool.

1. See how varying the parameters affects the results of the code. This includes varying a, b, c, and T.
2. How is this system substantially different than the previous few examples?

Pre-Vaccination

One way to limit the spread of a disease is by vaccinating against the disease. In terms of the ODEs, this is represented by having the R function not start at 0. Since R is the number of recovered individuals who are immune to the disease, having someone vaccinated means they start immune to the disease, which means they start in the R category and don't move out of it. In this case, we have the normal SIR equations

$$\frac{dS}{dt} = -aIS$$
$$\frac{dI}{dt} = aIS - bI$$
$$\frac{dR}{dt} = bI$$

but our initial conditions are $R(0) = p$, $S(0) = 0.99(1-p)$ and $I(0) = 0.01(1-p)$, i.e., 1 percent of the non-vaccinated people are infected with the disease.

1. Set $R(0) = 0$, and find a set of parameters a and b that give rise to an epidemic.
2. Increase the value of p until an epidemic no longer occurs.
3. For your values of a and b, compute $R_0 = a/b$. How does the p you found relate to this value?
4. To get an idea of comparison, the R_0 value for measles is between 12 and 18 and smallpox is between 5 and 7. Run the code for $a = 15$, $b = 1$ and figure out where the p needs to be to prevent an epidemic. This p is the critical percentage of population that needs to be vaccinated to prevent an epidemic. This is the idea of herd immunity.

Post-Vaccination

Another extension of this model is the idea of a vaccine to a disease being created once the epidemic is started. It is then a race to vaccinate the population before the disease spreads. This can be modeled by the following system of differential equation

$$\frac{dS}{dt} = -aIS - cS$$
$$\frac{dI}{dt} = aIS - bI$$
$$\frac{dR}{dt} = bI + cS$$

where c in this context denotes how much of the population is given the vaccine per time unit. Here, the initial conditions are $S(0) = 0.99$, $I(0) = 0.01$, $R(0) = 0$. Run some trials with an increasing value of c and see how this changes the results.

Zombies

A modified version of the SIR model can be applied to a zombie epidemic as well. The main difference here is that there is no recovery, but zombies still die off if they can not feed. Thus, we can start with the standard model

$$\frac{dS}{dt} = -aIS$$
$$\frac{dI}{dt} = aIS - bI$$

There are a few modifications that can be made to this to make it seem more like a zombie model. The first of those is that the zombies are organized, so the spread is more aggressive than that of a normal disease. This can be repesented by

$$\frac{dS}{dt} = -aS\sqrt{I}$$
$$\frac{dI}{dt} = aS\sqrt{I} - bI$$

Since $I < 1$, $\sqrt{I} > I$, so this is a faster rate. A second modification is that the zombies don't die off on their own, but can be killed by the survivors. This results in a system that looks like

$$\frac{dS}{dt} = -aSI$$
$$\frac{dI}{dt} = aSI - cS^2I$$

where c describes how quickly and efficiently survivors can kill off zombies. This last term represents the fact that if there are multiple survivors interacting with a zombie, they have a chance of killing it before they get converted. These could also be combined to give more complicated models. Run this for a variety of different parameters and see how they affect if the people survive or how long they last.

Assignment

By next Monday, each group needs to write up an exploration of one of the "Other Models." Your write-up should include some graphs of the system at a variety of parameters, an explanation of how changing the parameters affects the behavior of the system.

MATH 252 - Student Generated Test Questions

Matt Charnley

June 29, 2018

Purpose

The purpose of this assignment is to give you the opportunity to think about what we have learned in this class and formulate this into questions that could be seen on an exam. It will also result in you all having a collective list of problems that can be used as a review for the final.

Assignment

By Monday, July 2, everyone will be responsible for submitting at least 2 questions from a section or topic from this course, along with their solutions. When creating these problems, you should think about what types of problems you have seen on quizzes and exams up to this point, as well as the extension of those problems to what we have done more recently. You can also use the problems in the textbook as motivation for creating your problems. The plan will be to type up all of these problems and post them to Sakai so that you can use them for review. I will also post the solutions (once I check them over) so that you have those as well. You should use all of the resources at your disposal when writing the solution; these are (assuming you are ok with this) going to be posted for the rest of the class to view, and so should be correct.
Here's what I need you to turn in:

1. At least 2 questions written from your assigned section/topic.
2. The corresponding solutions. When writing the solutions, it would be very helpful if you could write them on printer paper. It'll make it easier for me to scan them in.

Ideally, all of this will be posted on Sakai during the morning of Tuesday, July 3, so you have plenty of time with them to review.

Topics

This is my initial outline of topics and how many people will be working on each. If you want to move these around, we can do that.

1. Verifying solutions (1 person)
2. Separable Equations (1 person)
3. Slope Fields + Existence and Uniqueness (1 person)
4. Euler's Method (1 person)
5. Autonomous Equation (1 person)
6. Bifurcations (1 person)
7. Linear Equations (1 person)
8. Linear Systems - Phase Portraits (4 people)
9. Trace-Determinant Plane and One-Parameter Systems (2 people)
10. Matrix Exponentials (1 person)
11. Second Order Equations - Undetermined Coefficients (2 people)
12. Mass on a Spring system - General Behavior of Solutions (2 people)
13. Mass on a Spring - Beats and Resonance, Amplitude of Steady State (2 people)
14. Non-Linear Systems (4 people)
15. Modeling with Equations (2 people)
16. Modeling with Systems (1 person)

A.5 Fall 2018 - Math 104

This section contains the following documents from my Fall 2018 section of Math 104:

1. The course syllabus
2. A worksheet I designed about counting problems, which was built from previous work with Pascal's Triangle
3. A worksheet I designed introducing the idea of continuous distributions and the normal distribution
4. A sample practice problem that students were given at the end of class
5. A sample "small quiz" that was given to students the class after turning in a homework set
6. A sample "big quiz" that was given to cover material from about two weeks worth of lectures
7. A sample homework solution that was given to the grader

MATH 104 - Section 02 - Fall 2018

Contact Information

Name: Matt Charnley
Email: charnley@math.rutgers.edu
Office: Hill 606

Office Hours

Mondays, 1:00 - 2:30 PM
Fridays, 10:30 AM - 12:00 Noon
or by appointment.

Prerequisites

Intermediate Algebra at the level of Rutgers Math 026 or 027, or equivalent. A real mastery of elementary algebra and other basic skills is crucial to success in this course.

Course Description

Math 104 gives a mathematical yet accessible and concrete introduction to probability. Most of the course is devoted to understanding how probability works, and how it is applied in a number of areas, including medical testing and financial decision making. The end of the course introduces some statistics and applications. You will never be left wondering, "What is this good for? What does this have to do with real life?"

Math 104 is not intended for majors in STEM fields. Credit will not be granted to those who have already taken, or are concurrently taking, a higher level probability course (including but not limited to 01:640:477, 01:198:206, 01:960:381, 14:332:226, and 14:540:210).

Learning Goals

Over the course of this semester, students will learn how to answer the following general questions:

1. What is probability and where in my life can it be used?
2. What is expected value and how does/should it affect what I decide to do?
3. How can I compute probabilities?
4. What is the normal distribution and what is it good for?

In particular, after this course, students will be able to

- understand the difference between theoretical and experimental probability
- understand expected value as it arises in a variety of real-life contexts, and understand its role in decision making
- use Venn diagrams and other related tools to compute probabilities in a variety of contexts
- recognize and avoid common fallacies in probability, including the gambler's fallacy and the base rate fallacy
- understand the relevance of and rationale behind probability simulations, and carry them out
- understand the difference between odds and probability, and convert between them

- understand independent and dependent events, recognize the difference between them, and carry out relevant computations
- understand conditional and unconditional probabilities, recognize the difference between them, and carry out relevant computations
- use the binomial probability model to compute probabilities in a range of settings
- have a concrete understanding of the rationale behind Pascal's triangle, the meaning of its entries, and the reasons why they appear throughout this subject
- work with z-scores and probability tables or the standard normal distribution, to compute probabilities of events arising from the normal distribution
- understand p-values as applied to experimental results and use this to carry out a hypothesis test using given data
- interpret a broad range of real life settings to which probability is relevant, and interpret what people mean when they make statements related to probability
- understand why they should resist the temptation to go to a casino, or gamble online, or even play the lottery

SAS Core Curriculum Learning Goals

Math 104 fulfills both the Quantitative Information (QQ) and Mathematical or Formal Reasoning (QR) learning goals of the SAS Core Curriculum:

QQ: Formulate, evaluate, and communicate conclusions and inferences from quantitative information.
QR: Apply effective and efficient mathematical or other formal processes to reason and to solve problems.

Textbook

Finite Mathematics: An Applied Approach, by Michael Sullivan, 11th Edition customized for Rutgers University, available at the bookstores. The customized edition contains only the chapters which are used in the course.

The 10th Edition is no longer used in this course; you really do need the 11th. WileyPlus is no longer used in this course. No solution manual is required. The custom edition of the textbook can also be bought as an e-book, at `https://store.vitalsource.com/show/9781119947172`.

Calculator

You will need a scientific calculator for homework. Calculators will be permitted on half of the exams; see the exam section for more information.

Class Meetings

Tuesday/Thursday, 1:40 - 3:00 PM, Tillett 207

Office Hours

I will be holding office hours in my office, Hill 606, on Busch campus. My current schedule for office hours is Mondays, 1:00 - 2:30 PM and Fridays, 10:30 AM - 12:00 Noon. Any changes to this schedule will be announced in class and on Sakai. I will also be holding informal office hours right after class both Tuesdays and Thursdays, and will have office hours by appointment. If the times I have scheduled do not work for you, feel free to send me an email and we can work out a different time to meet. Do not be shy about asking! All appointments should be scheduled at least 24 hours in advance. There will also be unofficial office hours in the Sakai Chat Room. You can post questions there for your fellow students or me to answer. I will check in the Chat Room around once a day (maybe more) to answer any questions there that are left unanswered.

Email and Contact

My email is charnley@math.rutgers.edu, and you should feel free to email me whenever you have questions or concerns about this class. I will respond to all emails within two business days, and if not, you are free to email me again to make sure I respond. However, I will not be checking my email constantly, and I make no guarantee that I will respond to emails after 6pm each evening. Since homework is due at 11pm, that means that if you have questions you want answered, you need to have worked on the homework in advance to ask them before 6pm on the day homework is due. I still might respond, but it is also possible that you will get a response the next morning. I will also try to respond to emails over the weekend, but there are some weekends where I may be travelling, and will be unable to respond.

In addition to teaching you probability, it is also part of my job to make sure you all are ready to enter the workforce, in whatever capacity that is, once you leave here. Part of that is being able to send professional emails that also get your point across and get the information that you want. A few pointers towards this end:

- All emails you send about course-related material should be sent from your Rutgers email (ScarletMail).
- All of your emails should have a subject line and some sort of salutation at the beginning.
- All emails should be written in complete sentences, and slang terms should be avoided.
- You should sign your name at the bottom of the email.

Online Resources

This course uses Sakai, accessible at `sakai.rutgers.edu`; login with your ordinary Rutgers NetID and password. Use Sakai to view announcements, submit homework via the Assignments feature, view solutions to homework problems via Resources, access the textbook, and participate in online discussions about the subject matter of the course in the Chat Room. You are also expected to check your Rutgers email account frequently, since that is the account to which Sakai sends all messages, and which your professors are expecting you to check. If you wish to set up email forwarding from that account to a different one, you can do so, but it is your responsibility to ensure that it works properly.

Attendance

You are expected to attend all class meetings, whose content will go far beyond a mere rehashing of the textbook. The classroom setting will involve a fair amount of active learning, and your full participation and engagement are necessary for you to learn effectively. **If you entertain the notion that you can succeed in the course by learning the material on your own, please think again.**

Grade Breakdown

Final grades for the class will be decided according to the following breakdown:

Homework and Quizzes	20%
Midterm Exams	40%
Final Exam	40%

Exam Schedule

- Midterm 1: October 9, 2018, 1:40 - 3:00 PM
- Midterm 2: November 13, 2018, 1:40 - 3:00 PM
- Final Exam: December 17, 2018, 12:00 noon - 3:00 PM

Homework

There will be typically be a homework assigned after each class. Due dates for each assignment will be indicated on Sakai. Typically, you should expect 1-2 assignments due each week. These will generally be due on Tuesday and Saturday nights at 11:00 PM. **All written work must be submitted online via the Assignments tab of Sakai; it may not be submitted on paper, nor by email. There will be NO exceptions!**

Since the solutions to each homework assignment will be posted in the Resources section of Sakai shortly after the assignment is due, **late homework will not be accepted!** Finally, doing the homework is crucial to learning the subject thoroughly, and the system of electronic submission makes it possible for you to get feedback quickly on whether you have done the homework correctly.

Important guidelines for submitting homework:

1. To access Sakai, be sure that you have an active email account and know your NetID and password.
2. The order of the homework problems in your submitted work should be the same as the order in which they are listed in the assignment on Sakai.
3. If you upload a file, it must be in one of the following formats: .doc, .docx, .pdf, or .jpg. Files which are not in this format, and which the instructor cannot open, will be given a grade of 0.
4. If you need to scan your homework assignment, the campus computer labs and libraries do have scanners, and there are technical assistants there if you need help. Note: Scanned documents should be saved as pdf files.

5. A popular technique for producing images of handwritten work is to photograph each page, or each half of each page, using a cell phone. This is acceptable.
6. It is entirely your responsibility to make certain that the file you upload has the appearance you intend. Please check this by opening the file after you have uploaded it, and see that it does open, that the image is right side up and generally readable, etc. If the file format is incorrect or the image is blurry, you will be given a grade of 0.
7. If you ever have technical difficulties with Sakai, especially in uploading homework, please contact the excellent and very responsive Sakai help desk at sakai@rutgers.edu or 848-445-8721.
8. Do not wait until the last hour to upload your homework, since the system may be unexpectedly busy.
9. Whether you upload a file or type your answers into the textbox, be sure to hit submit at the bottom of the screen to submit your assignment.
10. You will very quickly get a confirmation email from the Sakai system that your assignment was submitted. Make sure that you get this confirmation email, because it is your responsibility to make sure that your submission goes through.
11. If you do not receive the confirmation email, your work probably did not go through, in which case you should log back in to confirm that your work was posted, and if it wasn't posted then you should resubmit.

Unless specified otherwise, **you must write your answers in complete, grammatically correct English sentences**. Being able to do this is a crucial aspect of quantitative literacy, which goes beyond mere computational proficiency.

You are permitted, and in fact encouraged, to work together on homework problems, but all written work which you submit must ultimately be your own. It is also super important for you to stay on top of the homework, and ideally you should be working on the homework a little bit every day. This will allow you to better digest the material and get a chance to ask your questions well in advance of the due date for the homework.

Quizzes

Students should expect a quiz each class day. Thus, attendance is crucial. If you have completed and understood the homework (both the graded and ungraded homework), you should not find the corresponding quiz terribly difficult. These quizzes will take place at the start of each class (after about 5 minutes for questions) on the most recent homework that has been turned in, and will be done without calculators. Please note that **there are no make-up quizzes!** However, I do intend to drop at least one of the lowest quiz grades. There will also be practice problems at the end of each class that you will need to turn in before you leave. These will count towards a portion of your homework grade, but if you do not turn in a problem, your quiz grade for that day will be affected. Therefore, it is necessary for you to stay until the end of each class meeting. There will also be 5 larger quizzes over the course of the semester that will take place at the end of class, covering the previous two weeks of material. These will replace the quiz at the start of class, be announced in advance, and included on the course schedule.

Exams

There will be two "midterm" exams and a comprehensive, cumulative final. The exams will be closed book and student-prepared formula sheets will **not** be permitted. The exams will have two parts; the first of which will be done without calculators, and the second will allow the use of calculators. As the first exam approaches, there will be another handout with more details on how the exam will work. **If you dont show up to an exam, you will receive a zero!**

Missing an exam is a serious matter, and should only occur as a result of a genuine, verifiable emergency situation. "Verifiable" means that there should be a doctor's note, notice of court appearance, etc. indicating that you were unable to attend at the time of the exam. If circumstances beyond your control prevent you from attending an exam, it is important that you contact the instructor as quickly as possible.

Makeups

There are no makeup quizzes or exams. As noted above, under truly compelling circumstances an absence from a quiz or exam can be excused, but instead of having a makeup, the portion of the final exam pertaining to the content of the missed midterm exam will count correspondingly more heavily.

Classroom Setting

Using your cell phone or other device to send or view texts, or to surf the internet, or for any other purposes not directly related to your in-class work, is not acceptable. Doing so is very distracting to you and to others, and is truly inappropriate. This is excellent news for you, and everyone else, since you have all paid a significant amount of money to be here, and there are ample opportunities for conversation and texting and social media outside of class time.

Academic Integrity

All Rutgers students are expected to be familiar with and abide by the academic integrity policy (`http://academicintegrity.rutgers.edu/policy-on-academic-integrity`). Violations of this policy are taken very seriously. During exams, cell phones, tablets, laptops and any other WiFi or cellular capable devices must be turned off (not just silenced), and completely put away; **having a cell phone, tablet, or laptop visible during the exam will automatically be reported as an academic integrity violation, with a minimum penalty of receiving a 0 on the exam.** Moreover, during an exam, if you leave the room you must turn in your exam paper, and will not be able to return to continue working on it.

Extra Help

If you are having difficulty, please take advantage of the opportunity to visit office hours. Also, please do not hesitate to ask questions by email, or in the Sakai Chat Room. The Rutgers Learning Centers also provide drop-in, free tutoring for Math 104, according to a schedule accessible at `http://lrc.rutgers.edu/content/tutoring`.

A few friendly words of advice:

1. Never fall behind in a math course!!!!! The ideas we will discuss need time to sink in, and are very difficult to learn quickly right before an exam, so it is important to clear up your confusions sooner rather than later.
2. An excellent way to improve your understanding of the subject is to study together with classmates. Explaining mathematical ideas to others is often the most effective way to sort out your own confusions and clarify your understanding; you don't know just what it is that you don't know until you try explaining it to someone else.
3. You are also warmly invited to ask questions in class, which students are far too hesitant to do in math courses, or in office hours, or by email, or in the Sakai Chat Room! I very much want you to succeed in this course.

Disability Accommodations

Rutgers University welcomes students with disabilities into all of the University's educational programs. In order to receive consideration for reasonable accommodations, a student with a disability must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: `https://ods.rutgers.edu/students/documentation-guidelines`. If the documentation supports your request for reasonable accommodations, your campuss disability services office will provide you with a Letter of Accommodations. Please share this letter with your instructors and discuss the accommodations with them as early in your courses as possible. To begin this process, please complete the Registration form on the ODS web site at: `https://ods.rutgers.edu/students/registration-form`.

Adjustments

All information in this syllabus is subject to change at any time. Any changes will be announced on Sakai, changed on this document, and announced in class.

Student Wellness Services

Just In Case Web App

http://codu.co/cee05e

Access helpful mental health information and resources for yourself or a friend in a mental health crisis on your smartphone or tablet and easily contact CAPS or RUPD.

Counseling, ADAP & Psychiatric Services (CAPS)

(848) 932-7884 / 17 Senior Street, New Brunswick, NJ 08901/ www.rhscaps.rutgers.edu

CAPS is a University mental health support service that includes counseling, alcohol and other drug assistance, and psychiatric services staffed by a team of professional within Rutgers Health services to support students efforts to succeed at Rutgers University. CAPS offers a variety of services that include: individual therapy, group therapy and workshops, crisis intervention, referral to specialists in the community and consultation and collaboration with campus partners.

Violence Prevention & Victim Assistance (VPVA)

(848) 932-1181 / 3 Bartlett Street, New Brunswick, NJ 08901 / www.vpva.rutgers.edu

The Office for Violence Prevention and Victim Assistance provides confidential crisis intervention, counseling and advocacy for victims of sexual and relationship violence and stalking to students, staff and faculty. To reach staff during office hours when the university is open or to reach an advocate after hours, call 848-932-1181.

Scarlet Listeners

(732) 247-5555 / http://www.scarletlisteners.com

Free and confidential peer counseling and referral hotline, providing a comforting and supportive safe space.

Tentative Course Schedule

Date	**Material**	**Textbook Sections**	**Deadlines**
September 4	Intuitions, Probability, Sample Space, Events	Section 7.4	
September 6	Real-World Examples, Relative Frequency, Experimental vs. Theoretical Probability	Notes	
September 11	Expected Value, Events as subsets	Section 7.6	
September 13	Basic Set Operations and Probabilities	Section 7.1	
September 18	Inclusion/Exclusion	Section 7.2	Big Quiz 1
September 20	Multiplication Rule and The Birthday Problem	Section 7.3	
September 25	Odds, Expected Value and Decision Making	Section 7.5	
September 27	Pascal's Triangle	Notes	
October 2	Pascal's Triangle	Notes	Big Quiz 2
October 4	Catch up and Review		
October 9	Midterm 1		
October 11	Binomial Coefficients	Section 8.5	
October 16	Binomial Probability Model	Section 8.6	
October 18	Expected Value for Binomial	Section 8.5	
October 23	Conditional Probabilities and the Monty Hall Problem	Section 8.1	Big Quiz 3
October 25	Conditional Round 2, False Positives, Law of Total Probability	Section 8.3	
October 30	Conditional Round 3, Independent events	Section 8.2	
November 1	Conditional Round 4, Tree Diagrams and Contingency Tables	Sections 8.1-8.3	
November 6	Conditional Binomial Probabilities	Sections 8.1-8.3, 8.6	Big Quiz 4
November 8	Catch up and Review		
November 13	Midterm 2		
November 15	Simpson's Paradox, Mean, Standard Deviation	Section 9.4	
November 20	Binomial to Normal Approximation	Section 9.6	
November 27	Empirical Rule and Z-Scores	Section 9.5	
November 29	Using Z-scores and Z-tables	Section 9.5	
December 4	Hypothesis Testing and p-values	Notes	
December 6	Hypothesis Testing Round 2	Notes	Big Quiz 5
December 11	Catch up & Review		
December 17	Final Exam		

Worksheet 8 - Binomial Coefficients

October 11, 2018

Name: ______________________________

So, it turns out that those $C(n,k)$ numbers that came out of Pascal's Triangle that I called 'binomial coefficients' are a lot more prevalent across math than I may have let on. Today, we're going to start an exploration into combinatorics to see what these numbers can do. A lot of this will pull back to the multiplication principle from before.

Example 0.1.

1. How many ways are there to get 5 heads in 12 flips of a coin?

2. How many ways are there to write a 12 letter word consisting of only H and T so that there are 5 Hs?

3. How many ways are there to determine and answer key to a 12 question true-false test with exactly 5 true answers?

4. How many ways are there to choose a committee of 5 people from 12 eligible members?

5. How many ways are there to pick 5 numbers out of 12 in a lottery drawing?

It turns out that these are all the same number! That's not a coincidence; they are all counting the same thing in different contexts. These last two give an indication as to why we read $C(n, k)$ as n choose k, as in we are picking k things out of a set of n. In terms of the coin flips, you are basically choosing which of the flips are going to be heads or tails.

Example 0.2. 1. How many ways are there to draw a 5 card hand out of a 52 card deck?

2. How many ways are there to give 6 identical prizes to different people in a group of 40?

3. How many ways are there to draw 20 numbers from a lottery pool of 80?

Let's consider that last one and look to develop a formula for these $C(n,k)$ that does not depend on using Pascal's Triangle. Let's reduce to 12 balls, choosing 5 of them, for simplicity.

Example 0.3.

1. How many ways are there to select the first ball? What about the second?

2. How many ways are there to pick 5 balls out of the group of 12 where the order of selection matters?

3. Now say that I want to forget about the order. How many 'ordered' selections are equivalent to the same 'unordered' selection?

4. How many ways are there to select the 5 lottery balls from a group of 12?

Using the notation that

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

we can write this as

$$C(12,5) = \qquad\qquad \text{or} \qquad\qquad C(n,k) =$$

Computationally, we can see that this is the same as the actual formula for $C(n, k)$ that is commonly used. Let's provide a full motivation for this formula.

Example 0.4.

1. Put all 12 balls in an order:

2. Draw a line between the chosen and unchosen ones:

3. Get rid of the ordering:

Now, let's look at one other application of thinking like what we have been doing so far today:

Example 0.5. 1. How many ways are there to order the letters in the word MUSIC?

2. What about HAPPY?

3. What about LIBRARY?

4. What about MISSISSIPPI?

These binomial coefficients can help count so many more things in a variety of different contexts. Here's another worked out example to illustrate this.

Example 0.6. Assume that you have a group of 15 people, 7 men and 8 women, from which to make a committee of 5 people.

1. How many ways are there to make the committee?

2. How many ways are there to make the committee if it only consists of women?

3. How many ways are there to make a committee that contains 2 men and 3 women?

4. How many ways are there to make a committee of 2 men and 3 women that does not contain Mr. Jones?

5. How many ways are there to make a committee of 2 men and 3 women if Mr. and Mrs. Jones will not serve on the committee together?

6. How many ways are there to make a committee containing either 2 men and 3 women, or 3 men and 2 women?

7. What is the probability of ending up with a committee of 3 men and 2 women if the committee is chosen at random?

In case we don't get far enough in class, here are the answers to the previous example.

1. This is the same as the problems we have considered so far in class. You are picking a committee of 5 people from 15, so the answer is C(15,5).

2. For this problem, instead of having the entire group of 15 to choose from, we need to choose only from the women, which gives us 8 options. Therefore, the number of ways to do this is C(8,5).

3. In this case, we need to pick 2 of the men to serve on the committee and 3 of the women to serve. This goes back to our multiplication principle from earlier: The number of ways to pick the full committee should be the number of ways to pick the men *times* the number of ways to pick the women. Since there are 7 men and we are picking 2 of them, there are C(7,2) ways to pick the men, and similarly there are C(8,3) ways to pick the women. Thus, the total number of ways to choose the committee under these conditions is
$$C(7,2) \cdot C(8,3)$$

4. If Mr. Jones will not be on the committee, we just have one less man taking part in the selection process. Therefore, we will be choosing 2 men from 6, and 3 women from 8. This gives a total number of ways of doing this as
$$C(6,2) \cdot C(8,3)$$

5. This problem can be solved in two ways. First we could add up the following three options: Mr. Jones is on the committee but Mrs. Jones is not, Mrs. Jones is on the committee but Mr. Jones is not, and neither of them is on the committee. If we want to force someone to be on the committee, we basically pick them first, and then choose the rest of the committee from the remaining people. For instance, if we want to force Mr. Jones to be on the committee, then the number of ways to pick the men on the committee is
$$C(1,1) \cdot C(6,1)$$
where we have first chosen Mr. Jones to be on the committee, and then picked one of the remaining 6 men to fill the other seat. In this case, we want Mrs. Jones to not be on the committee, so she needs to sit out, leaving C(7,3) ways to fill the women's seats on the committee. If we work this out for the other two cases, we get the total number of ways here is
$$C(1,1) \cdot C(6,1) \cdot C(7,3) + C(6,2) \cdot C(1,1) \cdot C(7,2) + C(6,2) \cdot C(7,3).$$
The first term is Mr. Jones on the committee, the second is Mrs. Jones on the committee, and the last is neither. We could also compute this solution by looking at the total number of ways to organize the committee, and subtracting off all committees where both Mr. and Mrs. Jones are on it. The total number of ways was part 2 above, and the number of ways with Mr. and Mrs. Jones both on it means that we need to pick each of them for their respective sexes, and then fill the rest of the committee. This can be done in
$$C(1,1) \cdot C(6,1) \cdot C(1,1) \cdot C(7,2) = C(6,1) \cdot C(7,2)$$
ways. Therefore, the total number of ways to not have both Mr. and Mrs. Jones on the committee is
$$C(7,2) \cdot C(8,3) - C(6,1) \cdot C(7,2)$$

6. For this part, we need to add the number of ways of having a committee with 2 men and 3 women and the number of ways to have a committee with 3 men and 2 women. This is given by
$$C(7,2) \cdot C(8,3) + C(7,3) \cdot C(8,2)$$

7. For this, we just need to take the number of ways to get this committee and divide it by the total number of ways the committee can be formed. This is
$$\frac{C(7,3) \cdot C(8,2)}{C(15,5)}$$

Worksheet 12 - The Normal Distribution and Z-Scores

November 27, 2018

Name: ____

1 Continuous Distributions

Last time we talked about binomial distributions and saw that we could represent them by graphs like this:

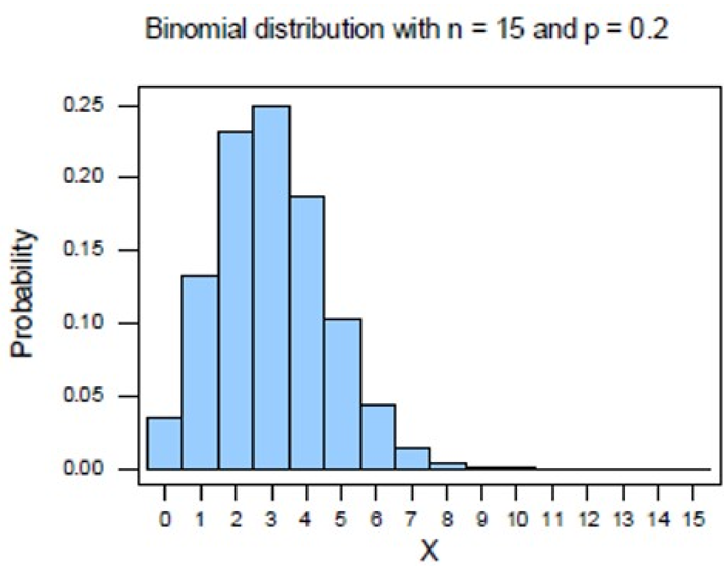


Figure 1: From: https://onlinecourses.science.psu.edu/stat414/node/70/

For this picture, we have

1. Heights of the bars:

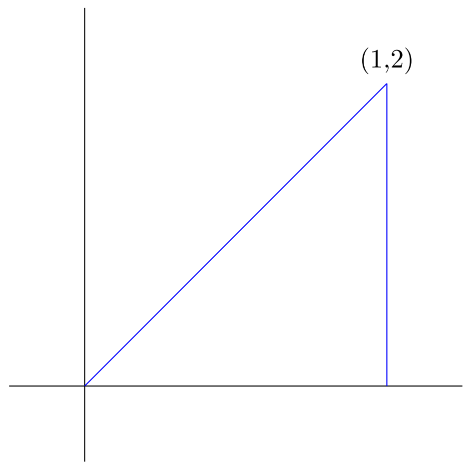
2. Finding the probability of some event:

3. Total area under the bars:

With this sort of picture, there's no reason why we need to have this be a histogram; we could do the same thing for any curve. The important thing we need is that the curve is always above the x-axis, and the total area under the curve is 1. We then find probabilities of events by computing the area under the curve on the appropriate range.

Example 1.1.

Example 1.2. Consider the distribution defined by the function $y = 2x$ on $0 \leq x \leq 1$.



1. Verify that the total area is 1.

2. What is the probability that an outcome here is less than 1/2?

3. What is the probability that an outcome here is greater than 2/3?

4. What is the probability that an outcome is between 1/3 and 3/4?

2 Normal Distribution

Now, we want to do the same as before with a different function:

$$y = \frac{1}{2\pi} e^{-\frac{x^2}{2}}$$

whose graph looks like

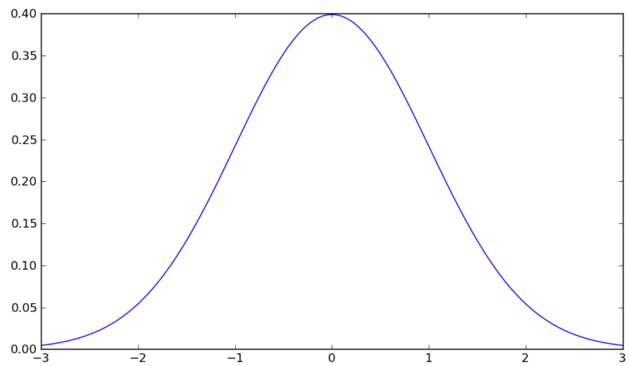


Figure 2: From https://stackoverflow.com/questions/10138085/python-pylab-plot-normal-distribution

Properties of this distribution:

With this, we can draw a diagram similar to the Empirical Rule for the normal distribution

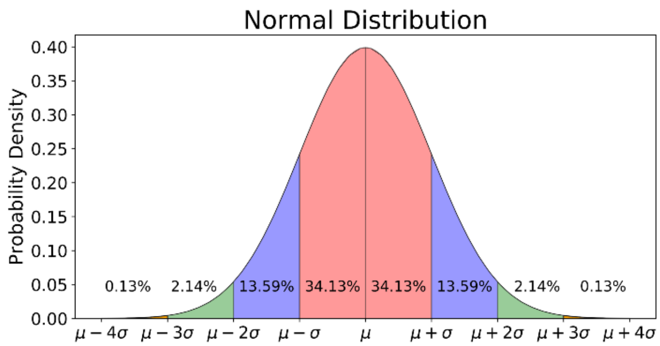


Figure 3: From https://towardsdatascience.com/understanding-the-68-95-99-7-rule-for-a-normal-distribution-b7b7cbf760c2

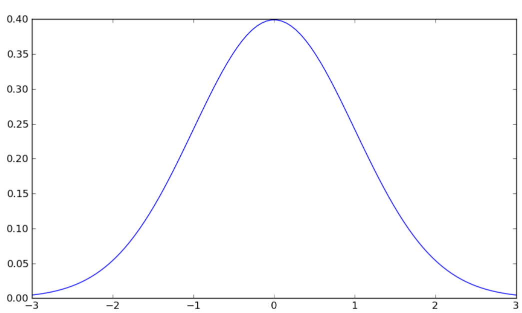
We can also think about a normal distribution with mean μ and standard deviation σ, which is defined by the graph of

$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

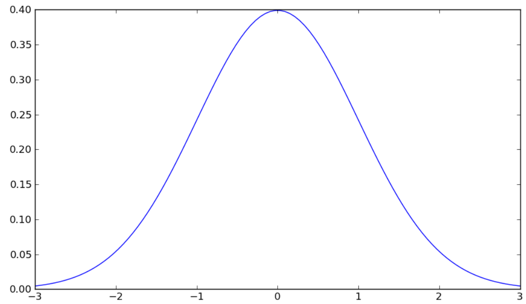
The empirical rule type numbers still apply, but now for a number of standard deviations away from the mean instead of the numbers from before.

Example 2.1. Assume that the mean height of a person is 70 inches with standard deviation 4 inches (this is probably not true), and that this is normally distributed (this is probably true).

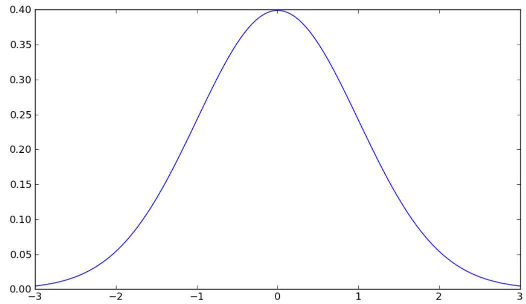
1. What proportion of people are between 66 and 74 inches tall?



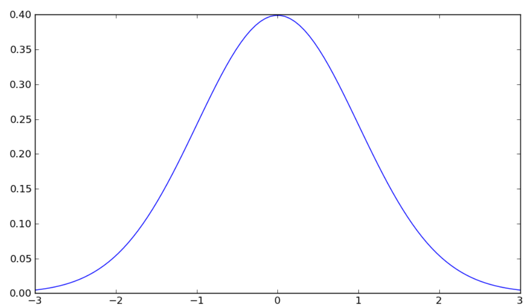
2. What proportion of people are between 62 and 70 inches tall?



3. If I pick a random person, what is the probability that they are between 66 and 78 inches tall?



4. If I pick a random person, what is the probability that they are more than 82 inches tall?



Now, to discuss the vague statement we made last time, about distributions being 'bell-shaped'. What we really mean is:

Example 2.2. Assume that you are going to look at a basketball player shooting free throws over the course of a season. Assume that this player shoots 300 free throws, and the probability of any one free throw being made is $\frac{3}{4}$, independent of all other shots.

1. How many free throws do you expect this player to make?

2. What is the standard deviation of the number of free throws made?

3. Write an expression for the exact probability that this player makes between 218 and 232, inclusive, free throws this season?

4. We know that this data is approximately normally distributed. Find the probability of this same quantity using the normal distribution.

3 Z-Scores

Why do we care about the normal distribution?

Why is the normal distribution nice?

Definition 3.1. Assume that we get a result of x from data that is normally distributed with mean μ and standard deviation σ. The **Z-score** associated to this value x is:

Example 3.1. Assume that scores are normally distributed with mean 70 and standard deviation 5. Find the Z score of the results 87, 68, 75.

Example 3.2. Under the same conditions, what is the probability that a student scores more than 75?

What about more than 78?

Z-tables:

Example 3.3. Assume that heights are normally distributed with mean 60 and standard deviation 7. What is the probability that someone is over 65 inches tall? What about that they are between 56 and 68 inches tall?

Example 3.4. Assume that a basketball player shoots 500 free throws over the course of a season, and the probability of them making each shot is 0.8, independent of each other attempt.

1. Find the mean and standard deviation of this distribution.
2. Use the normal approximation to compute the probability of the shooter making more than 420 free throws in the season.
3. Use the normal approximation to find the probability of the shooter making between 395 and 413 free throws over the course of the season.

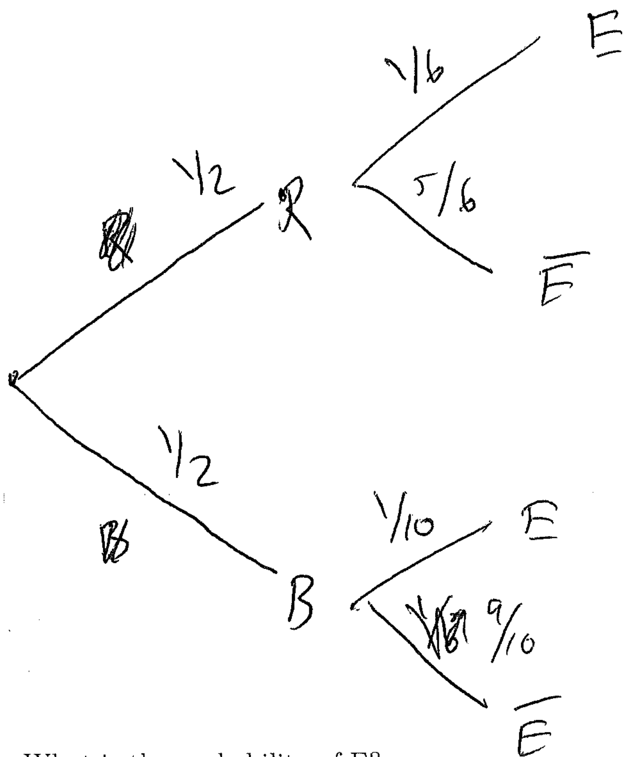
MATH 104 - Practice 11

October 25, 2018

Name: Key

Consider a two part experiment. In the first step, you draw a card out of a standard 52 card deck. If the card is red, you roll a fair 6-sided die, and if the card is black, you roll a fair 10-sided die. In terms of the die roll, we care about the event E = "the die roll is a 1."

1. Draw a tree diagram for this experiment, writing all probabilities on the branches.



2. What is the probability of E?

$$\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{10}=\frac{1}{12}+\frac{1}{20}$$

$$=\frac{5}{60}+\frac{3}{60}=\frac{8}{60}$$

$$=\underline{\frac{2}{15}}$$

MATH 104 - Quiz 10

October 25, 2018

Name: Key

You are playing a game where you are going to roll a fair 6-sided die 10 times, and will win based on the number of 6's you see.

1. What is the expected number of times you will see a 6 in the 10 rolls?

$$10 \cdot \frac{1}{6} = \frac{10}{6} = 5/3$$

2. Suppose the game has the following payouts:

Number of 6's	0-4	5	6-9	10
Winnings	\$0	-\$2	\$1	\$15

Write an expression for your expected winnings from this game.

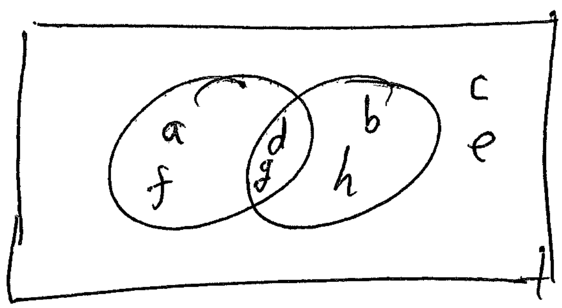
$$(-2)\left(\binom{10}{5}(1/6)^5(5/6)^5\right) + 1\left(\binom{10}{6}(1/6)^6(5/6)^4 + \binom{10}{7}(1/6)^7(5/6)^3 + \binom{10}{8}(1/6)^8(5/6)^2 + \binom{10}{9}(1/6)^9(5/6)^1\right) + 15 \cdot \binom{10}{10}(1/6)^{10}(5/6)^0$$

MATH 104 - Quiz 7

October 2, 2018

Name: Key

1. Let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, d, f, g\}$ and $B = \{b, d, g, h\}$. Find each of the following sets. Hint: Constructing a Venn Diagram may be helpful.



(a) $A \cap B$

$\{d, g\}$

(b) $\bar{B}$

$\{a, c, e, f\}$

(c) $\bar{A} \cup B$

$\{b, c, d, e, g, h\}$

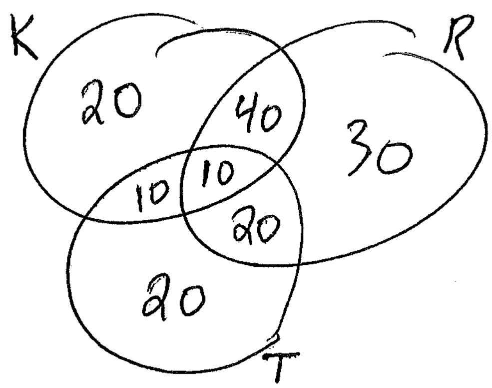
(d) $\overline{A \cup B}$

$\{c, e\}$

2. You survey 200 students as to what types of candy they like. You find that:

- 80 like Kit-Kat bars
- 100 like Reese's
- 60 like Twix
- 50 like both Kit-Kat and Reese's
- 30 like both Reese's and Twix
- 20 like both Kit-Kat and Twix
- 10 like all three.

(a) Construct a Venn Diagram for this situation.



(b) How many of the students surveyed liked none of the three candy bars?

$20+40+30+10+10+20+20 = 150$

$200 - 150 = \boxed{50}$

(c) How many students liked exactly 2 of the options?

70

(d) How many students liked only Kit-Kat bars out of the three options?

20

3. Consider developing a 6 character username that is only made up of lowercase letters.

   (a) How many such usernames are possible?

   26^6

   (b) How many are possible that do not repeat any letters?

   $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21$

   (c) How many are possible so that adjacent letters are not repeated? (that is, you can not have two consecutive letters that are the same)

   $26 \cdot 25 \cdot 25 \cdot 25 \cdot 25 \cdot 25$

4. Let U be the set of all students at Rutgers, A the set of all sophomores, and B the set of engineering majors. Using proper set builder terminology, describe the sets $A \cup \bar{B}$ and $A \cap B$.

$A \cup \bar{B}$ is the set of all students who are either sophomores or not engineering majors.

$A \cap B$ is the set of all students who are both sophomores and engineering majors (sophomore engineering majors)

5. (a) Assume that the odds of rolling a 4 on an unfair die are 4:1 against. What is the probability of rolling a 4?

$$\frac{1}{4+1} = \frac{1}{5}$$

(b) If the probability of rolling a 3 on this die is $\frac{2}{7}$, what are the odds for rolling a 3?

$$\frac{P(3)}{P(\bar{3})} = \frac{2/7}{5/7} = \frac{2}{5}$$

So the odds for rolling a 3 are $\boxed{2:5.}$

MATH 104 - Homework Set 8

Assigned: September 27, 2018

Due: October 2, 2018, 11 PM

Assigned Problems

1. Section 7.6, Problem 24. (10 points)

   The idea of this problem is to figure out how many ski sets to be stored will give the highest expected value. We need to compute the expected value for each number individually. The point is, if we do not have enough sets for the customers that show up, we lose out on the profit. That is, if we only store 92 sets of skis and 94 customers show up, we can only rent 92 skis and lose out on the potential extra profit for the last two customers. For example, if we store 93 skis, we compute the expected value as follows:

   - If 90 customers show up, our profit is $90 \times \$20 = \$1800$, and this occurs with probability 0.01.
   - If 91 customers show up, our profit is $91 \times \$20 = \$1820$, and this occurs with probability 0.10.
   - If 92 customers show up, our profit is $92 \times \$20 = \$1840$, and this occurs with probability 0.20.
   - If 93 or more customers show up, our profit is $93 \times \$20 = \$1860$, and this occurs with probability 0.69, because we can only rent up to 93 sets of skis, because that's all we have in stock.

   In all of these cases, our expenses are $93 \times 6 = \$558$. Thus, our expected value is

   $$E_{93} = \$1800 \cdot 0.01 + \$1820 \cdot 0.10 + \$1840 \cdot 0.20 + \$1860 \cdot 0.69 - \$558 = \$1293.40$$

   Computing this for all possible numbers of skis, we find that

Number of Skis	90	91	92	93	94	95	96	97
Expected Value	\$1260	\$1273.80	\$1285.60	\$1293.40	\$1297.20	\$1295	\$1290.80	\$1285.60

   Therefore, the optimal number of skis to store is 94, because that has the largest expected value.

   Grader Comments: 4 points for setting up the expected value calculations correctly. 4 points for calculating all of the expected values in the table. 2 points for the final answer.

2. Section 7.6, Problem 27. (6 points)

(a) If the probability of success at each location is $\frac{1}{2}$, then the expected value for the first location is

$$\frac{1}{2}(\$15000) + \frac{1}{2}(-\$3000) = \$6000$$

and the expected value for the second location is

$$\frac{1}{2}(\$20000) + \frac{1}{2}(-\$6000) = \$7000$$

so the company should choose the second location.

(b) If the probability of success at the first location is $\frac{2}{3}$ and it is $\frac{1}{3}$ at the second, the expected value of the first location is

$$\frac{2}{3}(\$15000) + \frac{1}{3}(-\$3000) = \$9000$$

and the expected value for the second location is

$$\frac{1}{3}(\$20000) + \frac{2}{3}(-\$6000) = \$2666.67$$

and so the company should choose the first location.

Grader Comments: Each expected value computation gets 1 point. The final answer for each (a) and (b) gets another point.

3. There are three different games you could play involving rolling two fair 6-sided dice. The games are as follows:

   A. Pay \$6, and win the amount of money equal to the sum of the number of the dice.

   B. Pay \$8, and win twice the number on the larger of the two dice.

   C. Pay \$2, and win an amount of money equal to the smaller of the values of the two dice.

   For both B and C, if the die rolls are the same, you win the amount of money corresponding to that number. That is if you roll two fours, you would win 8 dollars in B and 4 dollars in C. Which game should you play? Hint: Start by writing out the sample space of this experiment in a table. (12 points)

   The sample space for this experiment is

	1	**2**	**3**	**4**	**5**	**6**
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Using this table, we can build tables for each of the three games, computing the amount we win for each roll. For game A,

\$	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

for game B

\$	**1**	**2**	**3**	**4**	**5**	**6**
1	2	4	6	8	10	12
2	4	4	6	8	10	12
3	6	6	6	8	10	12
4	8	8	8	8	10	12
5	10	10	10	10	10	12
6	12	12	12	12	12	12

and for game C

\$	**1**	**2**	**3**	**4**	**5**	**6**
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

Note that these tables do not account for the amount of money you need to pay for the game. The remainder of this calculation can be done in two ways: You can either compute the expected value of the game without worrying about how much you need to pay and subtract the cost off at the end, or you can subtract the cost from every entry in the table and then compute the expected value. Either way will get you to the same final answer. To find the expected value here, since every outcome in the table above has probability $\frac{1}{36}$, we can just add up all of the entries in the table and divide by 36 (like finding the mean of an ideal data set). This will give the expected value for game A as

$$\frac{2(1)+3(2)+4(3)+5(4)+6(5)+7(6)+8(5)+9(4)+10(3)+11(2)+12(1)}{36} - 6 = \$1$$

while for game B, we get

$$\frac{2(1) + 4(3) + 6(5) + 8(7) + 10(9) + 12(11)}{36} - 8 = \frac{322}{36} - 8 = 8.94 - 8 = \$0.94$$

and for game C

$$\frac{1(11) + 2(9) + 3(7) + 4(5) + 5(3) + 6(1)}{36} - 2 = \frac{91}{36} - 2 = 2.53 - 2 = \$0.53$$

So, while all of these games are positive for the player, game A has the highest expected value, and is the one you should choose to play.

Grader Comments: 2 points for the sample space (this doesn't have to be written explicitly), 3 points for getting the expected value of each game, 1 point for the final conclusion.

Appendix B

Full Student Feedback

Included are the full SIRS student feedback surveys for each of the classes for which I was the instructor. They have each been summarized in their corresponding sections, but the full results are shown here for reference. The classes are presented in the following order:

1. Summer 2015 - Math 251
2. Summer 2016 - Math 477
3. Summer 2017 - Math 244
4. Summer 2018 - Math 252
5. Fall 2018 - Math 104

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Summer 2015, 01:640:251:C1 — MULTIVARIABLE CALCULUS (index #01954) Enrollment= 23, Responses= 8 **Part A: University-wide Questions:**	**Student Responses**						**Weighted Means**			
	Strong Disagree 1				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner	0	0	0	1	7	0	**4.88**	4.66	4.51	**4.52**
2. The instructor responded effectively to student comments and questions	0	0	0	0	7	1	**5.00**	4.56	4.54	**4.49**
3. The instructor generated interest in the course material	0	0	0	1	7	0	**4.88**	4.40	4.27	**4.40**
4. The instructor had a positive attitude toward assisting all students in understanding course material	0	0	0	1	7	0	**4.88**	4.63	4.65	**4.56**
5. The instructor assigned grades fairly	0	0	0	2	6	0	**4.75**	4.51	4.48	**4.45**
6. The instructional methods encouraged student learning	0	0	0	1	7	0	**4.88**	4.46	4.29	**4.37**
7. I learned a great deal in this course	0	0	1	1	6	0	**4.63**	4.40	4.38	**4.38**
8. I had a strong prior interest in the subject matter and wanted to take this course	0	1	1	3	3	0	**4.00**	3.86	3.68	**3.55**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as	0	0	0	2	6	0	**4.75**	4.37	4.37	**4.41**
10. I rate the overall quality of the course as	0	0	1	2	5	0	**4.50**	4.23	4.25	**4.27**

What do you like best about this course?:

"Matt was an awesome instructor and very approachable! Always had answers to our questions!"

"It's very useful for describing motions. And it's closely relevant to physics, which makes it easier to understand intuitively. "

"I liked his way of teaching the best. It was easy to understand, especially as we progressed to the more difficult material. "

"The professor "

"I love to see teachers who are passionate about what they are teaching it makes me wish I could enjoy the subject as much as they do. I liked that he would do his best to present the material in the best way possible given the time constraints."

If you were teaching this course, what would you do differently?:

"Nothing."

"I would not do anything differently if I were teaching the course."

"There is very few things I would do differently. Mr. Charnley was excellent in explaining all the concepts, presenting examples and applications, and answering any questions I had."

"not a thing"

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

"He is always showing us how even the hardest of topics can be tackled in the simplest of ways."

"He tries to make everybody at the class understand the materials."

"The instructor helped me understand the fundamental material so that when we started to cover the more difficult material, I did not have a hard time understanding it."

"Mr. Charnley had a great, positive attitude throughout the summer. For 8 AM, it was awesome to have someone with such a great attitude."

"I was just trying to pass so it was kind of hard to be encouraged intellectually when my mind was set on finishing this class without wasting the thousands of dollars that was spent on taking these summer classes."

Other comments or suggestions::

"Matt did a really great job teaching this summer math session. He was always encouraging and made sure everyone had a fair chance at asking their questions as well as doing a great job answering them. By far the best Calculus course I've taken here at Rutgers. "

"Mr. Charnley was an excellent instructor. He was, by far, the best instructor I've had here at Rutgers. I hope that he is teaching Differential Equations next semester, because he can explain any math concept very well."

"This course was not easy for me considering the last math class I took was damn near 2yrs ago. I forgot a lot of things that my classmates breezed over but I realize that I can't complain since all that I needed to succeed was at my disposal. If I had taken this class during the semester with Matt I'm sure I would have done well. If I didn't need this for P-chem I wouldn't have taken it. Side Note:I hate filling these things out bc they always show up when I'm supposed to be preparing for the final exam. *sighs*"

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Summer 2016, 01:640:477:B1 — MATHEMATICAL THEORY OF PROBABILITY (index #02048) Enrollment= 25, Responses= 14	Student Responses						Weighted Means			
Part A: University-wide Questions:	Strong Disagree 1				Strong Agree 5	No response	Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner.	0	1	0	1	12	0	**4.71**	4.74	4.88	**4.63**
2. The instructor responded effectively to student comments and questions.	0	0	0	3	11	0	**4.79**	4.74	4.84	**4.50**
3. The instructor generated interest in the course material.	0	0	3	1	10	0	**4.50**	4.62	4.75	**4.42**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	0	14	0	**5.00**	4.93	4.93	**4.61**
5. The instructor assigned grades fairly.	0	0	1	4	9	0	**4.57**	4.74	4.74	**4.50**
6. The instructional methods encouraged student learning.	0	0	0	4	9	1	**4.69**	4.77	4.73	**4.39**
7. I learned a great deal in this course.	0	0	0	4	10	0	**4.71**	4.74	4.79	**4.48**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	1	2	6	5	0	**4.07**	4.30	4.27	**3.67**
	Poor				Excellent					
9. I rate the teaching effectiveness of the instructor as:	0	0	1	3	10	0	**4.64**	4.59	4.77	**4.41**
10. I rate the overall quality of the course as:	0	0	1	5	8	0	**4.50**	4.48	4.68	**4.31**

What do you like best about this course?:

“in class practice problems that reinforced what the class learned that day”

“Even though this was an 8 a.m. summer class and many of the students were tired and distracted, the instructor seemed to put a lot of effort and enthusiasm into the material he was teaching. He offered a lot of office hours which were very helpful. ”

“Although there was little time to learn the material, I think I acquired an accurate understanding of the subject matter.”

“The teacher is really patient and passionate.”

“There are enough review materials for preparing the exams. Easy to understand for the professor's explanation.”

“The practice problems will helped introduce the material and set a good foundation! ”

If you were teaching this course, what would you do differently?:

“go over the harder homework problems assigned. give more examples that were not in the textbook ”

“When presenting new material, I would give more simple examples to explain concepts. I felt like often, we would learn a new theorem or proposition, and be given one extremely simple example followed by one hard, overly complicated example. I understand that there is more of a time constraint when going through material during a summer course, but I think it would be more effective to give less complicated examples and more simple/moderate examples to demonstrate concepts. Additionally, when it comes to grading, I think quizzes should be a smaller percentage or equal percentage to homework/practice problems. I find it unfair that we only have 4 quizzes worth 20% while there were 18 homeworks only worth 10%. I did poorly on only 1 quiz and it pulled my entire grade down, while I spent hours competing all of my homework and do not have much to show for it. For future classes, I think either these grading policies should be adjusted, or one quiz grade should be dropped to compensate.”

“I found that the last three chapters seemed rushed. A longer focus on this material may be more beneficial. ”

“I would assign less homework; maybe one assignment every other day. Even though you should be studying (i.e. doing problems) every day in a summer course, some days it was hard to complete homework on time if you have other obligations in addition to the course.”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“The use of practice problems during every lecture definitely helped to learn the material”

“I've learned countless of new ways to think about probability and even find myself thinking about what I've learned during this course when I play card games with my family.”

Other comments or suggestions::

“This class was tough and had dry material but the professor definitely made it more bearable”

“The layout of the course was very effective with daily homework and weekly quizzes.”

“Overall, Matt Charnley is a really good instructor! He is really passionate about teaching and wants his students to do well. ”

“Maybe can be less homework and hope the exams are straight forward as what professor said.”

“The only problem I had with this course was the amount of formulas that we had to memorize. That might just be because I'm not great with memorization, but having a small formula sheet in my previous statistics courses really helped me personally. ”

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew Summer 2017, 01:640:244:C1 — DIFFERENTIAL EQUATIONS FOR ENGINEERING AND PHYSICS (index #00276) Enrollment= 25, Responses= 13	Student Responses						Weighted Means			
Part A: University-wide Questions:	Strong Disagree 1				Strong Agree 5	No response	Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	0	1	12	0	**4.92**	4.61	4.54	**4.68**
2. The instructor responded effectively to student comments and questions.	0	0	1	2	10	0	**4.69**	4.39	4.48	**4.56**
3. The instructor generated interest in the course material.	0	0	0	1	12	0	**4.92**	4.30	4.37	**4.54**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	0	13	0	**5.00**	4.63	4.60	**4.64**
5. The instructor assigned grades fairly.	0	1	1	2	9	0	**4.46**	4.30	4.43	**4.56**
6. The instructional methods encouraged student learning.	0	0	0	2	11	0	**4.85**	4.37	4.34	**4.50**
7. I learned a great deal in this course.	0	0	0	3	10	0	**4.77**	4.49	4.43	**4.55**
8. I had a strong prior interest in the subject matter and wanted to take this course.	1	2	3	3	4	0	**3.54**	3.57	3.75	**3.61**
	Poor				Excellent					
9. I rate the teaching effectiveness of the instructor as:	0	0	0	4	9	0	**4.69**	4.34	4.33	**4.53**
10. I rate the overall quality of the course as:	0	0	0	5	8	0	**4.62**	4.26	4.25	**4.41**

What do you like best about this course?:

“The flipped class setting alllowed for better and more learning.”

“The reverse classroom style”

“The method of doing video lectures at home then problems in class”

“The flipped classroom made learning easier. ”

“He posted his lectures as YouTube videos to be watched at home so we spent our time in the classroom practicing problems and working together which was a great way to learn the material. ”

“The format of learning and in class work.”

“I like the structure of the class. It is very different than the other calc classes I have taken at Rutgers. The way this class is set up encourages learning in a great new way. All math classes at Rutgers should be taught in this way. Active collaborative learning is the best!”

“I liked that the instructor posted videos of the course material online. It allowed me to learn on my own prior to the next class meeting. ”

“What I liked best about this course is the problems given to us. I felt like working on exercises everyday in class was an effective way to learn the material.”

“The online pre lecture videos were very beneficial to learning because most of class time was spent working on problems in groups ”

“I like the "flipped classroom" style.”

“The teaching style of this class was great for actually learning the material and understanding it.”

If you were teaching this course, what would you do differently?:

“Hace fewer write ups.”

“I would have five more partial credit on exams like all my past professors have done. If there was one silly mistake but the logic was correct I would still get 0 points of out 8, even though I knew what I was doing so it was frustrating at some times.”

“Nothing, done very well”

“Do an example before class to make sure students understand the videos”

“Nothing”

“Nothing”

“I believe that although there are online videos available, a brief lecture in class is still necessary. ”

“I would assign various problems that are easy, medium, and hard, than just one hard question for homework.”

“Nothing”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“This method of teaching worked very well and helped me understand the material much better.”

“Very helpful in class, encouraged questions, and was every enthusiastic about this course in class. ”

“I am better at articulating my answerings and recognizing graphing patterns when given a function or set of functions ”

“He has reinvigorated my passion for the subject.”

“The instructor had taught us how to apply math problems in a real world setting related to our majors. ”

"working on the material everyday has progressed and encouraged my intellectual growth."

"Group work and projects relating to applications "

Other comments or suggestions::

"Very well done"

"The use of canvas and how the class was taught in respect to the use of videos and other materials helped me learn like no other class before"

"Best class I took at rutgers "

"The videos were very helpful."

"Great teacher!!"

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew mpc163 Summer 2018, 01:640:252:B6 — Elem Diff Equations (index #00268) Enrollment= 27, Responses= 21	Student Responses						Weighted Means			
Part A: University-wide Questions:	Strong Disagree 1				Strong Agree 5	No response	Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	1	6	14	0	**4.62**	4.61	4.65	**4.62**
2. The instructor responded effectively to student comments and questions.	0	0	0	6	15	0	**4.71**	4.67	4.55	**4.54**
3. The instructor generated interest in the course material.	0	0	2	6	13	0	**4.52**	4.58	4.37	**4.40**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	6	15	0	**4.71**	4.67	4.58	**4.61**
5. The instructor assigned grades fairly.	0	0	1	8	11	1	**4.50**	4.50	4.46	**4.39**
6. The instructional methods encouraged student learning.	0	1	0	6	14	0	**4.57**	4.61	4.40	**4.40**
7. I learned a great deal in this course.	0	0	3	4	14	0	**4.52**	4.55	4.46	**4.45**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	1	5	7	8	0	**4.05**	4.09	3.72	**3.48**
	Poor				Excellent					
9. I rate the teaching effectiveness of the instructor as:	0	0	1	3	17	0	**4.76**	4.82	4.55	**4.45**
10. I rate the overall quality of the course as:	0	0	1	6	14	0	**4.62**	4.70	4.43	**4.26**

What do you like best about this course?:

“The combination of lecture with in-class practice of problems made it so there were few occasions in which I left class with questions remaining. ”

“I really enjoyed the active learning activities that we did. Each of the practice problems and mini quizzes helped reinforce what we learned and helped me have a better sense of what problems I understood and which I needed to review again. ”

“As a math major I love Calculus and the professors own love and enthusiasm for the topic made the course one of the best I have taken at Rutgers. ”

“The instructor explains the concepts very clearly with effective examples. He also does a great job answering student questions and clarifying any topics that may be confusing.”

“Matt is very enthusiastic.”

“the instructor put so much efforts to make sure we understand the stuff and the best thing is that he wants to make sure we absorb all of the in-class knowledge at the end of each class, which is really responsible. ”

“I like how the professor would engage the students and would always be receptive to questions. I also liked how I never was afraid to ask questions or felt stupid for asking. ”

“Everything is good.”

“How interactive everything was. Everything that was being taught, was easily understandable because we were constantly practicing them in class, at home, and through small assessments. ”

“I liked the active learning aspect of the course, I felt like being able to do the problems in class let me figure out what I really needed to work on and what I knew.”

“we have a lot of practices that helped me a lot to understand the materials.”

“Everything”

“The professor was excellent. He genuinely wanted students to learn. I appreciated his efforts. I also found that his class was engaging because of the problems we would receive at the end of each lesson. They were somewhat difficult problems that involved really understanding the material we learn in class so it got me to pay attention more. I really enjoyed the overall format of the class because it made me less lazy. It challenged me but at the same time allowed me to REALLY learn the material. I think the professor was very fair and it was actually refreshing watching him teach us. Professors with enthusiasm and effort like his allow me to feel more open to learning. Honestly, I think he is better than A LOT of the current full time mathematics professors at Rutgers; some of those professors are extremely lousy and do not deserve their positions. If I was on the administrative team... I would hire him in a heartbeat. ”

“This course helped me gain the foundations and understanding to take on more challenging upper level math courses. It broke down many concepts I couldn't understand and linked them back up in a more defined manner. ”

If you were teaching this course, what would you do differently?:

“Maybe shift the class towards being slightly more lecture based but I think the course worked well as is. Also, the Matlab component did not really aid in the understanding (outside of visualization), which I think is a result of the inconsistent software usage in Rutgers courses.”

“It was frustrating to wait until the last 5 minutes of class to get practice problems that sometimes took over 10 minutes to complete. The class period is already long enough. I didn't like staying after to complete a problem that could have been given to us just a few minutes earlier. I also think the Matlab assignments should have been due on Fridays. Having the Matlab assignments due the same day as our quizzes made it hard to find time to study. Some of the Matlab assignments took several hours to figure out so I would put off studying to work on them. If they were due on Fridays instead of Wednesdays we would have had more time to focus on one thing at a time. ”

“I honestly don't think I would do anything differently. I really enjoyed all aspects. ”

“Matlab assignments weren’t helpful since the assignments were intended to be done in order but we were skipping around. Seemed like there was important information explained in other assignments that we didn't do ”

“I would teach more examples.”

“I do not think the matlab thing really helps us because we never really learn about the basic principles and how to code in matlab and the HW just present the code and ask us to modify it, which is really harder than writing it by ourselves because we need to figure out what are the coder's thoughts. Besides, I believe that there is another way to organize the readiness assessments. It would be better for posting them online before everyday's class and then asking us to submit it at the beginning of the class.Because this book, honestly speaking, does not organize its thoughts very well and when we read about this book we feel so confused. Additionally, although we read about this book, we still do not know what the instructor wants us to know so we do not do good in RA. If the instructor can put the RA in advance, we can at least know what he want us to know and for the other confusing part, we can make it clear in the class because this instructor is clear enough.”

“I probably wouldn't give readiness assessments.”

“Maybe I wouldn't assign matlab, and have more workshop style assessments. ”

“While I really liked the active learning, I would do more example problems before giving us time to work on our own. I think if we were shown how to do one basic and one advanced type question before having to do them on our own it would've worked out a little better”

“none”

“nothing”

“I was not really pleased with the readiness assignments he gave us because they were unnecessary. I do not think we need to be tested on material based on the book that he still needed to teach us. Some students like myself have minor dyslexia and reading from the book does not necessarily help me grasp concepts very well. The professor even explained that some parts in the book were confusing yet still tested us on it. Maybe an alternative is to make readiness assignments based on concept videos to watch as homework as well? I also think some of the Matlabs were unfair. We should be tested on applying our knowledge on the CONCEPTS we've learned so far to the Matlabs; I think using intensive calculation-based numbers defeats the purpose. Furthermore, sometimes the professor gave us a problem when there were 5 minutes left in class... I do not think this was fair. ”

“Since there was a very restricted time limit on so many aspects, I wouldn't have made the course any different than the professor. He did a great job teaching and helping us practice the material. ”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“While Differential Equations is not my particular area of interest, it has proven to be useful in considering other kinds of problems, such as alternate means of comparing the relation between two factors in an experiment.”

“So many ways. I want to be a high school math teacher so every class I take I'm always learning classroom management and teaching techniques from the teacher even if it isn't an education class. This class deepened my love of calculus and my want to be a teacher. The professor is inspired, engaging and implemented many different learning techniques. ”

“Reading before class begins.”

“The quizzes and practices in this class. I have never done so much quizzes and in-class practices in this university.”

“active studying helps me a lot. ”

“He has shown me the importance of active learning in classrooms, and obviously helped me improve my knowledge of differential equations. ”

“This class encouraged my growth because the Instructor not only knew what he was doing but he knew how to teach it to the class well. Mr. Charnley is hands down one of the best instructors I've had at Rutgers because his teaching style was very easy to follow and made difficult concepts easier to grasp. ”

“good”

“This class constantly made me actively learning. Since we would have quizzes every day, I retained information better and when it came time for the midterms, I was not cramming or stressing prior to the midterms because all these quizzes and work problems really prepared me well. This is the first time in a while that I felt confident studying for a mathematics exams. On the first day of class, I remember writing that i wanted to gain my confidence for learning math back and getting over math test anxiety — I definitely did. I'm very thankful for Mr. Charney for helping me heal and get over this mental obstacle. As a human to human influence, this has really re-encouraged me to be more hopeful in this path. (:”

“I feel more driven to understand the concept of computational mathematics rather than just plug and chug. Matthew helped break down formulas and concepts which I struggled in. He helped relate things we learned in class to real life scenarios and put his heart in making sure we understood the concept. Great professor!”

Other comments or suggestions::

> “The course was well taught and ultimately more enjoyable than I would have suspected. ”
>
> “I really enjoyed this class and would recommend this summer section to anyone. It was fast passed but our professor broke it down to seem a lot less scary. I never felt afraid to ask questions or for help. This has been my favorite math class at Rutgers thus far. The professor made me not hate going to class on beautiful Friday nights, that takes a very good teacher. ”
>
> “If you do one more example after the explanations of the context will be better ”
>
> “For the quiz section of the grade add more quizzes or make it worth less, I think it's a little off balanced that the mini quizzes, practice problems, in class activities, and readiness assessments, and the Matlab's both account for the same amount of the grade but both areas had many more opportunities to make up for a bad assignment. ”
>
> “great class. ”
>
> “no”
>
> “Suggestion: Spend more times on showing students examples rather than giving them problems to try in class before the hand-in practice problem.”
>
> “No other comments or suggestions. ”

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew mpc163 Fall 2018, 01:640:104:02 — INTRO TO PROBABILITY (index #06597) Enrollment= 26, Responses= 13	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	0	1	12	0	**4.92**	4.87	4.37	**4.37**
2. The instructor responded effectively to student comments and questions.	0	0	0	2	11	0	**4.85**	4.87	4.21	**4.24**
3. The instructor generated interest in the course material.	1	0	1	0	10	1	**4.50**	4.77	4.08	**4.09**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	1	1	11	0	**4.77**	4.87	4.33	**4.35**
5. The instructor assigned grades fairly.	0	0	1	2	10	0	**4.69**	4.61	4.17	**4.19**
6. The instructional methods encouraged student learning.	1	0	0	2	10	0	**4.54**	4.71	3.98	**4.01**
7. I learned a great deal in this course.	0	1	1	1	10	0	**4.54**	4.68	3.96	**4.03**

Charnley Matthew mpc163 Fall 2018, 01:640:104:02 — INTRO TO PROBABILITY (index #06597) Enrollment= 26, Responses= 13	Student Responses						Weighted Means			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
8. I had a strong prior interest in the subject matter and wanted to take this course.	1	0	3	0	9	0	**4.23**	4.19	3.41	**3.56**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	1	3	9	0	**4.62**	4.71	4.02	**4.04**
10. I rate the overall quality of the course as:	0	1	1	2	9	0	**4.46**	4.58	3.85	**3.91**
Part B: Questions Added by Department or Instructor										
	Strongly disagree	**Disagree**	**Uncertain**	**Agree**	**Strongly agree**					
15. The course objectives were well explained.	0	1	0	2	10	0	**4.62**	-	-	-
16. The course assignments were related to the course objectives.	0	1	0	1	11	0	**4.69**	-	-	-
17. I understood what was expected of me in this course.	0	0	1	2	10	0	**4.69**	-	-	-
18. The instructor encouraged me to do my best work.	0	1	0	1	11	0	**4.69**	-	-	-
19. I learned perspectives, principles, and practices from this course that I expect to apply to new situations.	1	0	1	1	10	0	**4.46**	-	-	-

Charnley Matthew mpc163 Fall 2018, 01:640:104:02 — INTRO TO PROBABILITY (index #06597) Enrollment= 26, Responses= 13	Student Responses						Weighted Means			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
20. The course challenged me to think critically and communicate clearly about the subject.	1	0	0	2	10	0	**4.54**	-	-	-

What do you like best about this course?:

"Nothing."

"Perfect"

"The effective teaching method"

"The material was interesting "

"I love my prof Charnley as well as the course context."

"The fairness of Professor Charnley, and his dedication to making sure we learn all the material."

"The concept of probability is very interesting and it was just interesting to learn a little more about it. "

If you were teaching this course, what would you do differently?:

"Give homework as a participation grade as many points can be lost if you make even the smallest mistake. Same with practice problems at the end of class. Also much of the homework was not what we did in class. Difficulty went from 0-100."

"Nothing"

"I would give less assignments. Also if you missed one day of class, your grade would decrease significantly."

"I will apply the information I learned in the daily life."

"I think that I would assign chapters to read in the textbook as additional material."

"If anything outside resources would be valuable. If I didn't fully understand something I would have had liked somewhere else to go and be able to spend time trying to understand that topic. "

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

"The only upside I can think of is I have never put more effort both in and out of the classroom in order to pass."

"Just math knowledge "

"I am more interested in probability "

"Yes, he is very patient and full of enegetic which makes the class very vivid."

"I learned that I can persevere and still learn something new against all odds."

"I was not particularly interested in this course due to my not finding it useful, but by the end I can see how probability relates to the workforce and how it might be applied in my field of study. So this course expanded my understanding of the importance of probability. "

Other comments or suggestions::

"You go too fast in class, the only time I somewhat understood concepts was in office hours. Also stop making exams cumulative, it's literally the worst."

":))"

"Great teacher and interesting class"

"I love him, Charnley, the best teacher I have ever seen!"

"This class was truly amazing! I would highly recommend it."

"The course was well done. Professor was very motivated to have students understand the material. Good class overall. "
