Teaching Portfolio - Full Documentation

Rutgers University

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Contents

Chapter 1

Introduction

This document provides an in-depth summary of my experiences as a teaching assistant and instructor at Rutgers University. It begins with introductory statements, including my teaching philosophy in Section 1.1, which provides the motivation behind everything I do as an instructor, which felt like a good place to start this document, as well as research and diversity statements and my CV. After that, Chapter 2 will outline the various teaching experiences that I have had as a graduate student, and Chapter 3 will detail the evidence of these experiences that is included in the rest of the document. The Appendices to this portfolio contains a more detailed description of each of these teaching assignments, my reflection on those assignments, and sample materials from each of these classes. Appendix A covers the classes that I taught on my own while at Rutgers. Following that, Appendix B has the same material as the previous chapter, but related to the courses for which I was a Teaching Assistant. Appendix C contains a description of the other teaching related things I was involved with during my time at Rutgers. This includes mentoring undergraduate students in independent study projects and working for a variety of Summer programs. Finally, the last two appendices contain the sample materials from all of the courses earlier in the portfolio as well as my full student evaluations from each course.

1.1 Teaching Philosophy

One of the most important things in education, and particularly in math education, is the idea of self-efficacy—student's beliefs about their ability to complete a given task. In my opinion, a lot of students struggle with math because they go into classes with the mindset that math is hard or impossible, and if they were able to get past this, they could be more successful. For this reason, I have focused most of my teaching efforts on trying to improve self-efficacy in my classroom, helping students believe that they can succeed in my class, even if it seems challenging.

My commitment to the goal of improving self-efficacy can be seen throughout all of my teaching, but three types of experiences in particular both demonstrate this very well and have significantly impacted the way I think about teaching. These are

- the Summer 2017 Differential Equations class that I ran as a flipped classroom,
- the Summer 2018 Differential Equations class that I ran as a very active classroom with assignments and projects designed to encourage learning and self-efficacy, and
- the varied groups and communities dedicated to teaching that I have become involved with.

While there are many possible ways to attempt to address the problem of self-efficacy in the math classroom, two of the main techniques that I have tried to use to improve self-efficacy have been active learning and peer teaching. The idea of both of these is that getting students more involved in the learning of the material will help them to internalize it better and understand that it really is not that bad. These ideas became the foundation for the classes that I taught near the end of my time as a graduate student.

Summer 2017 - Math 244

During the summer of 2017, I taught a section of Rutgers' Differential Equations class for engineering majors. This is a 4-credit class that all engineering majors need to take before they get into the bulk of their major-specific classes. I decided to move the class to more of an 'active learning' format, and did so by running the section as a flipped classroom because I felt this would help the students understand the material better. I recorded video lectures for the entire class, and students spent the in-class time working on problem sets in groups, presenting problems to the rest of the class, or taking quizzes.

The goal in running the class this way was the same as the motivation for most flipped classrooms: students learn math concepts best by working problems, and this format gives them time to work on the homework-type problems in class, where I can fix any issues in understanding before they spend too much time doing problems incorrectly. By giving them this time to discuss things both with their classmates and with me, I had hoped to help them understand that things in this class were not too challenging, and it was something that they all could do. Overall, this class was successful. My experience in the class and student feedback seemed to indicate that some of the groups really enjoyed the format and took advantage of the time they were given to work on problems and did very well in the class. Some of the comments I received on the course were

- "The teaching style of this class was great for *actually learning* the material and understanding it."
- "He has reinvigorated my *passion* for the subject"
- "I like the structure of the class. It is very different than the other calc classes I have taken at Rutgers. The way this class is set up encourages learning in a great new way. All math classes at Rutgers should be taught in this way. Active collaborative learning is the best!"

On the whole, I saw a lot more ownership of the material in this class than I had seen in my previous classes, and I think it helped with some of the students being more confident in their ability to tackle math classes and problems in the future, but I also learned things about teaching a class like this that would help me to improve my future classes.

Summer 2018 - Math 252

One of the most common feedback comments I received from that class was my Summer 2017 I should lecture at the start of each class on top of the videos that the students had to watch before class. I took this feedback into consideration when designing my summer 2018 Differential Equations class, where I knew that I still wanted to use some of the active learning ideas that I had developed the previous summer, but also wanted to modify the course to make it better. In order to do this, I removed the video component of the course and added in condensed mini-lectures at the start of each section of material.

Due to the format of the course, this resulted in two mini-lectures each class day, with time between them for the students to work on problems from the section that I had just talked about. At the end of each problem session, the students would have to write up the solution to one problem from that section to turn into me that I would grade. I encouraged the students to work in groups during the problem set portions of the class and I think the atmosphere of the class and the amount of things they needed to turn into me made them more likely to actually participate. Within these groups, I saw the peer teaching that I was hoping to inspire develop on its own. Students would talk to each other and discuss their answers instead of asking me questions, and became more self-reliant in that way, trusting themselves and their groups to come to the correct answer instead of needing help or approval from the instructor. Student comments showed that they enjoyed this format:

- “I really enjoyed the active learning activities that we did. [...] helped reinforce what we learned and helped me have a better sense of what problems I understood and which I needed to review again.”
- “I liked the active learning aspect of the course, I felt like being able to do the problems in class let me figure out what I really needed to work on and what I knew.”
- “So many ways. I want to be a high school math teacher so every class I take I’m always learning classroom management and teaching techniques from the teacher even if it isn’t an education class. This class deepened my love of calculus and my want to be a teacher...”

I also encouraged peer instruction specifically with several activities in the course, including a Jigsaw-type activity, where the students were broken up in to groups, and each group had a distinct problem to work on. After all the groups were done, the groups were reshuffled so that each new group had one person in it that knew each problem, and they had to explain their solutions to the rest of the group. This helped to encourage students to discuss their work with each other throughout the course as a whole, and the need to explain things to someone else helped them to solidify that they could do these problems and they did understand this, and so they were able to succeed in a traditional exam setting, and would be able to take this confidence with them into future classes and their future career.

All in all, teaching is something that I have enjoyed for most of my life, and Rutgers has been a great experience in developing myself as a teacher. The various classes I have taught and groups I have been involved in have given me a foundation in and a passion for education that I am looking to expand on in the future. It has taught me that when students are confident and believe that they can complete the tasks you set out for them, it is fairly easy for them to succeed, and it is the job of the educator to set up an environment where this can happen. While we are just teaching a math class, the impact we can make on students goes well beyond that; inspiring confidence in a class that they find difficult will push them to tackle challenges that they face later in life. Calculus may be important, but the belief that a student can surpass whatever obstacles stand in their way is the most important thing that we can try to instill in our students.

1.2 Diversity Statement

Diversity considerations in the math classroom arise in two different contexts: those directly related to the course material, and those that arise from the classroom environment, indirectly related to the content of the course. The content related issues come out of diversity in levels of prior knowledge. This is particularly important when dealing with a class like Calculus 1 or with freshmen students, but applies to higher level classes as well. Students will be entering the classroom from all different types of high schools and previous learning environments, and this will affect the way they react in a college classroom. In higher level classes, students will have had different instructors and a different experience up to the point where they enter the class, and so will react to things differently. While an instructor can try to ask questions of their students to get at this information, but there is no accurate way to predict the amount of prior knowledge before a class starts. It is up to the instructor to analyze how things are going in the class and react accordingly to adapt to the particular set of students they have.

The considerations that arise indirectly related to the course content connect to the diverse backgrounds of the students that have arrived in your classroom. Math is math, no matter who you are, and so issues related to things like race and gender shouldn't really come up. However, people are still people, and the diversity issues that arise in other disciplines still show up in math because of it. These can show up either in student-student interactions, or in teacher-student interactions with the way material is presented. For the second one, instructors using a metaphor to describe a situation in class can alienate students who do not have the same background as the instructor and do not understand or relate to that metaphor. One of the largest issues that is especially prominent in math classes (and all of STEM) is that traditionally under-represented students in these fields do not feel like they belong in the class, and this can be amplified when an instructor says or does something in the classroom that emphasizes a type of normativity which that student does not identify with. This is still an issue in math classes, and instructors need to find a way to address it to make sure that everyone feels welcome in the class and can succeed to the best of their ability.

I have gained many enlightening experiences in diverse classrooms both at Rutgers and during a summer at a middle school in Detroit, and have tried to develop strategies to reach and accommodate diverse learners of all types. I have mixed different types of learning in my classroom, including lecturing, active learning, and group work, to attempt to work with all types of students in my classrooms. Within the last few summers, I have also tried to include a peer teaching component to my classes to help students with self-efficacy, believing that they can succeed and prosper in my class. The goal of all of this is to make sure that all students have a place and belong in the classroom, so that they will be free to learn, grown, and tackle what ever confronts them, in this class and beyond.

1.3 Research Statement

A well-established result in partial differential equations is the following: Let $B_1(0)$ be the unit ball in $\mathbb{R}^n$, $n \geq 2$, and consider the solution u_ϵ to the equation

$$\begin{cases} \nabla \cdot (\gamma_\epsilon \nabla u_\epsilon) = 0 & B_1(0) \\ u_\epsilon = \phi & \partial B_1(0) \end{cases}$$

for some $\phi \in H^{1/2}(\partial B_1(0))$ where

$$\gamma_\epsilon = \begin{cases} 1 & |x| > \epsilon \\ a & |x| < \epsilon \end{cases}.$$

Then, as $\epsilon \to 0$, the $u_\epsilon \to u_0$, where u_0 solves

$$\begin{cases} \Delta u_0 = 0 & B_1(0) \\ u_\epsilon = \phi & \partial B_1(0) \end{cases}$$

with an error bound of the form $||u_\epsilon - u_0|| = O(\epsilon^n)$. The idea of this result comes from the fact that as $\epsilon \to 0$, the domain where $\gamma_\epsilon \neq 1$ converges to a single point at the origin, and since a single point in $\mathbb{R}^n$ has zero capacity, functions that are harmonic in $B_1(0)\setminus\{0\}$ and bounded in $B_1(0)$ are the same as harmonic functions in $B_1(0)$. The proof of the error bound relies on asymptotics expansion of the solution around the points where the conductivity is not 1, and the volume of the inhomogeneity enters the equation, resulting in a factor of ϵ^n.

The natural question to ask from there is if there are possible extensions of this result. There are two fairly easy ways to try to extend it: allowing the constant a in the conductivity γ_ϵ to change with ϵ and modifying the region $|x| < \epsilon$ to something more generic sets ω_ϵ whose Lebesgue measure goes to zero as $\epsilon \to 0$. It turns out that changing the parameter a alone doesn't alter the result, and as long as the inhomogeneous regions are contained within balls that are collapsing to zero, the result stays the same as well. One of the simplest regions that leads to an interesting change of the underlying problem is when the inhomogeneous region is a tubular neighborhood of a hypersurface σ, where the width of the neighborhood goes to zero with ϵ.

From here on out, we only look at $\mathbb{R}^2$, so that our hypersurfaces are curves, and we generalize the unit ball $B_1(0)$ to a general domain Ω. The reason why this case becomes interesting is that the limiting singularity set is a curve σ, which is only of codimension 1, and so needs an additional boundary condition to uniquely determine a harmonic function. In addition, if the parameter a is allowed to vary as well, having a very large or very small results in the tubular neighborhood being very conductive or very resistive, which results in distinctly different behavior near σ. This distinct behavior tells us that a single background solution u_0 will not be enough to encompass all of the different possible behaviors of the solutions u_ϵ as $\epsilon \to 0$. Therefore, we need to define a different kind of approximate solution u_0^ϵ that satisfies the following:

- u_0^ϵ is defined on the domain $\Omega \setminus \sigma$.
- u_0^ϵ depends on ϵ and a_ϵ through the boundary condition on σ
- $u_\epsilon - u_0^\epsilon$ goes to zero as $\epsilon \to 0$.

The first two of these conditions mean that it works as a replacement for a background solution, and the third means that it is an effective approximation to the solution u_ϵ. The work in this area is then to determine the appropriate form of the approximate solution u_0^ϵ and prove how well it approximates u_ϵ as $\epsilon \to 0$.

One of the main developments in this area is in the paper [2], where the case of closed curves σ is considered. This paper uses an energy approach, finding an energy functional for u_0^ϵ that approximates the one for u_ϵ and use this to prove that the functions u_ϵ and u_0^ϵ must be close. This leads to a PDE with a somewhat non-standard boundary condition depending on ϵ and a_ϵ along the curve σ, where the solution to this PDE replaces our u_0 from the initial problem.

Our paper [1], currently in preparation, looks at the case where the curve σ is open. The main issue here that needs to be dealt with are the endpoints of the curve; outside of that, the previous work in [2] applies. Our goal with this paper is to prove that the same u_0^ϵ from the previous paper works as an approximation to u_ϵ when the curve has endpoints. In order to work out the details of how u_0^ϵ behaves near the endpoints of σ, we consider the specific domain $\Omega = B_2(0)$ and $\sigma = (-1, 1) \times \{0\}$. Since σ is a subset of the curve $y = 0$ and Ω is symmetric around $y = 0$, we can use this symmetry to further simplify the problem.

In this context, the differential equation we derive is

$$\begin{cases} -\Delta u = 0 & \Omega \setminus \sigma \\ \frac{\partial u^+}{\partial y} + \epsilon a_\epsilon \frac{\partial^2 u^+}{\partial x^2} - \frac{a_\epsilon}{2\epsilon}(u^+ - u^-) = 0 & \sigma \\ \frac{\partial u^-}{\partial y} + \epsilon a_\epsilon \frac{\partial^2 u^-}{\partial x^2} + \frac{a_\epsilon}{2\epsilon}(u^+ - u^-) = 0 & \sigma \end{cases}$$

where u^+ and u^- denote the values on σ approaching from the top and bottom respectively. To make use of the symmetry of the problem, we define

$$u_o(x, y) = \frac{1}{2}(u(x, y) - u(x, -y)) \qquad u_e(x, y) = \frac{1}{2}(u(x, y) - u(x, -y))$$

as the odd and even parts of u, and see that they solve

$$\begin{cases} -\Delta u_o = 0 & \Omega_+ \\ \frac{\partial u_o}{\partial y} + \epsilon a_\epsilon \frac{\partial^2 u_o}{\partial x^2} - \frac{a_\epsilon}{\epsilon} u_o = 0 & \sigma \\ u_0 = 0 & \{y = 0\} \setminus \sigma \end{cases}$$

and

$$\begin{cases} -\Delta u_e = 0 & \Omega_+ \\ \frac{\partial u_e}{\partial y} + \epsilon a_\epsilon \frac{\partial^2 u_e}{\partial x^2} = 0 & \sigma \\ \frac{\partial u_e}{\partial y} = 0 & \{y = 0\} \setminus \sigma \end{cases}$$

where, due to the symmetry, we only need to define these solutions on the upper half ball $\Omega_+ = B_2(0) \cap \mathbb{R}^2_+$, and we are ignoring the condition on $\partial B_2(0)$ while we analyze the local behavior of the solution near the curve σ.

The first problem we consider is the even symmetery. In this, using a careful Sobolev space analysis, assuming that $\epsilon a_\epsilon > m > 0$ as $\epsilon \to 0$, and tools from elementary complex analysis, we determine that the solution to a related problem is almost in $H^4(\Omega_+)$, which, using Sobolev embeddings and a Taylor series approximation, tells us that the u_e, near one of the endpoints of the curve, is of the form

$$u_e = b_0 + b_1 r^{1/2} \sin(\frac{\theta}{2}) + b_2 r \cos(\theta) + e(x,y)$$

where (r,θ) denote polar coordinates around an endpoint of σ, and $e(x,y)$ is controlled by remainder estimates from Taylor's theorem. In particular, everything outside of the $r^{1/2}$ terms is at least $C^{1,\eta}$ near σ, for any $\eta < 1$. Using this explicit form of the solution, we are then able to derive estimates for how close u_0^ϵ is to u_ϵ, working through the same energy formulation.

The key step in the process of deriving the error bound involves finding an appropriate test function in both the primal and dual energy formulations that can be compared to u_ϵ, which we know is the minimizer of the full functional, to show that the energy difference is small. The distinct difference between this result and the one in [] is that the adjustment factor needed to make the desired u_0^ϵ fit into the same function space as u_ϵ to be tested against it in the energy formulation has to be dealt with in a different way. It turns out that dealing with it in the same way (making the adjustment inside the inhomogeneity) results in a non-vanishing change to the energy of that function, while moving it outside the inhomogeneity does not change the energy, allowing us to prove that the energy gap between u_ϵ and u_0^ϵ is small. Therefore the same energy formulation tells us that u_ϵ and u_0^ϵ need to be close, but we don't get the same error bounds that we had before, achieving a bound of $\epsilon^{\eta/2}$ instead of the order ϵ error that was in the previous paper.

It turns out that this version of the problem (even symmetry with $\epsilon a_\epsilon > m > 0$) contains most of the difficult work needed for solving the problem in all cases. By a duality argument particular to two dimensions, done by rotating the gradient of the function, the problem with odd symmetry and $\frac{a_\epsilon}{\epsilon} < M$ can be connected to the problem that we had just solved, and the results from before carry over to this case. The problem is easier in the other two cases; in these cases, we can define a background solution that is independent of epsilon and show that both the solution to the reduced problem and the full problem are close to it, thus implying that the reduced and full problem are close to each other with an appropriate bound. Finally, a supremum argument lets us prove that in all cases, and for any a_ϵ, this reduced problem is an appropriate approximation to the full problem with an error bound. The extension to non-symmetric domains happens fairly smoothly via an energy argument.

In addition, numerical experiments have shown that this behavior is to be expected; for most boundary conditions and certain values of the conductivity parameter a_ϵ, the solution to both u_ϵ and u_0^ϵ display an $r^{1/2}$ behavior near both endpoints of the curve. The experiments also illustrate the convergence of u_0^ϵ to u_ϵ away from the curve σ. We see convergence on σ in most cases, but in certain situations, namely odd symmetry with $\frac{a_\epsilon}{\epsilon}$ bounded, we know we can not expect convergence on σ, but none of our work so far has been able to prove convergence on σ in any case.

My thesis expands on some of these results. I compute an explicit formula for the coefficient of the $r^{1/2}$ term using the properties of the function u_0^ϵ and use this expression to determine how this coefficient depends on a_ϵ. In particular, this allows it to be determined that this coefficient vanishes unless ϵa_ϵ stays bounded away from zero, which is exactly what is shown in the numerical results. I also analyze the problem on σ, discussing when we can prove convergence and what happens in the case when we can not. In addition, my thesis discusses what needs to happen to extend this results to non-straight σ and dealing with non-symmetric situations.

For future work in this area, one of the clear next step is to try to extend this result to the Helmholtz equation or other second-order linear operators. The general direction of this extension would be to work towards applications in scattering theory and electromagnetics. Thin devices of this form show up in a variety of applications, and understanding how the far field reacts to inhomogeneities of this form could help in designing them to behave properly. In addition, the original problem (where the inhomogeneity is a ball collapsing to a point) has applications to cloaking, where the cloak makes an object of unit size look like one of size ϵ, and the error bound then says that this close object 'looks like' the background solution, up to a term of order ϵ^n. This problem here shows directly that the solution to the full problem is 'close' to some relevant background solution, and so if an object is cloaked to look like a tubular neighborhood of a curve, then we know what the far field solution will be close to. This may have extra uses in directional cloaking, making use of the fact that this domain is no longer forced to be symmetric, and allowing for non-symmetric cloaked domains may allow for the cloak to be just as effective from certain directions without requiring as anisotropic of materials in building the cloak.

Once both open and closed curves are well understood, it would also be interesting to look into a varying conductivity parameter a_ϵ within the inhomogeneity ω_ϵ. Since we have seen distinct behavior depending on how ϵa_ϵ and $\frac{a_\epsilon}{\epsilon}$ behave as $\epsilon \to 0$, having a variable conductivity that exhibits different types of these behaviors on different subsets of σ could make for a very interesting problem. It would also be a problem that has more direct applications, as the thin devices used in electronics do not generally have a constant electrical conductivity. Finally, the 3-dimensional problem would be interesting to consider. I believe that the results should scale up just fine, but the proofs for these method distinctly relies on the fact that the region is 2-dimensional.

Bibliography

[1] Matthew Charnley and Michael Vogelius. Uniform asymptotic expansions of solutions to the 2d conductivity problem with non-closed thin filaments. *In preparation*, 2018.

[2] Charles Dapogny and Michael Vogelius. Uniform asymptotic expansion of the voltage potential in the presence of thin inhomogeneities with arbitrary conductivity. *Chinese Annals of Mathematics Series B*, 38:293–344, 2017.

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EDUCATION

Doctor of Mathematics,
Rutgers University, Piscataway, NJ
May 2019 (expected)
Concentration: Partial Differential Equations
Advisor: Michael Vogelius

Bachelor of Science, Mathematics and Chemical Engineering
University of Notre Dame, Notre Dame, IN
May 2013

HONORS AND AWARDS

Spring 2016 - Invitation to RASTL
Spring 2018 - TA Teaching Excellence Award, Rutgers University Math Department
Fall 2018 - TAPCast, Invited Guest, Season 1 Episode 4: Active Learning, with Sandra Medina (Spanish). Released October 15, 2018. `http://go.rutgers.edu/y87h66ik`

TEACHING EXPERIENCE

Fall 2018 - Introduction to Probability
Summer 2018 - Differential Equations
Summer 2017 - Ordinary Differential Equations
Summer 2016 - Introduction to Probability
Summer 2015 - Calculus 3

TEACHING ASSISTANT EXPERIENCE

Spring 2018 - Numerical Methods in PDEs/Numerical Analysis 2
Fall 2017 - Advanced Calculus for Engineers
Fall 2016 - Graduate Real and Complex Analysis
Spring 2016 - Numerical Methods in PDEs
Spring and Fall 2015 - Calculus 3
Fall 2014 - Calculus 1

PROFESSIONAL DEVELOPMENT

Rutgers Academy for the Scholarship of Teaching and Learning (RASTL) — Fall 2016 - Spring 2019
Rutgers University, New Brunswick, NJ

- Met with graduate students from various departments at Rutgers to discuss pedagogy and other issues with teaching.
- Led workshops for the group on ‘Engaging Students and Managing Discussions’ and ‘Classroom Expectations’.

Rutgers TA Project — Fall 2016 - Spring 2019
Rutgers University, New Brunswick, NJ

- Ran and assisted in workshops run by the TA Project for graduate students at Rutgers interested in learning more about teaching.
- Discussed topics such as ‘Teaching a Summer Course’, ‘Teaching Non-Majors’, and ‘Providing Feedback that Matters’.
- Presented to new graduate students at TA Orientation, informing them of how the TA program in the math department works and where they can go to get extra teaching resources.

Rutgers Math Teaching Group Fall 2016 - Spring 2019
Rutgers University, Piscataway, NJ

- Planned a weekly seminar where graduate students from the math and math education departments could discuss teaching issues.
- Invited speakers from other departments at Rutgers to discuss programs that exist to support teaching.
- Participated in a semester-long discussion of the Calculus sequence at Rutgers and ways that it could be improved.

Math TA-At-Large Program Organizer Fall 2017 - Spring 2018
Rutgers University, Piscataway, NJ

- Organized the online office hours for the TA-At-Large program.
- Gathered class information from the TAs to provide to the technology staff to get the classes set up in the online system.
- Scheduled the 12 sets of office hours in the single technologically-capable office each of two semesters.
- Provided an outline to the department on how to run this program in future semesters.

MENTORING EXPERIENCE

Directed Readings Program Fall 2014 - Spring 2018
Rutgers University, Piscataway, NJ

- Mentored advanced undergraduates in independent study projects, showing them a little bit about what advanced math looks like.
- Advised students on giving a 15 minute presentation about the semester's work at the end of the project.
- Supervised 6 projects related to the Hydrogen Atom, Fourier Analysis, Functional Analysis and the δ-function, and Markov Chains.

Summer Session Head TA Summer 2016 - Summer 2018
Rutgers University, Piscataway, NJ

- Served as a peer reviewer for other graduate students teaching classes over the summer.
- Provided a formative assessment to each graduate student near the start of their summer class.
- Advised peers on how to improve their teaching before the summative review later in the summer by a full-time faculty member.

PUBLICATIONS

1. Charnley, M and M. Vogelius. In preparation.
2. Charnley, M and A. Wood. In preparation.
3. Charnley, M. "The average of a polygon is an ellipse." 2018. MAA Mathematics Magazine. (submitted)
4. Charnley, M. and A. Wood. "A linear sampling method for through-the-wall radar detection." 2017. DOI: `10.1016/j.jcp.2017.06.035`.
5. Charnley, M. and A. Wood. "Through-the-wall radar detection analysis via numerical modeling of Maxwell's equations." 2016. DOI: `10.1016/j.jcp.2016.01.039`.

EXTERNAL TALKS

1. "A Modern Approach to Gårding's Asymptotics Result." Rutgers Camden Several Complex Variables Learning Seminar. Rutgers University - Camden Campus. March 30, 2018.
2. "An energy lemma and an application to thin inhomogeneities." NYS Regional Graduate Mathematics Conference. Syracuse University. March 24, 2018.
3. "Flipped Classrooms for Higher Level Mathematics." NE RUME Conference. Montclair State University. November 11, 2017.
4. "A Linear Sampling Method for Through-the-Wall Radar Detection." Colloquium at the Air Force Institute of Technology. March 2, 2017.
5. "Numerical Simulation of Maxwell's Equations for Radar Detection Analysis." Brown Bag Seminar at the Air Force Institute of Technology. August 27, 2015.

OTHER RESEARCH

ORISE Summer Researcher Summer 2015 - Present
Air Force Institute of Technology, Dayton, OH

- Continued work from a Master's Thesis on "Through-the-Wall" imaging.
- Developed techniques for generating data and analyzing the results numerically, implementing both existing research and new ideas.
- Presented findings at AFIT at several of their seminars.

Chapter 2

Teaching Experience

Overall, I would put my teaching experiences at Rutgers into 3 categories: Early Teaching (which is mostly teaching assistant assignments), Miscellaneous Assignments, and Late Teaching. While there are things to be learned from all three of these areas, I feel like they show how my teaching has developed over my time as a graduate student.

The early teaching assignments took place during my first few years of teaching. These consist of my first teaching assistant assignment, Math 135 (Calculus 1 for the Social and Biological Sciences), the two times I was a teaching assistant for Math 251 (Calculus 3), and my first summer teaching assignment, also Math 251. These were my first experiences with teaching at the college level and where I started feeling out my personality as an instructor. For each assignment as a teaching assistant, I was in charge of running three recitation sessions, each with between 20 and 35 students, with a variety of responsibilities that changed with each course. In general, it consisted of answering student questions, writing and grading quizzes, and holding office hours to help the students succeed in the class. My first teaching assignment of Math 251 consisted of a summer class that met 4 days a week for 2 hours a session over a period of 8 weeks. There were 25 students in the class, and I was in charge of everything for the class, from the lectures to quizzes, exams and the course structure as a whole. This was the first time that I felt like I had control over what was going on in the classroom and tried to implement some changes. During this first assignment, I only made a few small changes to the course, but it was a great experience and led to the changes I tried to make throughout the rest of my teaching endeavors.

The miscellaneous assignments fill in the middle part of my time as a graduate student and bridge the gap into my later and more involved teaching assignments. These consist of the two times I worked as a teaching assistant for Math 575 (Numerical Partial Differential Equations, in the Masters in Math Finance program), my semester running problem sessions for Math 501 and 503 (first semester graduate courses in Real and Complex Analysis), a semester as a teaching assistant for Math 421 (Advanced Calculus for Engineers), and a summer teaching assignment of Math 477 (Theory of Probability). The assignment for Math 575 had about 20 students in them each, and for the first assignment I only served as a grader, but the second time around I also ran problem sessions and taught lectures for the last month of the course. The graduate course problem sessions consisted of working on homework problems and qualifying exam problems with the 10 students in the class and helping them learn the material from both classes. Math 421 was a different assignment; as a TA-At-Large, I was assigned to two sections of around 70 students each, and my main responsibilities for this assignment were to hold in-person and online office hours, run review sessions, and grade exams. This assignment was not the most rewarding in terms of teaching experience, but I tried to make the most of it. Finally, the probability class was another summer session that met for 2 hours a day, 4 days a week, for 6 weeks, and contained around 25 students. I ran this class very similar to how I ran the Calculus 3 class the previous summer, and had another varied experience running my own class.

The late teaching assignments are the ones that I am most proud of in terms of what I tried to bring to the class and how I put my own spin on everything. I feel like these three classes really show who I have become as an instructor. These are the two differential equations summer assignments (Math 244 and Math 252) and the introduction to probability class that I taught during Fall 2018 (Math 104). The first of these differential equations, Math 244, I ran during Summer 2017 as a flipped classroom, where I recorded videos for the entire class and the in-class time was spent with the students working problems in groups. There were 25 students in this class, and it followed the same schedule as Calculus 3; 4 days a week for 2 hours a session over a period of 8 weeks. The main things I added to this class were the videos, as well as a few projects that I designed to add to the course. The second differential equations class, Math 252, was run the following summer as an active classroom, where I would give short lectures in class followed by time for the students to work on problems in groups. There were 26 students in this class, and it met three days a week for 2 hours and 45 minutes over a period of 6 weeks. In addition to restructuring the class to allow for in-class group work, I also crafted many different projects, activities, and worksheets for the students to work on to help them learn the material. Finally, the Math 104 assignment was the first time I taught my own class during the semester, which was a very different experience from teaching over the summer. This class had 23 students in it, and met on a more standard schedule of two days a week for 80 minutes. This was a very different experience from all of the other classes I had taught, as this class was at a much lower level, and things needed to be explained differently. I also designed several new worksheets for teaching topics throughout this class and tried to add my own flavor to it, even though a lot of material for it had already been created by previous course instructors.

In addition, I have also been involved with several other projects and groups throughout my time at Rutgers that have done a lot to improve and develop my teaching. I have been a part of the Directed Readings Program for many years, working with advanced undergraduate students on independent study type projects and helping them prepare presentations on the material. I have also been a part of the Active Learning Community at Rutgers, as well as the Rutgers Academy for the Scholarship of Teaching and Learning, both of which have contributed discussions that have greatly added to my teaching abilities and beliefs. I also have some experience with helping to organize the TAs at large for two semesters and discussing possible course reform with the Math Teaching Group that I also helped to reorganize. I feel like all of these different experiences will help in how I approach classes in the future.

Chapter 3

Evidence of Teaching Development

The appendices to this document contain a variety of sources of evidence for how my teaching has grown and developed over my time at Rutgers. The first appendix contains course descriptions and reflections for all of the courses that I taught under my own direction. These were classes that I had a large amount of control over and could design them to run in what I felt was the best way possible. The second appendix goes through the same information for the classes where I was a teaching assistant, and so had to mostly stick to what the instructor wanted me to do in the course. The next appendix goes into more detail on all of the additional projects related to teaching that I took part in during my time at Rutgers. Appendix D contains sample materials from all of these classes. These include course syllabi, sample problem sets or quizzes, and any new materials that I personally designed for the course. Finally, Appendix E contains all of my student evaluations, again sorted by class. I have chosen all of the material in these appendices carefully to highlight what I think are my best qualities as an instructor.

Appendix A

Teaching Assignments

For graduate students who want to teach, Rutgers gives us the opportunity to teach our own classes during the Summer session, provided we have been a teaching assistant for at least one semester. Classes can run for 6, 8, or 12 weeks over the summer. My experience has only been with the 6 and 8 week classes, where, in general, the 6 week classes correspond to 3 credit-hour classes, and the 8 week ones correspond to 4 credit-hour classes. Classes meet between 2 and 4 times a week for between 2 and 3 hours each session. The classes taught over the summer are supposed to be equivalent to those taught during the semester, so generic syllabi are provided by the department. However, outside of following these syllabi, we are mostly free to do whatever we want with the summer classes. I took advantage of that with the classes I taught over the last two summers, working in fairly significant active learning components to these classes. These experiences of running my own classes and getting to plan them from the ground up will be very helpful as I move on to plan and design classes in the future.

A.1 Summer 2015 - Math 251

A.1.1 Class Overview

Math 251 is Rutgers' version of Multivariable Calculus. The difference for this instance is that it was run over the summer. During the summer, Rutgers runs compressed classes for students who are either trying to get ahead or catch up on classes. This class in particular met 4 days a week for 8 weeks, with 2 hour sessions every day. The format of these courses makes running them very interesting, because in addition to covering material very quickly, the lecture sessions are also longer than classes during the semester, and keeping the students' focus is also an issue. Running classes over the summer also involves planning out a syllabus for the course and deciding how you are running the class, because it is not identical to classes that are run during the semester. The material is the same, but the structure is up to the instructor to decide. As this was my first class, I kept things fairly simple, but it was still a good exercise in planning and organizing a class.

A.1.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.1. Included in this section are

1. The course syllabus
2. A sample practice problem with its solution
3. A sample quiz with its solution
4. A review sheet handed out before the final exam

As this was my first summer class, I stuck fairly close to the standard syllabus and class format. The only main thing I introduced were the Practice Problems that were done in the middle of class.

A.1.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix E, at the end of this document. A few selected comments from these surveys are

- "I love to see teachers who are passionate about what they are teaching it makes me wish I could enjoy the subject as much as they do."
- "He is always showing us how even the hardest of topics can be tackled in the simplest of ways."
- "He was always encouraging and made sure everyone had a fair chance at asking their questions as well as doing a great job answering them."

- “Mr. Charnley was excellent in explaining all the concepts, presenting examples and applications, and answering any questions I had.”

A.1.4 Class Reflection

This was my first time teaching a class for myself, and for a first experience, I think it went really well. I had an 8 AM class time, which I know can be difficult for students, but I tried to make sure to bring as much energy to the class as I could in the morning. The students seemed very responsive to the early class, much more than I had anticipated, and I think that fed into the atmosphere of the classroom. Even though I was doing a lot of lecturing during the class time, I tried to encourage discussion and questions from the students to help break up the class period. Twice a week, the second half of class was devoted to a recitation-type period, where students were encouraged to ask homework questions, and I would work out solutions on the board. I think this also helped to break up the aggressive schedule of meeting four days a week for two hours at a time.

As the summer courses are supposed to be equivalent to those taught during the semester, the topics on the syllabus were pretty much set for me from the start. In addition, since this was an 8 week class, there were actually more class meetings than a lecture would get during the semester, so it was pretty easy to figure out how to schedule everything for the summer. This also meant that, because the semester courses have recitations, there was also supposed to be some time for that built into the schedule. My general plan for the course just came from putting all of that information together. Thankfully, it worked out nicely, but it was good practice for the more complicated courses I needed to plan out in future semesters.

A component that I added to this class that is not usually seen during the semester were my ‘Practice Problems.’ These were generally given out around the middle of class for two purposes. They were meant to give the students a chance to practice what I had just been talking about, and they were also a way to break up the 2 hour lectures. I would tend to lecture for around an hour, then give the students a practice problem to work on, followed by a break. Depending on the class, after the break would consist of more lecture, recitation time, or a quiz. I felt like these worked out well, but I should have worked them more into the lecture. That is, I should have given them time to work on the problem, and then discussed it afterwards. I did this a few times, with some of the harder sections, but it probably should have been done more often. These practice problems continued to be present in all of my later classes, and I tried to make them more involved in the structure of the class as a whole.

Overall, this class was a lot of work, but a great experience. I had been a TA for the same class the previous semester, so I knew the material very well, but being in front of the class the entire time, and being the main lecturer, was new to me. I very much enjoyed the experience and felt like I better understood the dynamics of a classroom environment after this class. I think I did a good job promoting the type of environment that I wanted to see in the classroom, but it could have been better. This is something that I have striven to improve on in all of my future classes.

A.2 Summer 2016 - Math 477

A.2.1 Class Overview

Math 477 is a class on the Theory of Probability. This is the senior-level version of probability, requiring Calculus 3 as a prerequisite, which goes into both discrete and continuous probability. It covers the general axioms of probability, a basic introduction to combinatorics (in terms of counting outcomes for probabilities), independence, conditional probability, expectation, and the same concepts for continuous random variables. It is also a prerequisite for the classes on stochastic processes and mathematical statistics.

During the summer, this class was run as a 6-week course, meeting 4 days a week for 2 hours each meeting. With only 6 weeks, the class moved very quickly, and it was a struggle to keep up with everything. With this, there was room for only one midterm along with the final exam, and only 4 quizzes. There would generally be a lot more of these during the semester, but there really isn't time for it over the summer. This was also the second class I had taught over the summer, so I had some experience with this format, but the class itself was very different.

A.2.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.2. Included in this section are

1. The course syllabus
2. A Calculus 3 review that was given to the students early on in the class to make sure they were ready for the second half of the course
3. A sample homework solution that was posted for the students
4. A sample quiz with its solution
5. A topic list for the final exam that was given to the students
6. A set of review problems for the final exam

A.2.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix E, at the end of this document. A few selected comments from these surveys are

- "in class practice problems that reinforced what the class learned that day"
- "go over the harder homework problems assigned. give more examples that were not in the textbook"

- "This class was tough and had dry material but the professor definitely made it more bearable"
- "I found that the last three chapters seemed rushed. A longer focus on this material may be more beneficial."
- "I would assign less homework; maybe one assignment every other day. Even though you should be studying (i.e. doing problems) every day in a summer course, some days it was hard to complete homework on time if you have other obligations in addition to the course."

A.2.4 Class Reflection

This class was one of the most difficult experiences I had teaching a class. The main reason for this was that I have never taking a college level probability class, and so had to learn the material right before teaching the class. This meant that not only was I not as confident in my knowledge of the material as I should have been, but I also didn't know any of the common pitfalls or issues that students would run into with the material over the course of the summer. There were many instances over the summer where immediately after class, I knew that it didn't go as well as I would have liked, and I knew what I needed to change to make it better. It was a frustrating experience, but by the time the class started, there wasn't really an opportunity to make up ground and fix the problem because of how fast the course progresses. The midterm is three weeks in, and by that point, the class is already done with discrete random variables and moving through continuous probability. I felt like I was behind where I wanted to be the entire class and didn't really have the ability to make it up, which can be seen in the fact that at least one student felt that the last three chapters were rushed. On top of the issues with the material, I felt like this course had a fairly distinct end point (getting to the law of large numbers), which meant that even though I felt like I was behind, I wanted to get to that ending, causing me to rush through the last part of class.

I also had difficulty with structuring this class properly because of the schedule. Since this is only a three credit class, it only runs for 6 weeks and there is no built-in time for recitation sessions. Thus, for a class like this, it is supposed to be basically all lecture over the summer, which is impossible and doesn't work very well in a class that meets 4 times a week for 2 hours at a time. It was very difficult to cover the necessary material within that time, with also trying to allow for breaks and the practice problems that I used the previous year. My intention at the start of the summer was to lecture for about an hour, give a practice problem, and then lecture for 40 minutes or so after that. In reality, since I was not as strong with the material, the first part of lecture regularly took 70 to 80 minutes, meaning that after the practice problem (which also took longer than expected because I didn't know how to write good and easy problems on the material) left like 20 minutes to wrap up the class. I never really got much done in this time, but felt like I needed to keep pushing on, otherwise I was not going to make it through all of the material that I wanted to for the class.

This class really taught me how important it is to be prepared for a class; not just in terms of knowing the material, but knowing how to teach it. I felt like I knew a decent amount of probability going into this class (since I had worked through the book on my own to learn it) but I didn't really know how to teach it. I didn't know where the common mistakes were going to be and what problems to assign to flush them out. I didn't have an array of extra examples in my head that I could bring out to use at any time, so I mostly stuck with using the examples in the book because I didn't know any better. The class still went ok, and I had plenty of students do well, but I knew that I could have done better with it.

A.3 Summer 2017 - Math 244

A.3.1 Class Overview

Math 244 is one of Rutgers' two classes on Differential Equations. This is a four-credit version of the class that mostly consists of students from the School of Engineering. For some of them, it is the last math class they need to take, while the rest of the students will go on to take Math 421. The class covers the basics of differential equations: Solving first-order equations, second and higher order equations, linear systems, and analysis of non-linear systems. As the course is meant for engineers, it tends to focus on applications, showing the engineers how the material in this class will be used in their future courses and jobs. It also includes an introduction to linear algebra, since that is not a prerequisite for this course and is required in order to analyze the linear systems that they will see. Since they will see more linear algebra in 421, this is a very brief introduction, only covering the material needed to do the problems from this class.

As a four credit class, the course lasted for 8 weeks, meeting 4 days a week for 2 hours. My version of the class covered the same material as during the semester, but my course looked substantially different because I flipped my class, recording video lectures and posting them for the students to watch before they came to class. In class, they would then work on problems in groups, discussing the material with their classmates and me, and then class would end with either in-class presentations or a quiz. While watching the videos and reading the sections in the book, students would have a worksheet to fill out that they would bring to class before starting on the group problems, but these were eventually replaced with quizzes at the start of class due to poor attendance at the start of class. The idea with this class format was to focus the in-class time on doing problems, which, in my opinion, is the most important part of Differential Equations, and how a student will best learn the material. I also developed two new projects for this class since I felt that projects (and more involved applications of the material in general) were important for engineers to see while they were in my class. All of this went into how I decided to run my class over the summer.

A.3.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.3. Included in this section are

1. The course syllabus
2. The Fluid Flow project that I created
3. The Bifurcation project that I made for the class
4. A problem set that was done in the library to give them practice with numerical methods

5. A fairly standard problem set to be worked on in class
6. A sample worksheet that the students would do while watching the videos
7. The worksheet quiz that corresponds to the previous worksheet that would be completed when the students arrived to class

The first three documents here after the syllabus are the things I am most proud of from this course. These are three project type activities that I designed in addition to the content needed to flip the course. The first two of these were designed to be completed by the students individually over the course of a week. The first, on Fluid Flow, was meant to introduce another application of differential equations, modeling of fluid flow, to the class, and give them an opportunity to play with it more. The second, the Bifurcation project, was meant to let them explore the idea of bifurcations, which are not generally covered in this class, through numerical experiments. The last of these assignments was supposed to let them practice numerical methods in a computer lab, where they could actually code the methods, but didn't work nearly as well as I would have hoped.

The last two documents show how the videos were integrated into the class, with the worksheet that the students would need to complete while watching the videos and the corresponding quiz that was given at the start of the next class. In addition, all of the vidoes that I created for this course are also on YouTube. The channel can be found here:
`https://www.youtube.com/channel/UCWmjNk4wyUW98C-SrctULQQ`.

A.3.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix E, at the end of this document. A few selected comments from these surveys are

- "The teaching style of this class was great for actually learning the material and understanding it."
- "Very helpful in class, encouraged questions, and was every enthusiastic about this course in class."
- "He has reinvigorated my passion for the subject"
- "I like the structure of the class. It is very different than the other calc classes I have taken at Rutgers. The way this class is set up encourages learning in a great new way. All math classes at Rutgers should be taught in this way. Active collaborative learning is the best!"
- "I believe that although there are online videos available, a brief lecture in class is still necessary."
- "Do an example before class to make sure students understand the videos"

A.3.4 Class Reflection

This class was a fantastic experience. It was a lot of work, even more than I had expected, but as my first foray into the world of Active Learning, I think it went very well. On they whole, the class seemed to buy in to what I was trying to do and participated in the activities and group work in class. I got several comments, both on the end of semester surveys and during the class itself that students enjoyed the videos and like the way the class was run. I feel like the students learned a lot in this class, and I gained a lot more experience in running an active classroom. I also learned what needs to go into preparing and recording video lectures, which, as the way things are heading with technology in the classroom, will likely be very useful in the future. Recording lectures caused me to think a lot more about what I was saying or writing and why I was doing it. Trying to keep videos under 10 minutes when recording lectures for a differential equations class, which has long computations involved, is difficult, and it made me realize what was really important in the lecture and what I could cut out. It also encouraged extra thought into the structure of the lecture, because even though the lectures ended up being 3 or so videos per section, they still needed to be distinct videos, containing some form of complete information on a topic. I know this practice of planning lectures carefully has already come into play with more recent classes I have taught, and it will continue to do so.

The semester was not without its issues, however. At the beginning of the semester, there was a student who contacted the Summer Section coordinator to tell him that I was 'not teaching' the class, because I was having the students watch videos outside of class and in class had them working on problems. The coordinator responded and took care of the situation and I never heard anything else after that. I also had an issue of students not arriving to class on time in the morning. My assumption to this end was that, since I was not lecturing in class, the students who were showing up late didn't feel like they were missing out on anything by showing up late. It got to the point where I needed to change something, because students were showing up late enough to throw off the class. This motivated me to change the worksheets that students would do outside of class while watching the videos to worksheet quizzes that the groups would have to do at the start of class. These quizzes were graded as a group (resulting in less grading for me) and needed to be turned in within the first 15 minutes of class, forcing students to be on time. I could have also moved some of the actual quizzes to the start of class, which is something I decided would happen in my future classes, not only for making sure students arrive on time, but also give them time to practice the material outside of class before the quiz. I had a few students make comments to that end during the class, and I feel like that's something that could be implemented fairly easily. I did not want to do this initially because my class started at 8 AM, but I could (and probably should) have done it anyway.

In addition with getting practice with a more active classroom, this summer also afforded me the opportunity to start working towards designing my own assignments and classes that were not quite identical to the ones that are run during the semester. The Fluid Flow project was a little bit of a struggle for the students to get through, but I think it went well on the whole. I'm fairly certain the students did not enjoy having to present problems in front of the class, but I think most of them gained something out of the experience. Overall, the combination of designing my own assignments as well as running a flipped classroom is an experience that will pay dividends as I continue to improve as an instructor in years to come.

A.4 Summer 2018 - Math 252

A.4.1 Class Overview

Math 252 is Rutgers' other version of Differential Equations. This one is more directly aimed at math majors (or non-Engineering majors) and requires Linear Algebra as a prerequisite. This allows the class to cover the same amount of material as Math 244, while only being a 3 credit class. This works out fairly well during the school year, but as a summer class, it means everything is even more compressed than it was in Math 244. In addition, this class ran 3 times a week, for 2 hours and 45 minutes a session, which is a significantly different schedule than what I had taught for summer courses in the past. In addition to the accelerated rate of covering material, this assignment also brought with it the challenges of how to handle a class period that was twice as long as a normal lecture during the semester, while still covering the material that would be discussed over a full week of classes.

In order to do this, I ran this course as a very active classroom. The goal was to minimize the amount of time I was lecturing to the students and maximize the amount of time they would spend working on problems during class. They were free to work in groups and discuss these problems, and I hoped that this format would allow them to get more comfortable talking about math, as well as help them understand the material better. To facilitate spending time on problems in class, I needed to move some of the introduction of material to outside of class. This was done via assigning sections of the textbook to read, which was tested via open-book Readiness Assessments at the start of class. In addition, to cut down on the amount of grading I had to do for the class (and prevent the academic integrity issues I saw the previous summer), no homework was collected. Instead, problems were recommended, and these were assessed via Mini-Quizzes that happened at the start of every class that didn't contain a larger assessment. Examples of these will be presented on the following pages.

A.4.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.4. Included in this section are

1. The course syllabus
2. A sample Mini-Quiz with its solution
3. A sample practice problem with its solution
4. A sample Readiness Assessment with its solution
5. A fairly standard example of a problem set that would be worked on in class
6. Another problem set that had more of an activity component to it

7. The set of review problems that made up the Gallery Walk activity on the first day of class

8. Assignment sheet for the bifurcation jigsaw activity

9. Assignment sheet for the oscillators workshop activity

10. Assignment packet for the SIR modeling activity

11. Assignment sheet for the end-of-semester student generated test questions

After the syllabus, the next four documents outline the general procedure of each class: a mini-quiz, followed by a readiness assessment, the problem set, and then a practice problem. The rest of the materials beyond that are different projects and activities that I developed to add more to the active components of the course. The next problem set was given out as a standard problem set, but the second half of it has more of an activity component, where the students worked on a problem in groups and wrote their final answer on the chalkboards around the room so that everyone could see it. The next set of problems was put around the room on large poster paper on the first day of class, and everyone did one of the problems to help review for the prerequisite midterm that they had on the second day of class.

The next three documents are three of the new activities that I designed for this class. The first is a jigsaw activity based around bifurcations, where each group of students would work one a different bifurcation problem, and then the groups were mixed so that each new group had someone from each old group, and they all discussed the problems they had previously solved. The next activity was an exploration of harmonic oscillators, with the goal of them getting to dive deeper into these calculations on their own to get a feel for how it works. The last one, on SIR models, gave them a bunch of examples of models similar to SIR, and this was provided with MATLAB code that ran these models, so the students could play with the models and see how they work. The final document included in this section is an assignment for student generated test questions, which is something new that I tried for this class, and then gave these questions back to the class as a review sheet.

A.4.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Chapter E, at the end of this document. A few selected comments from these surveys are

- "I really enjoyed the active learning activities that we did. Each of the practice problems and mini quizzes helped reinforce what we learned and helped me have a better sense of what problems I understood and which I needed to review again."

- "I like how the professor would engage the students and would always be receptive to questions. I also liked how I never was afraid to ask questions or felt stupid for asking."
- "Maybe shift the class towards being slightly more lecture based but I think the course worked well as is."
- "I was not really pleased with the readiness assignments he gave us because they were unnecessary."
- "So many ways. I want to be a high school math teacher so every class I take I'm always learning classroom management and teaching techniques from the teacher even if it isn't an education class. This class deepened my love of calculus and my want to be a teacher. The professor is inspired, engaging and implemented many different learning techniques."

A.4.4 Class Reflection

Overall, this class was a success and a great conclusion to my run of teaching summer classes at Rutgers. The structure of the course caused me to think a lot about how I planned my lessons and what was really important to discuss in class. I also had the pleasure of meeting with an instructional designer to talk about how to plan out a class that meets for almost three hours at a time, and how to structure it in a way that allows for active learning in the classroom. This led me to figure out what the main components of the course were and design my lessons and weekly plans around getting these topics across to the students. I feel like this could have been done a little bit better, but it was a good start to the process, and a good exercise that I will continue to do in future classes. I felt that by the end of the class, I could have used some extra time, and a little extra foresight would have helped me speed things up at the start of the course to get this extra time at the end. I also noticed that, when structuring the class to involve short lectures and in-class problem solving, the material covered in the lectures gets reduced to only the essential elements, which means there isn't time to talk about more of the side material or personal experiences. There were several moments in the course where I felt like I could have spent quite a bit of time talking about related ideas, but just did not have the time with this format. It was an interesting dilemma, and something that I think requires more consideration in the future.

This was also the first class where I attempted the Readiness Assessments and Mini-Quiz approach. I think, on the whole, the Mini-Quizzes were successful. They did eat up a little bit of class time, but I feel like they did the job of making sure the students had, at least to some degree, understood the material from the previous class before we moved on to new material. It made for extra preparatory work on my part to get these problems ready, but slightly reduced the grading, as I only needed to look at one problem for each student instead of a full homework set. The Readiness Assessments, on the other hand, were less successful. The goal of these was to make sure that students had read the sections in the textbook before coming to class, so I only needed to give a brief lecture on the material before they would be ready to work on problems. After the first one didn't go so well, I decided to make the rest of the open book, open note, but timed, so that if you hadn't read the sections in advance, you likely would not be able to complete the assessment in time. Grades on these stayed relatively low throughout the semester, and I have a few ideas as to why. The first is the possibility that the class was just not reading the textbook before class, and besides making these worth more, there's not too much to be done about that. Another possibility is that they didn't understand what I was expecting them to get out of the reading. I had hoped that this would sort itself out as the class went on, but I'm not sure if it did. This could be remedied by providing notes on the section(s) in addition to the textbook, or switching the in-class assignments to worksheets like I had the previous summer. The last option is that I was writing the Assessments too hard or too complicated that the students could not complete them before class. I feel like this may have been the case for a few of the assignments, so making them simpler may have been more beneficial to the students. The worksheets might be the way to go here; while they would give me less control over the environment in which the students were completing the assignment, it would make it more likely that they could find the answers and get things sorted out before coming to class. In addition, the student feedback showed that they were also not thrilled about the readiness assessments, which I think was a combination of the fact that they did not go well overall and that students were unsure how to do well on them. Both of these could be improved in the future by making my expectations more clear and writing better assignments.

Another addition that I made to this class was a prerequisite midterm on the second day of class. Since this course requires linear algebra as a prerequisite, I wanted to make sure the students were aware of this and that I was expecting them to be able to do linear algebra when we came to that point of the course. I think the prospect of having an exam on the second day of class scared the students a bit, but it did make sure they were ready for the class. Scores on this exam were high (as they should have been) and I believe the exam got my expectations across to them more clearly than anything I could have said on the first day of class. I also introduced Exam Rewrites to this course, mostly copied from another colleague at Rutgers. With this, students were able to rewrite any problem from either of the midterms (the prerequisite exam or the actual midterm) that they did not get full credit on for a chance to earn back half of the points they missed. In order to do so, they needed to write up a completely correct solution to the problem, but could use their textbook, classmates, or me to make sure that this happened. For the prerequisite midterm, this went very well, but there were a few problems on the second midterm that several students did not manage to write correctly. While these were difficult to grade, I feel like giving the students the opportunity to review their mistakes (and encouraging them to do so with points) was worthwhile.

In addition to the standard 'active learning' problem solving sessions, I also created activities to take place during the Wednesday classes after quizzes.

These included more group-centered activities like jigsaws or group presentations. The goal with these was also to encourage discussion and help the students to become more confident in discussing math with each other. I feel like the activities did a fairly good job of this, but they could have been better designed to accomplish more at the same time. For instance, I did not anticipate how difficult it would be to write a jigsaw activity that would allow all of the groups to finish at approximately the same time. The other activities were similarly successful, but had their issues with bringing the class back together as a whole after working on individual problems in groups. This is something that I feel like I will get better at personally with more practice.

I feel like a lot of these experiences are things that I will take with me as I move on to teach more classes in the future. I think the active components of the class went well, and will definitely keep trying to implement them in my future classes. It was also a good experience to see that not everything works the first time I try it out. While the Readiness Assessments are something that I took from other instructors, they had not really been used in math classes before. Conceptually, they are a good idea, but the implementation may not have been the best, and this takes trial and error. The same goes for any new component I try to add to a class; the way I try to include it may not work perfectly well the first time, and that is ok. It is still worth it to try them out and see what kind of feedback I get about it, so that it can be improved the next time around. The end results of this class also taught me about sticking to the numbers in terms of assigning final grades. Even though students do well and are active participants in class doesn't necessarily mean that they deserve an A in the class. They have to put in the work for it and do well enough on the exams. I had several students this summer that were active participants and were involved throughout the entire class, but ended up with a B based on exam and other scores. I felt like I wanted to give them a higher grade, because they were good participants, but the numbers said otherwise, and so I had to stick with those grades. All in all, I think this class taught me a lot about myself as well as how stepping out of the lecture role doesn't necessarily mean the students learn less in the class. They may actually learn more this way, and I definitely plan to take that with me in the future.

A.5 Fall 2018 - Math 104

A.5.1 Class Overview

Math 104 is an introduction to probability class directed at students in the humanities and social sciences. There are a few majors, including Human Resource Management, that require this class, but for most other students, they are in this class because they are interested in probability and it meets the university's quantitative requirements. It is possibly the hardest class, outside of the calculus sequence, that meets these requirements, so it poses an interesting set of challenges to teach. The course is basically an introduction to discrete probability, counting problems, conditional probability, and the normal distribution and Z-scores.

This was a very interesting course to teach because it is at a much lower level than anything I had taught up to this point. Everything else I had taught involved calculus, and so the students involved are at a level of mathematical maturity that has gotten them to that point. For this class, however, there are many students who have not taken a math class in several years and think about problems significantly differently than those who are taking calculus. It was also a good experience teaching this class as two of the other instructors in the department who had taught this class many times before provided me with most of their in-class materials. Preparing for this class then consisted of looking over the textbook and both sets of notes from the two other instructors and trying to combine them into something that I enjoyed and that worked with me as an instructor.

This class was also the first time I was teaching a class that met on a standard two days a week for 80 minutes schedule, which I had to adjust to. After the first few weeks, I settled in to how much material I could actually cover in an 80 minute class and determined the best way to bring the practices I had developed from my summer classes into this shorter class. I decided to bring back my practice problems from before, but these would take place at the end of class instead of the middle. The beginning of class would consist of a short homework quiz on the set that had been turned in previously (and which the students already had the answers to), followed by an interactive lecture, with the class wrapping up with a practice problem. I think this format worked well for this class in terms of keeping people interested in the material and showing them how to do problems before they have to do them for homework.

A.5.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.5. Included in this section are

1. The course syllabus
2. A worksheet I designed about counting problems, which was built from previous work with Pascal's Triangle

3. A worksheet I designed introducing the idea of continuous distributions and the normal distribution

4. A sample practice problem that students were given at the end of class

5. A sample "small quiz" that was given to students the class after turning in a homework set

6. A sample "big quiz" that was given to cover material from about two weeks worth of lectures

7. A sample homework solution that was given to the grader

A.5.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Chapter E, at the end of this document.

A.5.4 Class Reflection

I feel like teaching this class was a great experience for me on several levels. First of all, teaching a class that is separate from the Calculus sequence was a very interesting challenge, especially near the end of the class, where the topics covered in class were things that I had learned using Calculus. I had to think about different ways to present the material in order to avoid using both the methods and terminology from Calculus when discussing things like normal distributions. For instance, the concept of "area under a curve" isn't really something that they have seen before, so discussing continuous probability distributions (and, in particular, the normal distribution) becomes a much trickier prospect. After a decent amount of planning, I believe I was able to cover the topics in a way that was accessible to the class, but also covered them in a way so that if they were to see them again later, they would be able to adapt their knowledge to the new setting. This class taught me a lot about how to present things to students and attacking problems in ways that are different from the way that I would normally approach them in order to accommodate the different prior knowledge and skills of my students.

Secondly, this is the first time that I taught a class in 80 minute blocks, with another class immediately before and after mine. In the previous classes I had taught, my room was either empty before or after my class, so I didn't have to be too worried about when I got there or exactly when I ended. In addition, my summer classes were at least 2 hours long, so there was plenty of time to do a variety of activities, and I had a feel for when the class would be getting close to done. With the 80 minute blocks, I did not have this intuition, and so had to make sure to keep an eye on the clock to make sure I did not go over time. I also had to recalibrate how much material could be covered in a class, because it was substantially less than during a two-hour class over the summer. It took some getting used to, but I got better at this during the semester, and it's something that I know I will continue to get better at as I teach more classes under these kind of time constraints.

Finally, this is the first time I taught a class during the semester, where the class runs over a 14 week period instead of 6 or 8 weeks over the summer. There is a very different type of planning that needs to happen for a semester long class instead of a summer class, particularly in how time needs to be budgeted. Over the summer, I tended to spend almost all of my time working on class preparation and grading during the short time the class was happening, and then would do my other work over the rest of the summer. That is not possible for a semester class, so I needed to learn how to ration my time and not spend all of it on teaching, because I had other things that needed to get done while the class was running. It took some getting used to, but I figured out a way to organize my time to allow me to both prepare for the class as much as I felt I needed to, as well as get the other work done that needed to happen. I know that in future jobs, I will likely be teaching multiple classes at the same time, and being able to organize my time will be a very important part of staying on top of everything that I need to get done. Outside of the experience with the actual class material, this practice with organizing my time was one of the most important skills I learned this semester.

Appendix B

Teaching Assistant Assignments

For graduate students at Rutgers, semester stipends are generally provided by teaching assistant positions. These positions generally involve the graduate student working along side a professor or instructor, running an extra meeting for the class to go over homework or provide extra time for the students to practice problems. Assignments generally involve either recitations, which contain mini-lectures on the material from class and discussions of homework problems, or workshops, where a sizable amount of time is spent with the students in groups working on more complicated and advanced problems. There is also the option for a TA-at-Large assignment, which will be discussed in more detail in the appropriate section. These assignments provide graduate students who want to go into teaching an initial opportunity to interact with undergraduates and get a feel for how the classroom works. These experiences became very valuable when I moved on to teach my own class over the summer.

B.1 Fall 2014 - Math 135

B.1.1 Class Overview

MATH 135 is Rutgers' version of Calculus for Non-Majors. This class serves students in some of the science and business majors who need Calculus for their programs, but do not need the rigor of the MATH 151 sequence. Students in these classes are also generally not able to go on to Calculus 3 (MATH 251) unless they jump to the engineering/majors version of Calculus 2 first. When I taught this class, the TA responsibilities took place in 55 minute recitation sections, consisting of around 35 students each, and all 105 of them had a single lecture twice a week. Since then, the 135 recitations have been extended to 80 minutes. but the general setup has remained the same. During the semester, I was also responsible for creating and grading the quizzes that were given each week in class.

B.1.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.6. Included in this section are

1. My syllabus for the recitation sessions
2. The set of practice problems that I handed out at the first recitation
3. A full solution to a quiz that was posted online for the students
4. A document sent to the students to help them review the topics for the common final exam

B.1.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix E, at the end of this document. A few selected comments from these surveys are shown here.

- "Teacher is very friendly; he closes the distance by memorizing every single students' name after one or two classes..."
- "Matt did a great job of answering out questions, and helping us through the hard concepts. He was very relatable."
- "The TA is helpful and supportive of [finding things out and linking concepts], even when its not during his office hours I have emailed him and received a quick response with help but not a totally detailed solution – he helps you solve the problem."
- "It isn't an easy subject for me to understand, but he made it easy for me to understand."

- “He worked to make sure that every student’s questions were answered to their satisfaction. He helped me achieve a deeper understanding of calculus, a topic I expected to struggle with.”

B.1.4 Class Reflection

I feel like this was a great class for a first teaching assistant assignment. Since the material was Calculus 1, particularly for non-majors, I did not need to spend time reviewing the content for the class, and could spend more of my time working on how I was going to present the material. With only 55 minute class periods, I had to be particular with how I spent my time, since there was nowhere near enough time to talk about everything that I wanted to in each class. I think having these short recitations helped me learn how to prioritize the important topics within a class session. That is definitely something that has persisted throughout all of my other teaching assignments, even though all of them have had a longer amount of time in each class period.

I also think I began to discover my personality as an instructor during this class. As can be seen in the comments from students, I tend more towards the friendly side with my class. I believe strongly in the interactive side of teaching: that students learn best by doing problems, talking about them, and being told not only when their answers are wrong, but also what things they are doing correctly. I tried to emphasize this with my students this semester, and I think it went very well. Finally, this class affirmed how much I enjoy teaching at the college level, and motivated me to continue to pursue teaching positions both as a TA and as a full instructor during summer courses.

B.2 Spring 2015 - Math 251

B.2.1 Class Overview

MATH 251 is the Calculus 3 class offered at Rutgers. This class is taken by people in engineering, physics, and math majors, and follows in the standard calculus sequence. The teaching assistant duties for this class included running 80 minute recitations for three sections of the class each week, each containing around 20 students. I was also responsible for writing and grading quizzes for the class. This class also introduces students to Maple, a computer algebra system that can be used to visualize surfaces in three dimensions. They complete five labs over the course of the semester that walk them through setting up programs to look at things like curvature, Lagrange multipliers, and vector operators. As the TA, I was also in charge of helping the students complete the labs and understand how Maple works.

B.2.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.7. Included in this section are

1. My recitation syllabus handed out during the first week of class
2. Solutions to a sample quiz given during recitation
3. An example document on Clairaut's theorem that was distributed to the class to illustrate that the conditions of this theorem are necessary

B.2.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix E, at the end of this document. A few selected comments from these surveys are

- "Matt has a very upbeat personality and really seems like he enjoys teaching."
- "stop being condescending."
- "The recitations were a perfect supplement to the lectures... Matt usually had a much clearer organized way of teaching and explaining things."
- "... You made the material so much easier to understand and you made calc 3 as fun as it could possibly be."

B.2.4 Class Reflection

This class was another great experience for me. I very much enjoyed the calculus 3 material, since it is more similar to what I am interested in for research. As the students were slightly more advanced and I have a background in engineering, I was able to hint at things that they would see in later classes, which I think they found useful. The recitation sections were 80 minutes long instead of 55 minutes with my first assignment, I had more time to go over material in class, but I still had to be careful with my time, since problems in calculus 3 take much longer to solve than problems in calculus 1, especially those at the 135 level. It was also my first introduction to Maple, as I had never used it before I was assigned to teach this class.

An important experience from this class for me is the fact that not everyone in a class is going to be a fan of how I teach and how I present things. I try to be more on the friendly side with my students, but sometimes my personality may not rub off on people in a positive manner. No matter what I do in the classroom, I will not be able to please everyone at all times. I can try as hard as I want, but eventually, I will have to make a decision about how things will be handled in the class and go with it. This class overall helped me to be more confident in the decisions I make with how I run a classroom, which definitely played a role later on in how I designed my classes over the summer.

B.3 Fall 2015 - Math 251

B.3.1 Class Overview

This was more or less the same class that I had been a teaching assistant for the previous Spring. The only difference is that it was an honors section of Calculus 3. This meant that all of the students in the class were members of the honors college, the class could ostensibly be taught at a higher level, and all of the sections were smaller. There were only about 50 students total between all three of my recitation sections, as opposed to the 75 I had during the Spring semester. Overall, the course material was identical to what had been covered in the previous semester, so I was covering the same material with a smaller and more involved class of students.

B.3.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.8. Included in this section are

1. My recitation syllabus handed out the first week of class
2. My written-up solution to part of the midterm that was given during my recitation session
3. Images of the lecture notes for the first section of extra video lectures that I recorded to make up for missed material in class

The most interesting of these is the last one: the set of lecture notes from the video lectures. This was my first endeavor into recording content online that students would be able to view on their own schedule. I also had to create the notes for these lectures, which was a great experience of itself.

B.3.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix E, at the end of this document. A few selected comments from these surveys are

- "It cleared up anything I didn't understand in lecture. Was very clear and easy to follow."
- "I liked how Matt was always positive in class and clearly explained the material."
- "He made the material much easier to learn, which made me more motivated to listen and stay attentive in class."
- "... Matt always made himself available to help us learn more outside of the classroom. He knows exactly what he is talking about."

- “Matt was one of the best math TAs I’ve ever had. He was detailed and extremely clear with all the lessons he taught.”
- “Matt has really sparked my interest in mathematics and made multivariable calculus an easy to understand and fun course.”
- “Class was not only informative and interesting but fun as well, making me excited for calculus class each week.”

B.3.4 Class Reflection

This semester of Calculus 3 was another growing experience. With the smaller class size and the fact that the students were substantially more involved with the class, I felt like I got to know my students better over the course of the semester. This semester, I was also coming off of having taught Calculus 3 the previous summer, which was a very interesting transition. While I liked being in the teaching assistant role, being much less involved with the class was a little difficult for me. There are things that came up over the course of the semester that I would have done differently, but as the teaching assistant, I could only do so much. It was a good lesson in knowing how to fill my role in a class and knowing where the boundaries are in what I can and should do.

At the end of the semester, the lectures for my class had ended up about 4 lectures behind, skipping some material at the end of the semester. In particular, the class had missed out on Stokes’ Theorem and the Divergence Theorem, both of which are concepts that show up in later engineering classes. In order to compensate for this, I decided to do a series of online lectures during Winter break that the students could watch to learn about these concepts. I recorded the lectures from my house and posted them, along with the notes I had written up during the lecture, to my website. I don’t know if any of the students watched the lectures or learned anything from them, but I enjoyed making them. At the time, I thought it was something nice to do for my students, but in retrospect, I feel like this was a first step towards the flipped classroom I ran during the Summer of 2017. Without making these short lectures, I don’t know if that class would have happened in the form that it did.

B.4 Spring 2016 - Math 575

B.4.1 Class Overview

MATH 575 is a class on Numerical Methods in PDEs for students in the Mathematical Finance Masters Program. It is generally taken by students in their first or second year of the program. They are required to take a Numerical Analysis course in the fall semester, and then can either take Numerical Analysis 2 or Numerical Methods in PDEs during the Spring. The course covers finite difference and finite element methods, containing both theoretical exercises and programming problems. The goal is that after leaving this class, students will be able to understand how numerical methods work, and potentially be able to write programs to do so for their financial jobs in the future.

B.4.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.9. As mostly a homework grader, I didn't have too much of a role in generating content, outside of homework solutions. Included in this section are

1. A sample homework solution done in MATLAB
2. A sample homework solution with more conceptual type problems
3. A sample piece of Python code from another homework set
4. A document outlining some properties of variational formulations, which I noticed the students were having trouble with

B.4.3 Student Feedback

There was no student feedback provided for this course.

B.4.4 Class Reflection

For this semester, I served as a grader for the class. My responsibilities were to hold office hours, grade the weekly homework sets (including coding exercises) as well as helping to proctor the exams. The teaching assignment as a whole was not all that interesting. I got some more experience with coding in Python, which some of the later assignments involved, but not much else came out of the assignment.

B.5 Fall 2016 - Math 501

B.5.1 Class Overview

During this semester, I served as a teaching assistant for the two first year graduate analysis classes: MATH 501-Real Analysis I and MATH 503-Complex Analysis I. My responsibilities included completing the homework sets each week so that I could present them to the class, prepare problem sets consisting of exercises from the written qualifying exams, and run recitation sessions where I would go over all of these with the students. Each week, I would run a single 80 minute recitation session where I would cover material from both classes. I also provided extra assistance to the students during office hours if necessary.

B.5.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.10. Included in this section are

1. A problem set from the middle of the semester, containing written qualifying exam problems
2. The last problem set of the semester, containing a topic review to help them prepare for the finals

B.5.3 Student Feedback

There was no student feedback provided for this course.

B.5.4 Class Reflection

This class was a nice experience for me. I was allowed to review the material from the two first year classes and got a chance to present higher level material to other graduate students. It required a decent amount of preparatory work on my part in order to get ready for each of the sessions, but it was worth it for the experience of running this class.

One of the big things that came out of this semester was the fact that I need to have stronger boundaries with my students. Over the course of this semester, I felt like several of the students took advantage of the fact that I was always around my office and only a few feet away from the first year office to come by very frequently to ask for help. In semesters since then, I have tried to make sure that my office hours are explicitly laid out in my syllabus and tried to make myself much less available outside of those times. I am still willing to set up appointments with students, but I try not to meet with students outside of these times.

B.6 Fall 2017 - Math 421

B.6.1 Class Overview

MATH 421 at Rutgers is a class called Advanced Calculus for Engineers. It serves as the last math class that some of the engineering majors need to take for their major. The class covers Laplace Transforms, Matrix Algebra, Fourier Series, and Separable PDEs. It is an assortment of topics that the engineering department decided that they wanted their students to know. This class was also a unique assignemnt as it was a 'TA-At-Large' position. I did not have to run recitation sessions for this class, but was instead assigned to two 90-student sections of MATH 421. I held two sets of in-person office hours each week along with one set of online office hours. These online office hours were recorded and posted onto the Learning Management system so that students could view them later. The last bit of my responsibilities were to run review sessions for each of the sections before each of their midterms and the final.

B.6.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.11. Included in this section are

1. A worked out homework exercise that was posted to the students after they were having trouble with these types of problems
2. A description of Sturm-Liouville problems to help with something the students were not understanding correctly
3. A review sheet handed out for the second midterm
4. Images of the whiteboard that was projected to the students during one of my online office hours
5. The handout for a workshop that was run at the end of the semester to help review for the final.

The whiteboard screens provide a decent picture into what online office hours looked like. I generally had one or two students show up to each online office hours, and I feel like those students got a lot out of it. I don't know how many of them watched the office hours later, but I don't think it was that many. The last document is something that the other TA for MATH 421 and I did entirely on our own, and was beyond what we needed to do for the course. We held a workshop-type session where students from all 4 sections were able to come to work on problems with the two TAs there to help them figure things out before the final exam.

B.6.3 Student Feedback Summary

The full set of responses from the student surveys for this class can be found in Appendix E, at the end of this document. A few selected comments from these surveys are

- "Matt is the best TA I have evern had and is one of the smartest people I have even talked to. No matter how poorly you word a question, Matt somehow knows what you mean and has an answer that goes above and beyond."
- "It would have been nice to see Matt in class more because he was always so helpful..."
- "Always helpful and showed me the 'behind the scenes' of how we could get to certain conclusions, made me more interested in math."
- "Good sense of humor with a very positive attitude toward students."
- "Matt gives the most practical essence of the course."

B.6.4 Class Reflection

This class was a very interesting experience for me. The 'TA-At-Large' assignment was different than anything I had ever experienced as an instructor. I felt like I had a very minimal connection with the students in the classes throughout the semester. Up until the first review session, I only interacted with the few students that decided to attend office hours, and was feeling fairly unfulfilled with my teaching assignment. After the first review session, a few more students decided to come to office hours, since they realized I was a person who existed and could help them, and things got a little better. I also felt like I had very little impact on the class, and had to again remind myself that it was ok if I was not in control of everything, especially with this semester coming on the heels of my flipped classroom for MATH 244. I think it was in this semester that I learned how much I value the connection that can be established between an instructor and students, and how far this can go to foster learning of the material. Once I was able to actually talk with students about the material, they seemed to be able to understand it better.

At the end of this semester, the other TA for MATH 421 and I decided to run workshop sessions for the students to cover the material that was going to be on their final exam. We prepared packets for them on each of the 4 main topics that were covered in the class for them to work through. The actual sessions were 3 hours long, and there were two sessions run for each of two weeks, with each week covering two of the topics. After an entire semester of not really interacting with students, the workshops were fantastic. We had about 100 students in total (out of about 300) show up to at least one of the sessions, and it seemed like they got a lot out of it. We also very much enjoyed getting to actually help students through problems instead of them just bugging us for answers when they didn't get something right. It was also nice to be in charge of a session like this because we knew exactly what was going to be covered, as opposed to the normal behavior during the semester, where we would be almost guessing as to what was covered in class, and didn't really have any definitive information to give to students. In addition to the importance of the connection between instructor and student, I feel like these workshops reinforced my belief in collaborative learning, and how working in a group setting helps everyone involved to learn and understand everything better.

B.7 Spring 2018 - Math 575

B.7.1 Class Overview

For this semester, I was given the Masters in Math Finance assignment again. This time however, there were actual recitation sessions involved and it was connected to both the Numerical Methods in PDE (575) class and the Numerical Analysis 2 (574) class. The recitations were once a week for each class. In one of the classes (575) I was responsible for collecting homework, going over homework solutions, and giving the students a quiz. For the second class (574) I was only responsible for going over homework solutions for the assignment that had been collected the day before. I was not responsible for grading this time around, but the recitations sessions made up for that.

B.7.2 Sample Course Materials

Sample materials that I created for this course can be found in Appendix D.12. My main responsibilities for this course were creating quizzes and homework set solutions for the course. Included in this section are

1. My recitation syllabus from Math 574
2. A solution set to an optional homework set for Math 574 at the end of the semester that was posted for the students to use for review
3. My recitation syllabus from Math 575
4. A diagnostic linear algebra quiz that was given to students early in the semester
5. A sample homework set solution that was given to the students and the homework grader

B.7.3 Student Feedback

Student feedback is included in Appendix E. There aren't any comments of note to include here.

B.7.4 Class Reflection

I was not entirely thrilled with this assignment, but I tried to make the most of it. Each week, I was responsible for going over the homework solutions with the class. I would be responsible for writing up these homework solutions with basically no guidance from the instructors. In addition, because the class is largely international students, they had a strong desire to see the solutions written out in full detail, and so I was also responsible for typing, editing, and posting the solutions to the course website. As this was either material that I had barely learned 3 years ago or had never seen before, writing up these solutions was no simple task. On several occasions, I had to look up books or other resources to learn about the material before I could actually write solutions and solve the problems. It took up a very large portion of my time, and seemed on the whole like it didn't really have an impact on the students in the class.

As to the student population, it was challenging to get them to be involved in class while I was trying to run recitations. At this point, they had already turned in the homework, and so either understood the assignment and didn't need my help, or were past it and ready to move on, and didn't want to understand it. I also had issues with students trying to talk during quizzes, which was not even a problem in my basic calculus classes. This was the first time that recitations had been run for this class, so I think there is some adjustment to be made in this area, both on the end of the instructor/teaching assistant and from the students.

This semester also ended with me filling in for the instructor of the 575 for lectures while continuing to run recitations, write up homework solutions, and give quizzes. While the instructor was still preparing the lectures and problem sets, since I was not too knowledgeable about the material, it took a significant amount of time to get the lecture ready for class. The last few weeks of the semester were hectic with this addition, but I got through it, and I think teaching a master's level class for a month is an experience that will stick with me into the future. In particular, it showed me how different levels of student need to be treated differently, and somehow not in the ways I would expect from the outset. It also taught me the importance of preparing my own lecture notes for a course, because the lectures where I had enough time to somewhat prepare my own material went much smoother than the ones where I was reading straight from the professor's lectures.

Appendix C

Teaching Development Activities

This chapter contains summaries of the other teaching related activities that I participated in during my time at Rutgers. These include the Directed Readings Program, where advanced undergraduate students meet one-on-one with a graduate students to work through a topic of their choosing. The topics are generally not covered in the curriculum that the students would see normally over the course of their undergraduate careers, so this gives them a look into material they would not get to experience otherwise. In addition, this chapter will cover two different summer programs that I was involved in, the DIMACS REU, and the Rutgers Youth Scholars Program. It will also discuss the Math Teaching Group, which I co-organized for three years, and teaching-related presentations that I gave over the course of my Rutgers career.

C.1 Directed Readings Program

The Directed Readings Program at Rutgers provides an opportunity for motivated undergraduate students to explore topics outside the normal undergraduate curriculum in an independent study format. The leaders of these projects are graduate students, and pairings are made between undergraduate students and graduate student volunteers based on a survey that the undergraduate student completes when requesting to be enrolled in the program.

The general outline of the program is that during the first few weeks, the pair settles on a project that both the undergraduate is interested in and the graduate student has some knowledge in. After that, they meet for around 1 hour per week, talking about the topic, and the undergraduate student is expected to do around 2 to 3 hours of work on the project per week outside of the meetings. At the end of the semester, the undergraduate gives a 15 minute presentation on what they learned over the course of the semester.

I have had the privilege of working on 5 of these projects with fantastic undergraduate students. The topics of these projects were as follows.

C.1.1 Schrödinger Equation

The first DRP project that I worked on was with a student who was double majoring in math and physics. With an interest in PDEs and quantum mechanics, we settled on a project that would work through the Schrödinger equation and the model of the Hydrogen atom. The end goal for this project was a worksheet that I had received from Brian Hall (University of Notre Dame) during a workshop on Quantum Mechanics for Mathematicians. The worksheet presented a series of steps for solving the separated variables version of the Schrödinger equation for the Hydrogen atom. In order to attack this worksheet, the student and I needed to go over some basic material on PDEs, as well as the derivation and solution of the Heat and Wave equations. This provided a solid backing for talking about the Schrödinger equation for the second half of the semester. The last part of the semester involved us working through the entire worksheet, seeing where the fact that the quantum numbers of the atom needed to be integers, and the student's presentation outlined some of these concepts. I also helped the student with writing code that would plot some of the images from the work on the Hydrogen atom, showing the shapes of the orbitals that he had seen in Chemistry classes, and these were included on a handout given to the attendees of his presentation. I feel like this was a very successful first project and first foray into mentoring an undergraduate.

C.1.2 Fourier Analysis

My second DRP project was with a math major who wanted to learn about Fourier Analysis. We spent the semester talking about the basics of Fourier Series, both in terns of the standard sine and cosine functions and general orthogonal functions. We spent some time discussing the Gibbs Phenomenon and uniform convergence of Fourier Series. The semester then rounded out with a discussion of Fourier Transforms.

C.1.3 Functional Analysis

The third DRP project that I was a part of involved a student who was also a double major in math and physics. However, instead of an interest in the PDE aspect of physics, her concern was more with the δ function that had been used somewhat haphazardly in her physics classes. She wanted to see more of a rigorous justification of this 'function', and so we developed a project in Functional Analysis to get to this end. The basic trajectory of our project was to start with an introduction to Banach and Hilbert spaces as extentions of standard finite dimensional vector spaces over the real numbers. This gave the proper framework to talk about Linear Functionals, of which an example is the δ function. To wrap up the semester, we talked about general results in functional analysis, like the Hahn-Banach theorem, and discussed orthogonal polynomials, including the Legendre and Hermite polynomials, both of which she had also previously seen in physics classes.

C.1.4 Markov Chains

The fourth DRP project I helped mentor involved a math major who had already taken several probability classes and was fairly interested in that area. She brought forward a suggestion of looking at Markov chains for a project, and we went with that as a starting point. We spent the first part of the semester going over the basic definitions of Markov chains, as well as the subclasses of absorbing, ergodic, and regular Markov chains. We spent a decent amount of time going over the proofs of each of the different results so that she would be able to present them at the end of the semester. As the semester began to draw to a close, the student brought up the Metropolis algorithm as a potential application of Markov chains to discuss, which I had never heard of. After discussing the algorithm for a few weeks, we (mostly led by the student) figured out how and why it worked, and looked into other ways to apply the algorithm. The one we settled on was the Travelling Salesman problem, and code was written to simulate this procedure. I then helped the student prepare her presentation, which covered the basics of Markov chains, one major proof in the area, and discussed the application of this result to the Metropolis algorithm and approximate solution of the Travelling Salesman problem.

C.1.5 Fourier Analysis Round 2

During the summer, I served as a mentor for another undergraduate student who was looking to learn about Fourier Analysis. As this student was spending the summer away from campus, most of the meetings took place remotely, with the student sending me notes on the sections that I recommend, and me giving feedback on these notes. While the program started with talking about Fourier

Series, we moved on to Fourier Transforms, which provided a nice conclusion for the summer project.

C.2 Summer Programs

C.2.1 DIMACS REU

During the summer of 2015, I was, along with another graduate student, in charge of helping run the DIMACS REU that takes place every year at Rutgers. My role in this event was as an administrator. I helped make sure the students had places to stay before they arrived, organized the offices that the students would be working in while on campus, and planned several events over the summer. I was basically in charge of making sure that everything ran smoothly, including sorting out technical issues during the first week, ordering t-shirts for all of the attendees (35), and making sure students attended all of their presentations and turned work in on time.

A second component of the program involved a trip to Prague with 6 of the students at the end of the summer. Since I was going on this trip, I was also in charge of running the 'Bridge' sessions, which consisted of the 6 students going to Prague meeting with the 7 students who were at the REU from Prague to talk about topics in combinatorics. I ran a few of these sessions, the Czech students ran a few others, and the last ones were run by outside speakers from the math department at Rutgers. As I am not a combinatorist by trade, this was more challenging for me, but I was able to figure out the material well enough to teach the students who would be on the trip with me. The end of the summer resulted in a 2 week trip to Prague, where the first week consisted of workshop sessions with faculty from Charles University on combinatorics topics and the second week was filled with attending the Midsummer Combinatorial Workshop, held at Charles University.

While I am fairly certain I overbooked myself for this summer, the REU was a great experience. I got extra practice working with and mentoring undergraduates, including ones from all over the country. I got practice in conflict resolution whenever students had issues with each other, as well as dealing with maintenance and housing people when there were problems with the living arrangements. I also got exposed to a fair amount of combinatorics over this summer, which I think has done a lot to improve my well-roundedness as a mathematician.

C.2.2 Head TA

For the summers following my third, fourth, and fifth years as a graduate student, I served as the Head TA for the summer session. The role of the Head TA is to serve as a mentor for all of the graduate students who are teaching over the summer. Before classes start, we meet with graduate students, particularly those who had not taught in the past, in order to discuss their syllabi and how they plan to run the course. Once classes have started, we go around to observe all of the graduate student instructors and give them feedback on their teaching, meeting with them a few days later to discuss what we saw. While the feedback from the Head TA doesn't get recorded anywhere permanent, the goal of it is to help the graduate student improve their teaching before their official observation happens later in the summer. We also serve as a resource to all of the other graduate students for anything they would like to discuss about teaching or any issues that arise in their classes.

I feel like this position was a great experience, not only in that I was able to help out many other students in my department, but that I got to observe so many other students teaching, all of whom had a different approach to their class than I did. When I went to observe students, I took notes on what they were doing in class, not only to give them feedback later on, but also to remember the good things they did that I could potentially add to my own class. It also put me in direct contact with the other graduate students who cared about teaching and allowed us to combine and discuss ideas for classes, helping all of us to get better at it for future summers and future jobs. My summer classes have steadily gotten better not only because I have improved as a teacher, but also because I have had this opportunity to observe and discuss teaching practices with a large range of graduate students during my time as the summer Head TA.

C.2.3 RYSP

The other summer program I participated in was the Rutgers Youth Scholar's Program. This is a summer camp for high school students interested in math, where they learn about different topics in Discrete Mathematics from different instructors over the course of 4 weeks. I was brought in to teach the Combinatorics class during the second week of the program in Summer 2018. This was a very interesting experience for me because I was teaching a topic I didn't really know that much about to high school students. While the preparatory work took longer than I had expected because I didn't really know what to cover in a week of Combinatorics, but once I got all of that together, everything went very smoothly. The students were super motivated and picked up on everything I was trying to tell them very quickly, much faster than I anticipated. It was a good experience in writing note sheets for a group like this, since it was very different from anything I had done before for teaching. I am not sure how this experience will directly apply to in-class activities during a normal class, but it definitely showed me how these types of summer programs can run and that I would definitely be interested in being involved in programs like this in the future.

C.3 Teaching Group

During my 4th year at Rutgers, I, along with another graduate student, restarted the Math Teaching Group. This had been a seminar in previous years, but had stopped running due to either a lack of interest or lack of motivation. We decided that there should be a place where students in the math department could get together to talk about the issues that arise in teaching, from pedagogical decisions to dealing with students. There were seminars for subgroups in Analysis or Algebra, why shouldn't there be one for teaching? With that motivation, we emailed the administrators and got the seminar restarted.

For the first semester, we had the luxury of running alongside a MAA webinar about undergraduate education. This webinar met every few weeks, and gave some semblance of structure to a few of these sessions. Outside of that, we met to talk about whatever topics people found interesting. Usually this topics was decided on the week before, and someone (usually me) would come up with some notes to guide the discussion.

The second semester did not have the MAA webinar to follow along with, and with more members of the group getting more busy, several of the weeks just resulted in us getting together to talk about what was going on in our classes. While this was a nice discussion, it was not necessarily the most structured. Therefore, we attempted to fix this going into the next fall. We tried to have a more involved layout of topics, and pick topics at the beginning of the semester instead of waiting until the last minute. This helped quite a bit with the organization of the seminar and kept things on track. It also gave us the ability to invite outside speakers to talk, because things were planned out more than a week in advance.

Both the fall and spring of that academic year were much better for the teaching group. Topics ranged from the Honors Program at Rutgers, to Academic Integrity issues, to the TA Training program that first year graduate students partake in every year. We also started to have conversations with students from the Graduate School of Education, inviting them to join us in our discussions about teaching. We had outside speakers come in from the Learning Centers, the Graduate School of Education, as well two instances where former graduate students came back to talk about their experiences teaching at places other than Rutgers.

Overall, I think this teaching group has been a great experience for me, as well as a huge boost to my development as a teacher. It has gotten me to think more critically about how I design and handle things in the classroom, as well as given me specific opportunities to talk to other like-minded people about their teaching strategies, so that I can improve mine. I also do not think I would have been as motivated to try what I did in my classes had it not been for the Teaching Group and the discussions I had there.

C.4 TA-At-Large Program Organizer

In the Fall of my 4th year at Rutgers, I was assigned a role as a TA-At-Large (TAAL) for the semester. Part of this assignment was to hold both in-person and online office hours, and the online ones generally took place in a single room in the math building. I had been working to schedule my office hours, when it became clear to me that no one in the department was actively doing anything to get these courses set up. When I discovered that, I decided to take on this task and get the program set up myself.

The duties for being the *de facto* TAAL coordinator consisted of contacting all of the other TAALs to have them provide me with their rosters so that the students could be added to the corresponding online course. I was also in contact with the technology staff to help decide what platform we were going to use for the semester, as well as to provide them with the roster data and the information from the corresponding TA who needed access to be able to run the session. I helped to organize the training sessions, where new TAALs would be shown how to use the online platform, and scheduled the online office hours because all of the 12 TAs for the courses needed to use the same room for their sessions, and I served as the go-between for the TAs and the technology staff to make sure that everything worked out for both parties.

When the same problem arose at the start of the following Spring semester, a lot of the TAALs came to ask me what the situation was, because no one else had any information on how this program worked. Since the classes needed to get up and running, I took charge of the program again that semester, coordinating between the TAs and the technology staff to make sure the office hours happened, even though I was not a TAAL that semester. In order to shift these duties off of me before the following Fall semester, I prepared a document outlining the process of getting the online office hours started and passed that on to the Undergraduate Vice Chair. The hope of this was that the department would know what the process was and that which ever faculty member was assigned to this task would know how to do it as effectively as possible.

C.5 RASTL and TA Project

The Rutgers Academy for the Scholarship of Teaching and Learning (RASTL) is a invitation-based group of graduate students from various academic departments around Rutgers who meet on a monthly basis to discuss various issues in teaching and pedagogy. I have been a part of this group since May 2016 and have gotten a lot out of the opportunity to discuss these issues with students and teachers from other departments, as the problems they face in their classes are significantly different from those that I have seen. I have been able to take some of their experiences and adapt them to my own classes, changing the way I handle things in the classroom for the better. During my time in RASTL, I have also helped run a few of the sessions, leading discussions on 'Engaging Students and Managing Discussions' and 'Classroom Expectations'.

My time in RASTL has also led to my becoming more involved with the TA Project at Rutgers. This group organizes the TA Orientation for incoming TAs each fall and runs a workshop series every semester for current or future TAs to learn about different issues that may arise while they are teaching a class. For the last three years, I have been one of the TA Orientation leaders from the Math department, which means that I went to the orientation to give them advice on how the TA program works within the math department and how to make the most of it. I have also been a leader for several of the workshops run by the TA project, with topics including 'Teaching a Summer Course', 'Teaching Non-Majors', and 'Providing Feedback that Matters'.

Appendix D

Sample Course Materials

D.1 Summer 2015 - Math 251

This section contains the following document from my Summer section of Math 251:

1. The course syllabus
2. A sample practice problem with its solution
3. A sample quiz with its solution
4. A review sheet handed out before the final exam

MATH 251, Section C1: Summer 2015

Syllabus

Instructor – Matt Charnley

Email: charnley@math.rutgers.edu

Office: Hill 606, Busch Campus

Office Hours: Monday and Tuesday, 10 am – noon, or by appointment.

Note: Office Hours on June 1 will be moved to June 4.

Course Website: SAKAI and math.rutgers.edu/~mpc163/Courses/SM15_MATH251.html

Class Meetings

MTWTh, 8:00am – 10:00 am, SEC-218

Exams

First Midterm Exam – Thursday, June 11th

Second Midterm Exam – Thursday, July 2nd

Final Exam – Thursday, July 16th

Course Information

Information about this course can be found on SAKAI, as well as the course website. SAKAI will be updated very regularly, and this will be the best place to obtain course materials. My personal website may lag behind in updates.

Grade Breakdown

Grades will be assigned according to the following weights

Homework & Practice Problems	10%
Maple Assignments	5%
Quizzes	15%
Midterms	17.5% each, 35% total
Final Exam	35%

Textbook

Jon Rogawski; *Calculus: Early Transcendentals.*
Custom Edition for Rutgers University; 2011; ISBN: 1-4641-0376-3

Class Attendance

Attendance at ALL class meetings is mandatory. Attendance will be taken at each class, and will factor into final grade assignments in the form of the practice problems and quizzes. This class will move very quickly through material, even more so than in a normal Spring or Fall semester, and missing one day of class will put you significantly far behind. If you must miss class for any reason, you should let me know well in advance, and plan to stop by office hours to catch up on missed material.

Quizzes

Thirty minute quizzes will be given every Thursday at the end of class, covering the material from that week of class. These will be closed book, closed note quizzes. Calculators and other electronic devices will NOT be allowed on any of the quizzes. You will be provided with any formulas you may need. The goal of these quizzes is to give you more practice working problems in an environment similar to that of an exam. There will be 5 of these quizzes over the course of the class. There will also be smaller 'practice problems' that will need to be written up during most class sessions. These will be handed out in the middle of the lecture, and will be open-book, open-note exercises.

Homework

Homework will be collected almost every day at the beginning of class. If the homework is not turned in to me by 8:10, it is late, and will receive at most half credit. If I do not have it by 9:00, it will not be accepted. In order to get credit, you must show all your work. The problem sets due each day will be assigned in class the day before. A tentative schedule is outlined below, but the final schedule will be announced in class, since it may change due to the rate at which we cover material. The homework sets are all assigned at the back of this packet, and are taken from the normal online syllabus:

http://www.math.rutgers.edu/courses/251/syllabushw2nd.html

I would highly recommend doing all these problems, because it will give you more practice with the material and a better understanding of it. However, since it is a shorter class over the summer, not all of these problems will be assigned or graded.

Problem Sessions

Almost every week, we will have two problems sessions for the second half of the class period. This will be time for you to ask about any homework problems that have been causing difficulty or go back over topics that are confusing. I will not prepare anything specific to talk about in these sessions, but will leave the entire time for you to ask questions. This is to fill the role of recitations during the Spring and Fall semesters. If there are no questions from the class, then I will just continue lecturing on the next section.

Maple Assignments

Maple is a computing software commonly used to carry out complicated calculations or visualize problems in 3 dimensions. This course requires 3 Maple assignments in addition to an optional introductory one. The first Wednesday of class, we will have a brief introduction to Maple and how to use it. After that, there will be three labs due over the summer. Those dates are already available below, and all of the necessary files will be posted on Sakai. These labs will need to be given to me, as a hard copy, on the specified due date. Emailed labs will not be accepted.

Exams

There will be two midterm exams given on the dates above. Calculators and other electronic devices will NOT be allowed on any of the exams. These will be 80 minute exams, which will start at 8:00 each day. These will be closed book, closed notes, with a formula sheet provided by me. After the 80 minutes, we will go over the exam in class. The final exam will be **three hours** on July 16th, with the same procedure. The exact time of the final exam will be specified at a later date, but it will run either 7:00 – 10:00 or 8:00 – 11:00 am.

Make-Up Policies

Exams and quizzes can NOT be made up under any circumstances. If there is a legitimate excuse for missing an exam, i.e. a doctor's note, then we can discuss a way to make the grades work out in the end, but you will not be allowed to take the exam late. Homework and Maple assignments will not be accepted past their due date.

Academic Integrity

All students in this course are expected to be familiar with and abide by the academic integrity policy (http://academicintegrity.rutgers.edu/academic-integrity-at-rutgers). Violations of this policy are taken very seriously. In short, don't cheat, and don't plagiarize.

Disability Accommodations

I will be happy to provide appropriate accommodations for students who provide me with a letter of accommodation from the Office of Disability Services (ODS). For more information, see http://ods.rutgers.edu/.

Class Schedule

Date	Sections	Assignments Due
5/26	12.1, 12.2, 12.3	
5/27	12.4, Intro to Maple	HW 1 Due
5/28	12.5, QUIZ 1	HW 2 Due
6/1	13.1, Problem Session	HW 3 Due
6/2	13.2, 13.3	HW 4 Due
6/3	13.4, Problem Session	HW 5 Due
6/4	14.1, QUIZ 2	HW 6 Due
6/8	14.2, Problem Session	HW 7 Due
6/9	14.3, 14.4	HW 8 Due
6/10	Catch Up / Problem Session	HW 9 Due
6/11	EXAM 1, Go over Exam	Maple Lab 1 Due
6/15	14.5, 14.6	
6/16	14.7	
6/17	14.8, Problem Session	HW 10 Due
6/18	15.1, QUIZ 3	HW 11 Due
6/22	15.2, Problem Session	HW 12 Due
6/23	15.3	HW 13 Due
6/24	12.7, Problem Session	HW 14 Due
6/25	15.4, QUIZ 4	HW 15 Due, Maple Lab 2 Due
6/29	15.4, Problem Session	HW 16 Due
6/30	15.6	HW 17 Due
7/1	16.1, 16.2	HW 18 Due
7/2	16.3, 16.4	HW 19 Due
7/6	Catch Up, Problem Session	HW 20 Due
7/7	EXAM 2, Go over Exam	HW 21 Due
7/8	16.5, Problem Session	
7/9	17.1, QUIZ 5	Maple Lab 3 Due
7/13	17.2, Problem Session	HW 22 Due
7/14	17.3	HW 23 Due
7/15	Catch up, Review	HW 24 Due
7/16	FINAL EXAM	HW 25 Due

Homework Sets

Sections	**Problems**	**Tentative HW Set Number**
12.1	5, 11, 15, 21, 39, 45	1
12.2	11, 13, 25, 33, 53	1
12.3	1, 21, 29, 66, 71	2
12.4	5, 11, 22, 39, 41	2
12.5	1, 13, 17, 33, 51	3
13.1	4, 7, 15, 21	4
13.2	10, 27, 28, 31, 45	5
13.3	3, 11, 14, 18	5
13.4	3, 17, 19, 23	6
14.1	7, 19, 20, 23	7
14.2	7, 15, 22, 29	8
14.3	3, 19, 22, 33, 49, 55	9
14.4	1, 7, 15, 21, 25	9
14.5	7, 13, 38, 39, 44	10
14.6	1, 5, 13, 27, 39	10
14.7	1, 3, 10, 19, 21, 35	11
14.8	2, 7, 11, 13, 15	12
15.1	17, 27, 29, 41, 45	13
15.2	3, 11, 27, 32, 33, 49	14
15.3	5, 9, 15, 19, 35	15
12.7	1, 5, 23, 35, 42, 53, 59	16
15.4	1, 5, 13, 17	17
15.4	22, 23, 27, 33, 41, 47, 49	18
15.6	1, 5, 15, 29, 33,	19
16.1	1, 3, 10, 15, 25, 31	20
16.2	3, 15, 21, 35, 43	20
16.3	5, 9, 11, 15, 21	21
16.4	1, 5, 8, 17, 23	21
16.5	1, 9, 12, 15, 17, 23	22
17.1	1, 6, 9, 13, 23, 25	23
17.2	1, 5, 9, 11, 21, 23	24
17.3	1, 5, 7, 9, 16, 17	25

MATH 251: Practice 10

June 8, 2015

Name: Solutions

1. Compute the gradient of

$$f(x, y, z) = e^{xy} + 2xz^2 + \sin(yz)$$

$$\nabla f = \langle ye^{xy} + 2z^2,\ xe^{xy} + z\cos(yz),\ 4xz + y\cos(yz) \rangle$$

2. Find the directional derivative of f in the direction $\langle 1, 1\rangle$ at $(-1, 2)$ for the function

$$f(x, y) = x^2 + 2y^2$$

$$\nabla f = \langle 2x, 4y \rangle \qquad \nabla f \big|_{(-1,2)} = \langle -2, 8 \rangle.$$

$$\nabla f \cdot \vec{v} = -2 \cdot 1 + 8 \cdot 1 = 6.$$

$$D_{\vec{v}} f = \frac{1}{\|\vec{v}\|} \nabla f \cdot \vec{v} = \frac{1}{\sqrt{2}} \cdot 6 = \boxed{6/\sqrt{2}}$$

$$= \boxed{3\sqrt{2}}$$

MATH 251: Quiz 4

June 25, 2015

Name: Solutions

1. Integrate $f(x, y, z) = 2x + 3yz$ over the rectangular prism $0 \le x \le 2$, $1 \le y \le 5$, $0 \le z \le 1$.

/3

$$\int_0^2 \int_1^5 \int_0^1 2x + 3yz \, dz \, dy \, dx$$

$$= \int_0^2 \int_1^5 2xz + \frac{3}{2} yz^2 \Big|_0^1 \, dy \, dx$$

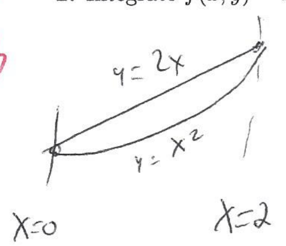
$$= \int_0^2 \int_1^5 2x + \frac{3}{2} y \, dy \, dx = \int_0^2 2xy + \frac{3}{4} y^2 \Big|_1^5 \, dx$$

$$= \int_0^2 8x + \underbrace{\frac{3}{4}(25-1)}_{18} \, dx = 4x^2 + 18x \Big|_0^2$$

$$= 16 + 36 = \boxed{52}$$

2. Integrate $f(x, y) = 2xy$ over the region between the graphs of $y = 2x$ and $y = x^2$.

/3



$$\int_0^2 \int_{x^2}^{2x} 2xy \, dy \, dx$$

$$= \int_0^2 xy^2 \Big|_{x^2}^{2x} \, dx$$

$$= \int_0^2 x(2x)^2 - x(x^2)^2 \, dx$$

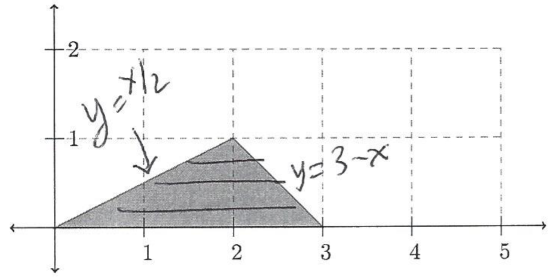
$$= \int_0^2 4x^3 - x^5 \, dx = x^4 - \frac{x^6}{6} \Big|_0^2$$

$$= 16 - \frac{64}{6} = 16 - \frac{32}{3}$$

$$= \boxed{16/3}$$

3. Integrate $f(x,y) = x + 2y$ over the triangle pictured below.

/4



$0 \le y \le 1$

$2y \le x \le 3-y$

$$\int_0^1 \int_{2y}^{3-y} x+2y \; dx\, dy$$

$$= \int_0^1 \frac{x^2}{2} + 2xy \Big|_{2y}^{3-y} dy$$

$$= \int_0^1 \frac{(3-y)^2}{2} + 2y(3-y) - \frac{(2y)^2}{2} - 2(2y)y \; dy$$

$$= \int_0^1 \frac{9}{2} - 3y + \frac{y^2}{2} + 6y - 2y^2 - 2y^2 - 4y^2 \, dy$$

$$= \int_0^1 \frac{9}{2} + 3y - \frac{15}{2}y^2 \, dy = \frac{9}{2}y + \frac{3}{2}y^2 - \frac{5}{2}y^3 \Big|_0^1$$

$$= \frac{9}{2} + \frac{3}{2} - \frac{5}{2} = \boxed{7/2}$$

4. Integrate $f(x, y, z) = x$ over the region in the first octant $[x \geq 0,\ y \geq 0,\ z \geq 0]$ bounded from above by the plane $x + 2y + z = 6$.

4

$0 \leq z \leq 6 - x - 2y$ $\qquad$ $6 - x - 2y = 0$

$0 \leq x \leq 6 - 2y$ $\qquad$ $x = 6 - 2y$

$0 \leq y \leq 3$ $\qquad$ $6 - 2y = 0 \Rightarrow y = 3$

$$\int_0^3 \int_0^{6-2y} \int_0^{6-x-2y} x \, dz\, dx\, dy$$

$18 \cdot 9 = 81 \cdot 2 = 162$

$27 \cdot 8 = 216$ $\qquad$ $36 \cdot 6 = 216$

$$= \int_0^3 \int_0^{6-2y} xz \Big|_0^{6-x-2y} dx\, dy$$

$\frac{64}{24} = \frac{\cancel{8} \cdot 6 \cdot 3 \cdot 3}{\cancel{8}} = 54$

$$= \int_0^3 \int_0^{6-2y} 6x - x^2 - 2xy \, dx\, dy$$

$$= \int_0^3 3x^2 - \frac{x^3}{3} - x^2 y \Big|_0^{6-2y} dy$$

$$= \int_0^3 3(6-2y)^2 - \frac{(6-2y)^3}{3} - y(6-2y)^2$$

$$= \int_0^3 3(6-2y)^2 - \frac{(6-2y)^3}{3} - y(36 - 24y + 4y^2)\, dy$$

$$= \int_0^3 3(6-2y)^2 - \frac{(6-2y)^3}{3} - 36y + 24y^2 - 4y^3 \, dy$$

Using $u = 6 - 2y$, $du = -2y\, dy$

$$= \left[-\frac{(6-2y)^3}{2} + \frac{(6-2y)^4}{24} - 18y^2 + 8y^3 - y^4 \right] \Big|_0^3 \qquad \boxed{27}$$

$$= 0 + 0 - 18 \cdot 9 + 8 \cdot 27 - 81 + \frac{6^3}{2} - \frac{6^4}{24} = -\cancel{162} + \cancel{216} - 81 + 108 - \cancel{54}$$

5. Convert $(x, y, z) = (0, 3, 4)$ to both cylindrical and spherical coordinates.

/3

Cylindrical: $(3, \pi/2, 4)$

Spherical: $(5, \pi/2, \cos^{-1}(4/5))$

6. Convert the following equations to spherical coordinates.

/3

(a) $z^2 = x^2 + y^2$.

(b) $z = x^2 + y^2$.

(c) $x^2 + y^2 + z^2 = 4$.

(a) $\cos^2\varphi = \sin^2\varphi \Rightarrow \boxed{\varphi = \pi/4.}$

(b) $\cos\varphi = \rho \sin^2\varphi \Rightarrow \boxed{\rho = \dfrac{\cos\varphi}{\sin^2\varphi}}$

(c) $\boxed{\rho = 2}$

Conversion Formulas

Cylindrical		Spherical	
$x = r\cos(\theta)$	$r = \sqrt{x^2 + y^2}$	$x = \rho\cos(\theta)\sin(\phi)$	$\rho = \sqrt{x^2 + y^2 + z^2}$
$y = r\sin(\theta)$	$\tan(\theta) = \frac{y}{x}$	$y = \rho\sin(\theta)\sin(\phi)$	$\tan(\theta) = \frac{y}{x}$
$z = z$	$z = z$	$z = \rho\cos(\phi)$	$\cos(\phi) = \frac{z}{\rho}$

MATH 251 Final Exam Review

Matt Charnley

July 7, 2015

These are some sample problems for the final exam. Look at these problems, the ones on previous reviews, as well as ones in the book to prepare for the exam. There may be things on the exam that are not directly listed here. The problems may be more complicated than the ones here.

1. Draw the following vector fields.
   - $\langle x, y\rangle$.
   - $\langle -y, x\rangle$
   - $\langle x^2, 2y\rangle$.
2. Check if the vector field $\vec{F}$ is conservative.
   - $\vec{F} = \langle yz, xz, xy\rangle$.
   - $\vec{F} = \langle x^2, xy, 3z^3\rangle$.
3. Compute a line integral (scalar and vector versions).
$$\int_{\mathcal{C}} \langle 2x, 3xy\rangle \cdot d\vec{s} \qquad c(t) = \langle \cos(t), \sin(t)\rangle \quad 0 \le t \le \pi$$
$$\int_{\mathcal{C}} x + y + z \; ds \qquad c(t) = \langle t, t^2, \frac{2}{3}t^3\rangle \quad 0 \le t \le 1$$
4. Use the Fundamental Theorem for Conservative Vector Fields to compute Line Integrals
$$\int_{\mathcal{C}} \langle 2x, 3y\rangle \cdot d\vec{s} \qquad c(t) = \langle \cos(t), \sin(t)\rangle \quad 0 \le t \le \pi$$
$$\int_{\mathcal{C}} \langle e^x + y, x + 2yz, y^2 + z^2\rangle \; ds \qquad c(t) = \langle t, t^2, t^3\rangle \quad 0 \le t \le 1$$
5. Find a potential function for $\vec{F} = \langle yz + 2x\sin(z), xz + 3y^2, xy + x^2\cos(z)\rangle$.
6. Find the equation for a normal vector for the parametrizations and surfaces.
   - $\mathcal{S}_1 = G(u, v) = (2u + 1, u + v, 3u + v)$, $\mathcal{D} = \{0 \le u, v \le 5\}$
   - $\mathcal{S}_2 = G(u, v) = (u\cos(v), u\sin(v), 1 - u^2)$, $\mathcal{D} = \{0 \le u \le 3, 0 \le v \le 2\pi\}$
   - $\mathcal{S}_3 = G(u, v) = (u, v, uv)$, $\mathcal{D} = \{0 \le u, v \le 5\}$.
7. Find the tangent plane to any of the above surfaces at a given value of u and v.
8. Compute the area of the above surfaces.
9. Compute scalar and vector surface integrals.
   - Integrate $x^2 + 3yz$ over $\mathcal{S}_1$.
   - Integrate $\langle xy, x^2, z\rangle$ over $\mathcal{S}_1$ with the upward normal.

- Integrate $\langle x, y, z\rangle$ over $\mathcal{S}_2$ with the upward normal.
- Integrate z over $\mathcal{S}_3$.

10. Use Green's Theorem to compute line integrals, area of curves, area integrals.

    - Integrate $\langle x^2 \sin(x) - x^2 y, ye^y + x^2 y\rangle$ over the triangle bounded by $x = 0$, $y = 2$ and $y = x$ oriented counterclockwise.
    - Find the area of the curve parametrized by $\langle \frac{3t}{1+t^3}, \frac{3t^2}{1+t^3}\rangle$, $0 \leq t \leq \infty$.
    - Find the integral of $\langle x^2(1 - y), xy^2 + y\sin(y)\rangle$ over the top half of the circle $x^2 + y^2 = 1$, oriented counterclockwise.
    - Find the integral of $\langle x^2 \sin(x) + y^2, xy\rangle$ over the three sides of the square $[0, 1] \times [0, 1]$, going from $(0, 0)$ to $(1, 0)$ to $(1, 1)$ to $(0, 1)$.

11. Identify the boundary of:

    - $\mathcal{S}_1 = \{x^2 + y^2 + z^2 = 4,\ z \geq 0\}$.
    - $\mathcal{S}_2 = \{x^2 + z^2 = 9,\ 1 \leq y \leq 3\}$.
    - $\mathcal{S}_3 = \{x^2 + y^2 + z^2 = 16\}$.

12. Use Stokes' Theorem to compute line and surface integrals.

    - Integrate $curl(\langle -y + z, z^2 + 2x, xyz\rangle)$ over $\mathcal{S}_1$, upward normal.
    - Integrate $\langle x^2, y^2, z + 5\rangle$ over $\partial\mathcal{S}_2$, oriented correctly with $\mathcal{S}_2$ having the outward normal vector.

13. Identify the boundary of:

    - $\mathcal{W}_1 = \{x^2 + y^2 + z^2 \leq 4\}$
    - $\mathcal{W}_2 = \{x^2 + y^2 \leq 9,\ 1 \leq z \leq 4\}$
    - $\mathcal{W}_3 = \{x^2 + y^2 + z^2 \leq 1,\ z \geq 0\}$.

14. Use the Divergence Theorem to compute surface and volume integrals.

    - Integrate $\langle 2x, 3y, z\rangle$ over $\partial\mathcal{W}_1$ with the outward normal vector.
    - Integrate $\langle -3y, 2x^2, z^2\rangle$ over $\partial\mathcal{W}_2$ with the outward normal vector.
    - Integrate $\langle -3xy, 2x^2, z^2\rangle$ over $\partial\mathcal{W}_3$ with the outward normal vector.

Formulas

If we want to decompose $\vec{u}$ into $\vec{u} = \vec{u}_{//} + \vec{u}_{\perp}$ with respect to $\vec{v}$, then we have

$$\vec{u}_{//} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} \qquad\qquad \vec{u}_{\perp} = \vec{u} - \vec{u}_{//}.$$

Area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$: $||\vec{v}\times\vec{w}||$.

Volume of the parallelepiped spanned by $\vec{u}$, $\vec{v}$, and $\vec{w}$: $|\vec{u}\cdot(\vec{v}\times\vec{w})|$.

Curvature Formulas:

$$\kappa(t) = \left|\left|\frac{d\vec{T}}{ds}\right|\right| \qquad \kappa(t) = \frac{||\vec{r'}(t)\times\vec{r''}(t)||}{||\vec{r'}(t)||^3} \qquad \kappa(x) = \frac{|f''(x)|}{(1+f'(x)^2)^{3/2}}$$

Linearization of a function at (a,b):

$$z = L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b).$$

Gradient of a Function $f(x,y,z)$:

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

Chain Rule for Paths: For a function $F(x,y,z)$ and a curve $\vec{c}(t)$,

$$\frac{d}{dt}F(\vec{c}(t)) = \nabla F(\vec{c}(t))\cdot\vec{c'}(t)$$

Directional Derivative of the function f in the direction of $\vec{v}$:

$$D_{\vec{v}}f = \frac{1}{||\vec{v}||}\nabla f\cdot\vec{v}$$

General Chain Rule: For a function $f(x,y,z)$ with $x = x(s,t)$, $y = y(s,t)$ and $z = z(s,t)$, then

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$$

Implicit Differentiation: If $F(x,y,z) = 0$ and z can be written as a function of x and y, we have that

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Second Derivative Test: If (a,b) is a critical point of f, then for

$$D = f_{xx}(a,b)f_{yy}(a,b) - f_{xy}(a,b)^2$$

- If $D > 0$ and $f_{xx}(a,b) > 0$, then (a,b) is a minimum.
- If $D > 0$ and $f_{xx}(a,b) < 0$, then (a,b) is a maximum.
- If $D < 0$, then (a,b) is a saddle point.
- If $D = 0$, the test is inconclusive.

Conversion Formulas

Cylindrical		Spherical	
$x = r\cos(\theta)$	$r = \sqrt{x^2+y^2}$	$x = \rho\cos(\theta)\sin(\phi)$	$\rho = \sqrt{x^2+y^2+z^2}$
$y = r\sin(\theta)$	$\tan(\theta) = \frac{y}{x}$	$y = \rho\sin(\theta)\sin(\phi)$	$\tan(\theta) = \frac{y}{x}$
$z = z$	$z = z$	$z = \rho\cos(\phi)$	$\cos(\phi) = \frac{z}{\rho}$
$dV = r\ dr\ d\theta\ dz$		$dV = \rho^2\sin(\phi)\ d\rho\ d\phi\ d\theta$	

Change of Variables Formula: If $G : \mathcal{D}_0 \to \mathcal{D}$ is a map given by $G(u,v) = (x(u,v), y(u,v))$, then

$$\iint_{\mathcal{D}} f(x,y)\ dx\ dy = \iint_{\mathcal{D}_0} f(x(u,v), y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right|\ du\ dv$$

where

$$\frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v}\end{pmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

Line Integrals: For a curve $\mathcal{C}$ given by $\vec{c}(t) = \langle x(t), y(t), z(t)\rangle$ for $a \leq t \leq b$, a scalar function $f(x,y,z)$ and a vector field $\vec{F}(x,y,z)$,

$$\int_{\mathcal{C}} f(x,y,z)\ ds = \int_a^b f(\vec{c}(t))||\vec{c}'(t)||\ dt$$

$$\int_{\mathcal{C}} \vec{F}(x,y,z)\cdot d\vec{s} = \int_a^b \vec{F}(\vec{c}(t))\cdot \vec{c}'(t)\ dt$$

Surface Integrals: For a surface $\mathcal{S}$ given by $G(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$ for $(u,v) \in \mathcal{D}$, a scalar function $f(x,y,z)$ and a vector field $\vec{F}(x,y,z)$,

$$\iint_{\mathcal{S}} f(x,y,z)\ dS = \iint_{\mathcal{D}} f(G(u,v))||\vec{n}(u,v)||\ du\ dv$$

$$\iint_{\mathcal{S}} \vec{F}(x,y,z)\cdot d\vec{S} = \iint_{\mathcal{D}} \vec{F}(G(u,v))\cdot \vec{n}(u,v)\ du\ dv$$

Green's Theorem: For a domain $\mathcal{D}$ with $\partial\mathcal{D}$ positively oriented,

$$\oint_{\partial\mathcal{D}} \langle F_1, F_2\rangle \cdot d\vec{s} = \oint_{\partial\mathcal{D}} F_1\ dx + F_2\ dy = \iint_{\mathcal{D}} \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\ dA$$

$$\text{Area}(\mathcal{D}) = \frac{1}{2}\oint_{\partial\mathcal{D}} \langle -y, x\rangle\cdot d\vec{s} = \frac{1}{2}\oint_{\partial\mathcal{D}} x\ dy - y\ dx$$

Stokes' Theorem: For a surface $\mathcal{S}$ with $\partial\mathcal{S}$ positively oriented with respect to the normal $\vec{n}$ of $\mathcal{S}$,

$$\oint_{\partial\mathcal{S}} \vec{F}\cdot d\vec{s} = \iint_{\mathcal{S}} \text{curl}(\vec{F})\cdot d\vec{S}$$

$$\text{curl}(\vec{F}) = \nabla\times\vec{F} = \left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial y} - \frac{\partial F_1}{\partial x}\right\rangle$$

Divergence Theorem: For a volume $\mathcal{W}$ with boundary surface $\partial\mathcal{W}$ given the outward normal,

$$\iint_{\partial \mathcal{W}} \vec{F} \cdot d\vec{S} = \iiint_{\mathcal{W}} \operatorname{div}(\vec{F})\ dV$$

$$\operatorname{div}(\vec{F}) = \nabla \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Integral Formulas

$$\int \sin^2(x)\ dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C$$

$$\int \cos^2(x)\ dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C$$

D.2 Summer 2016 - Math 477

This section contains the following documents from my Summer section of Math 477:

1. The course syllabus
2. A Calculus 3 review that was given to the students early on in the class to make sure they were ready for the second half of the course
3. A sample homework solution that was posted for the students
4. A sample quiz with its solution
5. A topic list for the final exam that was given to the students
6. A set of review problems for the final exam

MATH 477, Section B1: Summer 2016

Syllabus

Instructor – Matt Charnley

Email: charnley@math.rutgers.edu

Office: Hill 606, Busch Campus

Office Hours: Monday and Wednesday, 10 am – noon in LSH 102B, or by appointment.

By appointment office hours will be held in Hill 606, Busch Campus.

Course Website: Sakai. Make sure you can access this as soon as possible.

Class Meetings

MTWTh, 8:00am – 9:55 am, TIL-103A

Exams

Midterm Exam – Thursday, June 16th

Final Exam – Thursday, July 7th

Course Information

Information about this course can be found on Sakai. Sakai will be updated very regularly, and this will be the best place to obtain course materials. Calculus 3, or multivariable calculus, (Math 251 at Rutgers) is a mandatory prerequisite for this class. You will need to know how to deal with multiple integrals to get through the course. Please see me with any concerns about this.

Grade Breakdown

Grades will be assigned according to the following weights

Homework / Practice Problems	15%
Quizzes	20%
Midterm	25%
Final Exam / Project	40%

Textbook

Sheldon Ross: *A First Course in Probability*, **9th edition (2012)**,
ISBN # 978-0321794772, Prentice-Hall.

Class Attendance

Attendance at ALL class meetings is mandatory. Attendance will be taken at each class, and will factor into final grade assignments in the form of the practice problems and quizzes. This class will move very quickly through material, even more so than in a normal Spring or Fall semester, and missing one day of class will put you significantly far behind. If you must miss class for any reason, you should let me know well in advance, and plan to stop by office hours to catch up on missed material.

Quizzes

Thirty minute quizzes will be given every Thursday at the end of class, covering the material from that week of class. These will be closed book, closed note quizzes. Calculators and other electronic devices will NOT be allowed on any of the quizzes. You will be provided with any formulas you may need. The goal of these quizzes is to give you more practice working problems in an environment similar to that of an exam. There will be 4 of these quizzes over the course of the class. There will also be smaller 'practice problems' that will need to be written up during most class sessions. These will be handed out at some point during the lecture, and will be open-book, open-note exercises.

Homework

Homework will be collected every day at the beginning of class. If the homework is not turned in to me by 8:10, it is late, and will receive at most half credit. If I do not have it by 9:00, it will not be accepted. In order to get credit, you must show all your work. The problem sets due each day will be assigned in class two days in advance, that is, homework assigned on Tuesday will be collected on Thursday, homework assigned on Wednesday will be collected on Monday, etc. Problem sets will be posted on Sakai, and any significant changes will be announced in class. A tentative schedule as to the sections that will be covered each day is outlined below, and the homework assignments will follow the same schedule, but are always subject to change based on the rate at which we cover material. This schedule is also fairly flexible, so pay attention in class or to Sakai for updates. For each assignment, there will be recommended homework problems and a smaller subset of them that will be collected and graded.

Homework assignments will consist of two types of problems: "Problems" and "Theoretical Exercises". The Problems are more computational, direct applications of the material in the sections, while the Theoretical Exercises require a little more thought to solve. When turning in the problems, these should be turned in separately; one packet with the Problems, and a second one with the Theoretical Exercises. There are also Self-Test exercises in the book, which are another set of practice exercises. The solutions to the Self-Text exercises are in the back on the book, so you can do these on your own to make sure you know how to solve the problems.

I would highly recommend doing the suggested problems in addition to the assigned ones, because it will give you more practice with the material and a better understanding of it. However, since it is a shorter class over the summer, not all of these problems will be assigned or graded. All of the homework sets are subject to change, and I will announce any changes at the end of each class and post them on Sakai.

Exams

There will be one midterm exam, currently scheduled for June 16th. Calculators and other electronic devices will NOT be allowed on either the midterm or the final. The midterm will be a 100 minute exam, starting at 8:10, and will cover chapters 1-4. It will be closed book, closed notes, with a formula sheet provided. The final exam will be **three hours** on July 7th, with the same procedure. The exact time of the final exam will be specified at a later date, but it will run either 7:00 – 10:00 or 8:00 – 11:00 am.

Make-Up Policies

Exams and quizzes can NOT be made up under any circumstances. If there is a legitimate excuse for missing an exam, i.e. a doctor's note, then we can discuss a way to make the grades work out in the end, but you will not be allowed to take the exam late. Homework assignments will not be accepted past their due date.

Academic Integrity

All students in this course are expected to be familiar with and abide by the academic integrity policy (http://academicintegrity.rutgers.edu/academic-integrity-at-rutgers). Violations of this policy are taken very seriously. In short, don't cheat, and don't plagiarize.

Disability Accommodations

I will be happy to provide appropriate accommodations for students who provide me with a letter of accommodation from the Office of Disability Services (ODS). For more information, see http://ods.rutgers.edu/.

Tentative Class Schedule

Date	Sections	Assignments Due
5/31	2.1 – 2.5	
6/1	1.1 – 1.6	
6/2	2.5, QUIZ 1	HW 1 Due
6/6	3.1 – 3.3	HW 2 Due
6/7	3.4 – 3.5	HW 3 Due
6/8	4.1 – 4.4	HW 4 Due
6/9	4.5 – 4.7, QUIZ 2	HW 5 Due
6/13	4.8 – 4.10	HW 6 Due
6/14	5.1 – 5.3	HW 7 Due
6/15	5.4, REVIEW	HW 8 Due
6/16	EXAM 1	
6/20	5.5 – 5.6	HW 9 Due
6/21	5.7, 6.1 – 6.2	
6/22	6.3	HW 10 Due
6/23	7.1 – 7.2, QUIZ 4	HW 11 Due
6/27	7.2 – 7.4	HW 12 Due
6/28	6.4 – 6.5	HW 13 Due
6/29	7.5, 7.7	HW 14 Due
6/30	8.1 – 8.2, QUIZ 5	HW 15 Due
7/5	8.3 – 8.4	HW 16 Due
7/6	REVIEW	
7/7	FINAL EXAM	HW 17 Due

Calculus Review

Matt Charnley

June 20, 2016

(a) Compute the following integrals.

(i)

$$\int_0^5 x^3 - 5x^2 + 3 \; dx$$

(ii)

$$\int_0^{10} e^{-3x} \; dx$$

(iii)

$$\int_0^{\infty} xe^{-10x} \; dx$$

(iv)

$$\int_0^{2\pi} x\sin(x) \; dx$$

(v)

$$\int_0^{\infty} xe^{-x^2} \; dx$$

(b) Change the order of integration in the following integrals.

(i)

$$\int_0^2 \int_0^x f(x,y) \; dy \; dx$$

(ii)

$$\int_0^4 \int_{x^2}^{16} f(x,y) \; dy \; dx$$

(iii)

$$\int_0^{10} \int_{-y}^{y} f(x,y) \; dx \; dy$$

(c) Set up the integral of a function $f(x,y)$ over the regions sketched out on the final page.

(d) Compute the following double integrals.

(i)

$$\int_0^2 \int_0^x e^{x^2} \; dy \; dx$$

(ii)

$$\int_0^4 \int_0^3 x^2 + 3xy^3 \; dx \; dy$$

(iii)

$$\int_0^\pi \int_0^3 r^2 \cos(\theta) \; rdr \; d\theta$$

(iv)

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x^2 + y^2 \; dy \; dx$$

MATH 477: Homework 10 Solutions

Matt Charnley

June 22, 2016

1.

(a) We compute the integral

$$c\int_{-1}^{1}(1-x^2)\,dx = c\left[x-\frac{x^3}{2}\right]_{-1}^{1} = c\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)=\frac{4}{3}c$$

Since this must equal 1, we need to have $c=\frac{3}{4}$.

(b) For the cumulative distribution function, we know that it must be 0 if $x<-1$ and 1 if $x>1$. For everything in between, we have

$$F(x)=\int_{-1}^{x}\frac{3}{4}(1-x^2)\,dx=\frac{3}{4}\left[(x-\frac{x^3}{3})+\frac{2}{3}\right]$$

Therefore, the cumulative distribution function is

$$F(x)=\begin{cases}0 & x<-1\\ \frac{3}{4}\left(x-\frac{x^3}{3}\right)+\frac{1}{2} & -1\le x\le 1\\ 1 & x>1\end{cases}$$

.

2. First, we need to compute the constant C that makes this a probability density function. Thus, we need to compute integrals.

$$\begin{aligned}\frac{1}{C} &= \int_0^\infty xe^{-x/2}\,dx = -2xe^{-x/2}\,|_0^\infty + 2\int_0^\infty e^{-x/2}\\ &= 0-4e^{-x/2}\,|_0^\infty = 4\end{aligned}$$

Therefore, we have $C=\frac{1}{4}$. Then, we can compute that

$$\begin{aligned}P(X\ge 5) &= \frac{1}{4}\int_5^\infty xe^{-x/2} = \frac{1}{4}\left[-2xe^{-x/2}\,|_5^\infty + 2\int_5^\infty e^{-x/2}\,dx\right]\\ &= \frac{1}{4}\left[10e^{-5/2}+4e^{-5/2}\right]=\frac{14}{4}e^{-5/2}\end{aligned}$$

.

4.

(a) We compute this probability by integrating

$$P(X > 20) = \int_{20}^{\infty} \frac{10}{x^2}\,dx = -\frac{10}{x}\mid_{20}^{\infty} = 0 + \frac{10}{20} = \frac{1}{2}$$

(b) Changing the x to other numbers, we see that the cumulative distribution function is

$$F(x) = \begin{cases} 1 - \frac{10}{x} & x \geq 10 \\ 0 & x < 10 \end{cases}$$

(c) Using this, we see that $P(X \geq 15) = \frac{10}{15} = \frac{2}{3}$. Thus, if we assume that each of the 6 devices will last a certain amount of time independently, we have that, the probability of at least 3 of them functioning for 15 hours is

$$\binom{6}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 + \binom{6}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + \binom{6}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + \binom{6}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0$$

which simplifies to $\frac{650}{729}$.

.

6.

(a)

$$\begin{aligned} E[X] &= \frac{1}{4}\int_0^{\infty} x^2 e^{-x/2} = \frac{1}{4}\left[-2x^2e^{-x/2}\mid_0^{\infty} + 2\int_0^{\infty} xe^{-x/2}\,dx\right] \\ &= \frac{1}{2}\int_0^{\infty} xe^{-x/2} = -xe^{-x/2}\mid_0^{\infty} + \int_0^{\infty} e^{-x/2}\,dx \\ &= 0 + -2e^{-x/2}\mid_0^{\infty} = 2 \end{aligned}$$

(b) From problem 1, we know that $c = \frac{3}{4}$. Then, we can calculate that

$$E[X] = \frac{3}{4}\int_{-1}^{1} x(1-x^2)\,dx = \frac{3}{4}\int_{-1}^{1} x - x^3\,dx = \frac{3}{4}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{1} = 0$$

(c)

$$E[X] = \int_5^{\infty} \frac{5}{x}\,dx = 5\ln(x)\mid_5^{\infty} = \infty$$

.

7. From the information in the problem (the fact that we have a probability distribution and we know the expected value), we know that

$$1 = \int_0^1 a + bx^2 = ax + \frac{bx^3}{3}\mid_0^1 = a + \frac{b}{3}$$

$$\frac{3}{5} = \int_0^1 x(a+bx^2) = a\frac{x^2}{2} + b\frac{x^4}{4} \mid_0^1 = \frac{a}{2} + \frac{b}{4}$$

This becomes a system of equations of the form

$$3 = 3a + b \qquad\qquad \frac{12}{5}2a + b$$

Solving this, we see that $a = \frac{3}{5}$ and $b = \frac{6}{5}$.

.

8. The expected value is

$$E[X] = \int_0^\infty x^2 e^{-x}\ dx = -x^2 e^{-x} \mid_0^\infty + \int_0^\infty 2xe^{-x}\ dx = -2xe^{-x} \mid_0^\infty + 2\int_0^\infty e^{-x}\ dx = 2$$

MATH 477: Quiz 3

June 23, 2016

Name: <u>Solutions</u>

1. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} c\,(8 - x^3) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the constant c that makes this a probability density function.

(b) Calculate $P(X \ge 1)$.

(c) Find the expected value and variance of this random variable.

(a) $1 = \int_0^2 c(8-x^3)\,dx = c\left[8x - \frac{x^4}{4}\right]\Big|_0^2 = c\left[16-4\right] = 12c$

So $\underline{c = \frac{1}{12}}$

(b) $P(X \ge 1) = \frac{1}{12}\int_1^2 (8-x^3)\,dx = \frac{1}{12}\left[8x - \frac{x^4}{4}\right]\Big|_1^2$

$= \frac{1}{12}\left[16-4-\left(8-\frac{1}{4}\right)\right] = \cancel{\frac{1}{12}\left[8+\frac{1}{4}\right]}$

$= \frac{1}{12}\left[4+\frac{1}{4}\right] = \underline{\frac{17}{48}}$

(c)

$E[X] = \frac{1}{12}\int_0^2 x(8-x^3)\,dx = \frac{1}{12}\left[4x^2 - \frac{x^5}{5}\right]\Big|_0^2 = \frac{1}{12}\left[16 - \frac{32}{5}\right]$

$= \frac{1}{12}\left[\frac{80-32}{5}\right] = \frac{1}{12}\left[\frac{48}{5}\right] = \underline{\frac{4}{5}}$

$E[X^2] = \frac{1}{12}\int_0^2 x^2(8-x^3)\,dx = \frac{1}{12}\left[\frac{8}{3}x^3 - \frac{x^6}{6}\right]\Big|_0^2$

$= \frac{1}{12}\left[\frac{64}{3} - \frac{64}{6}\right] = \frac{1}{12}\cdot\frac{64}{6} = \frac{16}{18} = \underline{\frac{8}{9}}$

So $\cancel{\boxed{Var(X) = \left(\frac{8}{9}\right) - \left(\ \right)^2}}$ $\boxed{Var(X) = \frac{8}{9} - \left(\frac{4}{5}\right)^2}$

2. Assume that a person's height H is normally distributed with mean 68 and variance 9. Using the table at the back of the quiz, calculate

(a) $P(H > 72)$.

(b) $P(66 < H < 71)$.

(a) $P(H > 72) = P\left(\frac{H-68}{3} > \frac{72-68}{3}\right)$

$= P(Z > 4/3)$

$= 1 - P(Z \leq 1.33)$

$= 1 - .9082 = \underline{.0918}$

(b) $P(66 < H < 71) = P\left(\frac{66-68}{3} < Z < \frac{71-68}{3}\right)$

$= P(-2/3 < Z < 1)$

$= P(Z < 1) - P(Z < -2/3)$

$= .8413 - .2514 = \underline{0.5889}$

3. Assume that the amount of time an individual spends taking a psychology survey is uniformly distributed between 10 and 30 minutes.

(a) What is the probability that I take more than 25 minutes to complete the survey?

(b) If 5 people go to take the survey, what is the probability that exactly 4 of them finish the survey *before* 25 minutes have passed?

Uniform on $(10, 30)$ $\Rightarrow$ $f(x) = \frac{1}{20}$

(a) $P(X > 25) = 5/20 = 1/4$

(b) p (less than 25) $= 3/4$

$$P = \underline{\binom{5}{4}(3/4)^4(1/4)^1}$$

4. I have a biased coin that shows up heads with probability .8. Let X count the number of heads when I flip the coin 625 times. Use the normal approximation to the binomial distribution to calculate the probability that I see strictly between 485 and 510 heads; i.e. $P(485 < X < 510)$.

$np = 625 \cdot \frac{8}{10} = 625 \cdot \frac{4}{5} = 125 \cdot 4 = \underline{500}$

$np(1-p) = 500\,(.2) = \underline{100}$

$P(485 < X < 510) \sim P(485.5 < X < 509.5)$

$= P\left(\frac{485.5-500}{10} < Z < \frac{509.5-500}{10}\right)$

$= P(-1.45 < Z < .95)$

$= P(Z < .95) - P(Z < -1.45) = .8289 - .0735 = \boxed{.7554}$

5. Assume that the length of a card game is exponentially distributed and has an average length of 20 minutes.

(a) Describe this distribution.

(b) What is the probability of a game lasting longer than 30 minutes?

(c) If the game has already lasted for 15 minutes, what is the probability that it lasts for another 40?

(a) This is an exponential random variable with

$$\underline{\lambda = 1/20}$$

(b)

$$P(X>30) = \int_{30}^{\infty} \frac{1}{20} e^{-x/20}\,dx$$

$$= -e^{-x/20}\Big|_{30}^{\infty} = \boxed{e^{-3/2}}$$

(c) By the fact that the exponential distribution is memoryless, this is the same as

$$P(X>40) = -e^{-x/20}\Big|_{40}^{\infty} = \boxed{e^{-2}}$$

MATH 477: Final Exam Topic Review

Matt Charnley

June 29, 2016

Chapter 5: Continuous Random Variables

(a) Continuous Random Variables: What are they? How do we compute probabilities using these density functions? How do we check if something is a density function?

(b) Expectation and Variance of Continuous Random Variables. Expected Value of Functions of a random variable. Calculating these using integrals.

(c) Examples of Random Variables: Uniform, Normal, Exponential. Calculating probabilities and setting up word problems using these random variables.

(d) Normal approximation to the binomial distribution. How to set this up, using the continuity correction.

(e) Functions of a random variable. Using the cumulative distribution function to start the problem, then getting to the density function.

Chapter 6: Jointly Distributed Random Variables

(a) How do we talk about two random variables at the same time? What are joint distribution functions? How do we calculate probabilities when we have two random variables?

(b) What does it mean for two random variables to be independent? How does this help us compute probabilities?

(c) How do we add two random variables together? If we add two normal random variables, what is the result? What if we add two exponential random variables? What about Poisson or Binomial?

(d) How do we calculate discrete conditional distributions? What is the formula, and what does it mean?

(e) For the continuous case, what do conditional distributions look like? How can we calculate probabilities using them?

Chapter 7: Properties of Expectation

(a) Linearity of Expectation. What is it? How do we use it? How can we set up problems to use this to make things easier?

(b) Expectation of functions of two random variables. How do we do it, and what can we do with it?

(c) Covariance and Correlation. What is covariance? What does it mean, and how can we calculate it? There are two formulas here. How do we relate covariance to correlation? What are the properties of covariance that we can use to make it easier to calculate things? Some of the linearity stuff from earlier might show up here as well.

(d) Variance of a sum of random variables. Covariance shows up in the formula.

(e) Conditional Expectation. What is it? What does it mean? How can we use this to help with things?

(f) Computing Expectations using conditioning. What types of problems apply here? When is this easier than just computing the expected value outright? How can we just this to also calculate probabilities by conditioning, with Indicator random variables? When is this useful/what does this remind you of?

Chapter 8: Limit Theorems

(a) Markov and Chebyshev inequalities: When do they apply? How do you use them? Know the specific types of problems they show up in.

(b) Central Limit Theorem: Same Idea. Know how to identify a problem as a CLT problem, and then know how to get to the answer.

(c) Strong Law: It's cool, but not really any way to test it.

Overall Key Points

(a) Know the definitions of all of the terms and random variables so that you can use them in problems.

(b) Know how to break down a word problem to figure out what it is asking for, and what different techniques you need to apply from the course to solve the problem.

(c) Know how to set up the double integrals for evaluating probability appropriately, because once it is set up, the rest of the problem is just calculus.

(d) Think through problems carefully before you start to work on them. Sketch out a picture if you need to. Once you figure out what is going on and how it all works, putting together the proper probabilities becomes a lot easier.

(e) There are a lot of different ways and techniques to approach problems. Know how to figure out which technique you need, and which would be the best way to approach the problem.

MATH 477: Final Exam Review Problems

Matt Charnley

July 1, 2016

1. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} C(8 - x^2) & -2 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the constant C to make this a probability density function.
   (b) Calculate $P(X < 1)$, $P(|X| > 1)$.
   (c) Find $E[X]$ and $E[X^2]$.

2. Assume that the lifetime of a system component (in hours) is a random variable given by the density function

$$f(x) = \begin{cases} \frac{50}{x^2} & x \geq 50 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is the probability that a component lasts longer than 150 hours?
   (b) If we have 5 of these components, what is the probability that exactly 3 of them will need to be replaced within the first 100 hours?

3. Let X be the continuous random variable defined by the density function

$$f(x) = \begin{cases} \frac{x}{2} & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find $E[X]$.
   (b) Find $E[X^3]$.
   (c) Find $Var(X)$.
   (d) Find $E[e^X]$

4. There is a single cop car patrolling a stretch of road of length L, however, to be more efficient, he only actually moves back and forth between $L/4$ and $3L/4$. Assume that an incident happens at a point uniformly distributed along the full length of the road, and the cop's position is uniformly distributed in the range from $L/4$ to $3L/4$.

   (a) What is the probability density function of the position of the incident?
   (b) What is the average distance between the cop and the incident?

5. Assume that the length of a card game is approximately normally distributed with mean 30 minutes, and variance 9 minutes.

   (a) What is the probability of a game lasting longer than 35 minutes?

(b) What is the probability that a game lasts between 26 and 31 minutes?

6. Use the normal approximation to the binomial distribution to approximate the probability that, if I roll a 6 sided die 1000 times, that I see more than 200 5's.

7. Approximate the probability that, if I survey 400 people, I get at least 125 votes for a proposition, where 25% of the population approves of the proposition.

8. Assume that the length of a computer repair is an exponential random variable with parameter $\lambda = \frac{1}{10}$ minutes.

   (a) What is the average length of a repair?

   (b) What is the variance in the time a repair will last?

   (c) What is the probability that a repair lasts longer that 25 minutes?

   (d) If a repair has already lasted for 10 minutes, what is the probability that it will, in total, last for at least 25 minutes?

9. Let X be a continuous random variable with density function f and cumulative distribution function F. What is the density function of the random variable $Y = X^3$? What about $Y = e^X$?

10. Let X_1, X_2, Y_1, Y_2 be four random variables so that X_1 and Y_1 are jointly continuous with density function f_1, and X_2 and Y_2 are jointly continuous with density function f_2, as shown below

$$f_1(x,y) = \begin{cases} \frac{1}{64}xy & 0 < x < 4,\ 0 < y < 4 \\ 0 & \text{otherwise} \end{cases} \qquad f_2(x,y) = \begin{cases} \frac{1}{144}\left[x + 3y\right] & 0 < x < 4,\ 0 < y < 4 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Looking at X_2 and Y_2, calculate $P(X > 2, Y < 1)$ and $P(Y > 2)$.

   (b) One of these pairs is independent. Which one is it and why?

   (c) Using the independent density function you chose in part a), compute the marginal density functions of X and Y, $f_X(x)$ and $f_Y(y)$.

   (d) Calculate $P(X < 1|Y = 2)$ and $P(Y > 3|X = 2)$.

11. I have a bucket containing 6 red, 5 white, and 3 blue balls, from which I am going to draw 3. Let X be the number of red balls drawn, and Y the number of blue.

   (a) Find the joint probability mass function of X and Y.

   (b) Find the marginal distribution of X and Y.

   (c) Find the conditional distribution of $X|Y$ for any y.

12. Let X and Y be two independent normal random variables so that

$$E[X] = 10 \qquad Var(X) = 9 \qquad\qquad E[Y] = 15 \qquad Var(Y) = 16$$

   (a) What is the probability that $X \leq 12$?

   (b) What is $P(X > 14, Y < 13)$?

(c) What is the distribution of $X + Y$?

(d) What is the probability that $X \geq Y$?

13. Let X and Y be jointly continuous random variables with joint probability density function

$$f(x,y) = \begin{cases} 6e^{-2x}e^{-3y} & x > 0, \ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal distributions $f_X(x)$, $f_Y(y)$.

(b) Calculate $E[XY]$ and $E[X + Y]$

14. Consider a standard 52 card deck of cards, which has 26 red cards and 26 black. Assume I shuffle the deck and lay out all the cards. What is the expected number color changes in the deck, i.e., the number of times we have a red card followed by a black, or a black card followed by a red. Hint: Linearity of expectation.

15. A building has $N + 1$ floors, and anyone who enters the elevator is equally likely to go to any of the N floors above the first, independently of all other people in the elevator. Assume that k people enter the elevator on the first floor, and let X denote the number of floors that the elevator will stop at on the way up. Find $E[X]$. Hint: Linearity of Expectation. Let X_i be 1 if the ith floor is selected, and 0 otherwise. It may be easier to think about the probability of a floor not being selected.

16. 5 people are playing a game. Each player has their own hand of 10 distinct cards, and each player has the exact same hand. The game is played by each player picking a card from their hand, and revealing them all at once. A player wins if no other cards on the table match the one that he picked. There can be multiple winners in a given game. What is the expected number of winners each time this game is played? Hint: Linearity of Expectation.

17. I have a bag of balloons, containing approximately 1/3 red, 1/2 blue, and 1/6 green balloons. I am going to take out ten balloons, and there are enough balloons in the bag that each draw can be assumed to be independent of all others. Let R denote the number of red balloons I get, and G the number of green.

(a) What is the covariance of R and G?

(b) What is the correlation between R and G? Does this make sense?

18. I am going to roll a pair of 6 sided dice 30 times. Let X denote the number of times I see a pair on the dice, and Y the number of times I see a sum of 2, 5, 7, or 12. What is the covariance of X and Y? Can you say anything else about the events X and Y?

19. Let X and Y be jointly continuous random variables defined by the density function

$$f(x,y) = \begin{cases} \frac{1}{2}\left[2x + 3y^2\right] & 0 < x < 1, \ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal densities f_X and f_Y.

(b) Find $E[X]$, $E[Y]$ and $E[XY]$.

(c) Calculate $Cov(X, Y)$.

(d) Find the conditional density function $f_{X|Y}(x|y)$.

(e) Find $E_X[X|Y = y]$.

(f) Calculate $E_Y[E[X|Y = y]]$ and see that this is the same as $E[X]$.

20. You are playing a game with a pair of two 6-sided dice. You roll the dice. If a sum of 7 shows up, you win nothing and the game is over. If anything other than 7 shows up, you can either win that amount of money (the sum of the two dice), or you can roll again, following the exact same procedure. Your strategy for this game is to pick a number k, and stop as soon as you see a sum of at least k. What is your expected winnings, which will depend on k? Hint: You will want to condition on the initial roll of the dice. The result should look like the dumb miner problem.

21. Let X and Y be jointly continuous with density function

$$f(x, y) = \begin{cases} \frac{1}{y}e^{-y} & 0 < x < y,\ 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate $E[X]$, $E[Y]$ and $E[XY]$.

(b) Find $Cov(X, Y)$.

(c) Find $E[X|Y = y]$ and show that $E[X] = E[E[X|Y]]$.

(d) Calculate $E[X^3|Y = y]$.

22. A coin having probability p of landing on heads is flipped until both a heads and a tails has appeared.

(a) What is the expected number of flips?

(b) What is the probability that the last flip is heads?

23. Let X be a random variable that can only take on positive values and has expected value 10.

(a) What can be said about the probability that $X \geq 20$?

(b) What can be said about the probability that $X \geq 50$?

(c) What can be said about the probability that $X \geq 5$?

24. Let X be a random variable with mean 25 and variance 9.

(a) What can we say about the probability that $19 < X < 31$?

(b) What is the likelihood that I see a value for X less than 15 or bigger than 35?

25. Let X be a random variable with mean 50 and variance 16. I am going to sample X 100 times. Use the Central Limit Theorem to approximate the probability that the sum of all of these samples, $\sum_{i=1}^{100} X_i$ is between 490 and 520.

26. Let X be a normally distributed random variable with mean 10 and variance 4. I am going to sample from X 20 times, so that I get a sequence X_i from $i = 1$ to 25 of identical normally distributed random variables. I am interested in the probability that the sum is between 225 and 260.

    (a) Use Chebyshev's Inequality to approximate this probability.

    (b) Use the central limit theorem to approximate this probability.

    (c) Calculate this probability using the fact that the sum of normally distributed random variables is normal. What do you notice about these last two results?

D.3 Summer 2017 - Math 244

This section contains the following documents from my Summer section of Math 244:

1. The course syllabus
2. The Fluid Flow project that I created
3. The Bifurcation project that I made for the class
4. A problem set that was done in the library to give them practice with numerical methods
5. A fairly standard problem set to be worked on in class
6. A sample worksheet that the students would do while watching the videos
7. The worksheet quiz that corresponds to the previous worksheet that would be completed when the students arrived to class

MATH 244, Section C1 – Summer 2017
Syllabus

Instructor – Matt Charnley
Office: Hill 606, Busch Campus
Course Website - Canvas: canvas.rutgers.edu
Personal Website: math.rutgers.edu/~mpc163/Courses/SM17_MATH244.html

Class Meetings
MTWTh, 8:00 AM – 10:00 AM, Tillett 204, Livingston Campus

Office Hours
Monday and Wednesday – 10:00 AM – 12:00 noon, LSH 102C
By Appointment - Hill 606, Busch Campus

Exam Schedule
Midterm 1: Thursday, June 15 – In Class
Midterm 2: Thursday, July 6 – In Class
Final Exam: Thursday, July 20 – 8:00 AM – 11:00 AM

Course Information

The information for this course can be found on Canvas. Canvas is a newer Learning Management System that Rutgers is looking to implement in their classes. While I haven't used it for an actual class before, I feel like it's a lot better than Sakai, and I hope you'll like it too. The website is canvas.rutgers.edu. For the first week or so, I will also be posting links to everything on my personal website math.rutgers.edu/~mpc163/Courses/SM17_MATH244.html to allow everyone to keep up with the class if there are any issues with the Canvas site. After that, we will be exclusively using Canvas. If there are any issues, let me know as soon as possible.

Textbook

The textbook for this course is *Elementary Differential Equations*, **10th edition**. William Boyce and Richard DiPrima. ISBN: 978-0-470-45832-7.

Class Summary

This class is an introduction to Ordinary Differential Equations. In particular, this class is directed at engineering and physics students who will need knowledge of ordinary differential equations for future classes. To best do this, the class will focus on both the qualitative and quantitative aspects of differential equations, showing how both can be useful in different situations. There will be some instances where it may seem like the math has no connection to engineering or physics, but there will always be an attempt to emphasize the applications at every step. The topics covered in this class include:

- First Order Differential Equations
- Second and Higher Order Linear Differential Equations
- Systems of First Order Differential Equations
- Numerical Methods for Solving Differential Equations
- Non-Linear Differential Equations

Grade Breakdown

The final grades for this course will be calculated using the following distribution

Worksheets / Participation / Presentations	10%
Homework Writeups	5%
Quizzes (Including Syllabus Quiz)	10%
Projects and Maple Labs	15%
Midterms	28% (14% each)
Final Exam	32%

Note: No student will receive a final grade more than one mark higher than their average grade on the 3 exams, weighted in an appropriate manner. For instance, if your average exam grade is a C, you can receive no higher than a B for a final grade.

Class Structure

This class will be run as a flipped classroom. What this means is that the process of learning basic concepts and content (normally done via lecture) will be done on your own outside of class, and the practice of problem solving (normally done as homework) will be done in class when I am present to help you. The learning process outside of class will be facilitated by videos that I will be making over the course of the summer and worksheets that I will give you corresponding to both sections in the textbook and these videos. The worksheets will be due at the start of each class, at which point is expected that you will have read the sections in the textbook and completed the worksheet. These worksheets will contain questions about the important parts of the textbook sections as well as a few simple problems to get you started. Class will consist of a short lecture discussing the material and answering any questions from the previous night's reading, followed by group problem solving work. This will be similar to the workshop process in Calculus 1 and 2. The end of class will either consist of presentations or quizzes. The last page in this syllabus outlines the general process for a given class period.

Academic Integrity

All students in this course are expected to be familiar with and abide by the academic integrity policy (http://academicintegrity.rutgers.edu/academic-integrity-at-rutgers). Violations of this policy are taken very seriously. In short, don't cheat, and don't plagiarize. In terms of exams, it's fairly easy to understand what cheating/plagiarism is. However, this class is going to be heavily based on group work and projects. Everyone is expected to submit their own work, which means copying or borrowing answers from someone else in the class is plagiarism. Since you are expected to work together for some of the problems, this can be tricky. The general method that you should use in this class is that during the group work, you should only write notes about the problems, but don't work on the actual write-up. Then, outside of class, you can do the write-up using your notes, which will result in your write up still coming from the work you did in class, but will not be identical to your classmates. See the Canvas page for more information.

Attendance

Attendance at every class meeting is mandatory. Attendance will be taken in the form of the worksheets at the beginning of class and participation points for the problems solved in class. You are also expected to watch all of the videos that I link on the Canvas page. With the speed of all summer classes, missing any class will result in you falling significantly behind. If you must miss a class for any reason, come talk to me as soon as possible.

Problem Sets

The in-class problem sets will consist of three parts. The first, 'Warm-ups,' consists of problems that give a basic idea of the topics. All groups in the class should do all of these problems, or at least verify that they know how to do all of them. These are all fair game for quiz and exam questions. The second section, 'Exercises,' are a little more complicated and involved. These are the problems that will be presented at the end of class (see the Presentations section), and problems similar to these are around the level of exam questions. The final section, 'Problems,' are more involved and multi-step problems. These will be turned in as a part of the homework write-ups (see the Homework section).

Presentations

Whenever there is not a quiz at the end of class, there will be in-class presentations of the problems that were worked on that day. Each group will get a different problem to work through (assigned around the middle of class), and one person from the group will need to present it to the class in the last 30 minutes of class. The person presenting at the board will need to rotate every time a presentation is done, but the entire group can help in presenting the problem or giving guidance from their seats. The entire group can also work together to write the solution on the board before the presentation, and then only one person will talk through it to the class. The goal here is to build confidence in talking about the course material, as well as give everyone practice talking about math.

Quizzes

Every Tuesday and Thursday class (except exam days) will end with a quiz. This quiz will cover material from the previous two days of class, but can also be more cumulative depending on the situation. The problems on the quizzes will be on a comparable level to the in-class exercises. They will be closed book, closed note, individual quizzes. The first quiz is already posted on Canvas. It is a quiz about this syllabus and how the class is structured. It is worth triple of all of the other quizzes, and can not be dropped. It is due at the end of the day on Tuesday, June 6. You have unlimited attempts to get it right, and can use this syllabus and the Canvas website while you are taking the quiz.

Homework

Homework for this class will not be assigned in the typical manner. Before each class, you will be expected to complete a worksheet summarizing sections in the textbook. There will also be problems on here to be completed from the videos that you need to watch. This will be due at the start of class, graded during class, and returned to you the same day. At the end of each class, approximately one problem from each homework set covered that day will be assigned. These will need to be written up individually, although you will be doing the problems in groups, so you are definitely welcome to talk about the problems as you do them. The write-ups should be fairly complete, somewhere between workshops and normal homework. The write-ups should also be your own work, completed without collaborating with other students. These will be graded and returned to you.

Maple Assignments

There are 3 Maple labs that will need to be completed and turned in over the course of the summer. The dates are included in the tentative schedule below, and all of the necessary materials will be posted to Canvas. There will also be some introductory materials posted there if you need more information. If you have any issues with the Maple labs, come talk to me in office hours or send me an email. I will briefly mention each of them as they are assigned, but you will be overall responsible for completing them on your own, and coming to me with any questions. I will be spending minimal in-class time discussing the Maple labs.

Projects

There will be two projects assigned over the course of the summer. The due dates for these projects are in the schedule at the end of the syllabus. These projects will be somewhat similar to lab reports, in that math and writing will be incorporated together into a single document. You will need to both present solutions to the given math problems and discuss the implications of the results in an actual situation. The project description will make it clear what you are expected to talk about and how to use the mathematical results to do so. The idea with the projects is to show you how the math you do in this class is applicable to physical situations and understand yourself how to do these applications. The projects will be done individually, although you are allowed to discuss it with both me and your fellow students.

Exams

There will be two midterm exams and a final exam. The dates for these exams are posted above as well as on the tentative schedule below, but are subject to change. These will be exams in the standard sense, 80 minutes for the midterms and 180 minutes for the final, and will be taken individually. Calculators and electronic devices will not be permitted on the exams, and they will be closed book and closed note.

Make-Up Policies

There will be no make-ups for any of the in-class activities or homework assignments. In order to compensate for this, the lowest quiz, lowest 2 worksheets, and lowest 2 homework grades will be dropped at the end of the summer. Under no circumstances can an exam be made up after the fact. If there is a legitimate reason for missing an exam, i.e., doctor’s note, then we can discuss possibilities moving forward, but you will not be able to take the exam later. If there is an excessive need to miss class, talk to me about it sooner rather than later.

Disability Accommodations

I will be happy to provide appropriate accommodations for students who provide me with a letter of accommodation from the Office of Disability Services (ODS). For more information, see http://ods.rutgers.edu/.

Changes

This syllabus is subject to change at any point. Any changes will be announced in class and posted on the Canvas site.

Tentative Class Schedule

DATE	SECTIONS	DUE DATES
5/30	1.1, 1.3	
5/31	1.2, 2.2	
6/1	2.1	Quiz
6/5	2.3	
6/6	2.4, 2.8	Quiz
6/7	2.5	
6/8	2.6	Quiz
6/12	2.9	Project 1 Due
6/13	3.1, 3.2	Quiz
6/14	5.4, Review	
6/15	MIDTERM 1	
6/19	2.7, 8.1-8.3	Maple 1 Due
6/20	3.3, 3.4	Quiz
6/21	3.7	
6/22	3.5, 3.6	Quiz
6/26	3.8	Project 2 Due
6/27	Chapter 7, Day 1	Quiz
6/28	Chapter 7, Day 2	
6/29	7.5, 7.6	Quiz
7/3	7.8, 9.1	
7/4	NO CLASS	NO CLASS
7/5	C7, Day 3, Review	
7/6	MIDTERM 2	
7/10	7.7	Maple 2 Due
7/11	7.9	Quiz
7/12	9.2, 9.3	
7/13	9.4, 9.5	Quiz
7/17	9.7, 9.8	Maple 3 Due
7/18	Chapter 4 and 5 Summary	Quiz
7/19	Review	
7/20	FINAL EXAM	

Class Structure

Outside of Class:

1. Read assigned sections in the textbook.
2. Watch the corresponding videos through Canvas.
3. Fill out the worksheet and complete the problems from the videos.
4. Complete in-class problem write-up.

In Class:

1. Turn in worksheet, previous class's homework, and ask any questions from the previous sections.
2. Listen to brief lecture about the material.
3. Work on book problems or other assigned problems in groups.
4. Ask questions about the problems as needed.
5. Midway through class, end of class and homework assignment will be discussed.
6. Class will end with either presentations or a quiz.

Worksheets will be returned the same day they are collected. Homework assignments will be returned a day later.

Project 1: Fluid Flow

Matt Charnley

April 15, 2017

All fluids have a material property called viscosity, which basically measures how much the fluid likes to move or flow. For instance, honey is much more viscous (has a higher viscosity) than water. In this project, you will investigate how using ODE's to model fluid flow can help us to calculate the viscosity of a fluid in two types of viscometers.

The main equations we will be using is the Navier-Stokes equations, which are, in their original form, the following partial differential equation:

$$\rho\frac{\partial \vec{u}}{\partial t} + \rho(\vec{u}\cdot\nabla)\vec{u} = -\nabla P + \mu\Delta\vec{u} + \rho\vec{g} \tag{1}$$

where this is actually 3 equations, one for each coordinate direction, i.e., one of them is

$$\rho\frac{\partial u_x}{\partial t} + \rho(u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}) = -\frac{\partial P}{\partial x} + \mu\Delta u_x + \rho g_x \tag{2}$$

and the Laplacian Δ is

$$\Delta u_x = \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}$$

μ is the viscosity parameter we want to find, and ρ is the density of the fluid.

In each of the cases below, we'll be able to simplify the equation in order to reduce it down to a simple ODE that you can solve. This assignment will guide you through the simplifications needed, and allow you to do some analysis on this system. Note, the ODEs in this problem are really simple. Don't overthink it.

1 Torque Viscometer

For this first version of the calculation, we will consider fluid flowing between two parallel plates, the bottom plate being fixed in place, and the top moving at a velocity V.

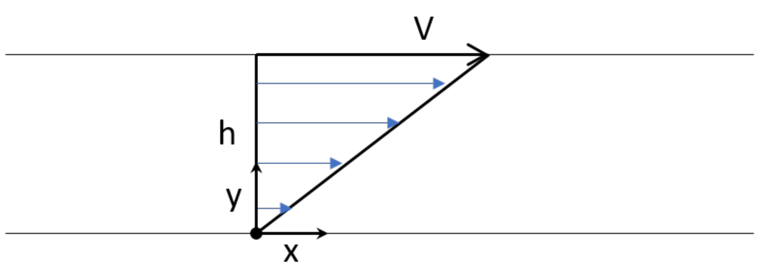


Figure 1: Sketch of the flow between parallel plates

We will assume that coordinates are aligned so that the origin is as marked in the image, the x direction is to the right, the y direction points up, and the z direction is back into the page. The assumptions we are allowed to make here are that the fluid only moves in the x direction, that is, $u_y = u_z = 0$ everywhere. We also assume that nothing depends on the z coordinate because the problem is infinite in that direction, that is, every derivative in z is zero. Finally, a different property, the continuity equation, lets us conclude that $\frac{\partial u_x}{\partial x} = 0$.

(a) Those assumptions tell us that we only need to care about the u_x equation from Navier-Stokes, i.e., the equation in 2. Using the above assumptions, plus the fact that the system is at steady state (time derivatives are zero) and gravity points in the y direction ($g_x = 0$), simplify 2 to get a simple ODE for u_x.

(b) What is the general solution of this ODE? Again, don't overthink it.

(c) Assuming the channel is of height h, the "no-slip" condition tells us that we must have $u_x(0) = 0$ and $u_x(h) = v$. Using these conditions, solve for the constants to get a specific solution.

(d) For this type of situation, the *stress* on the lower plane (something like the force the fluid imparts on the plane) is given by $\tau = \mu \frac{\partial u_x}{\partial y} \mid_{y=0}$. Compute this in terms of the given situation above.

(e) Now, we want to use this in a specific situation to compute viscosity. A torque viscometer is a pair of nested cylinders, where the fluid lies between them.

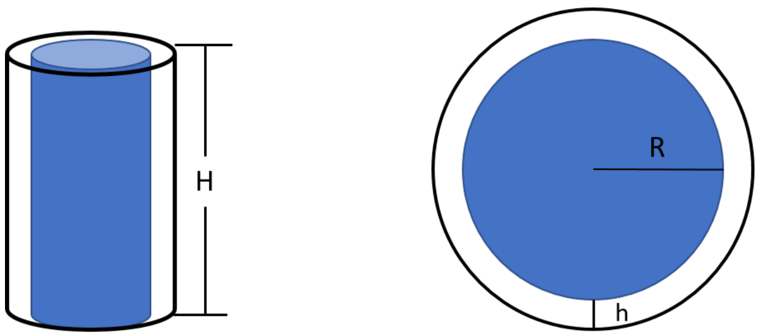


Figure 2: Sketch of a Torque Viscometer, both in profile and in cross-section

If the radius of the outer cylinder is really large compared to the gap between the cylinders, we can ignore the fact that the setup is curved, and pretend it is flat, which is what we just solved above. Assume we have a setup like this of height H in the z-direction. The torque T on the inner cylinder is given by the stress factor τ multiplied by the surface area of the cylinder (giving force) and then multiplied by the radius. Find a formula for the torque in terms of v, R, H, h, μ and other constants.

(f) Describe a procedure to use this system to calculate the viscosity of a fluid. You can leave all of the parameters of the problem in terms of variables or pick values for them. Determine which things are physical parameters of the system, which things you can measure, and how to calculate the viscosity from that.

We are a little bit off in the calculations here, because we assumed the round cylinder was flat, but it's only around a 1% error in most situations. So, a simple setup like this will allow us to approximate the viscosity of a fluid.

2 Capillary Viscometer

In this problem, we will consider flow in a cylindrical pipe.

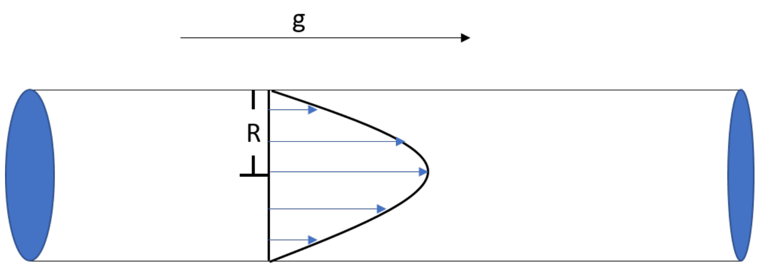


Figure 3: Sketch of flow in a cylindrical pipe

In order to find the equations here, we will want to use cylindrical coordinates, because that best fits the geometry of our system. We will assume that z points along the axis of the pipe, r is in the direction of the radius of the pipe, and θ points around the pipe. We will again assume that the fluid only moves in the z direction, so that $u_r = u_\theta = 0$. Thus, we only care about the u_z equation, which, in these coordinates is

$$\rho \frac{\partial u_z}{\partial t} + \rho(u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z}) = -\frac{\partial P}{\partial z} + \mu \left[\frac{1}{r}\frac{\partial}{\partial r}(r \frac{\partial u_z}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] + \rho g_z \tag{3}$$

(a) Making the assumptions above, along with the steady state assumption from part 1, and the additional fact that $\frac{\partial u_z}{\partial z} = 0$, we can simplify this equation. For this part of the problem, we will assume that $\frac{\partial P}{\partial z} = 0$, but $g_z = g$ is the normal acceleration due to gravity. Make the cancellations above to get an ODE for u_z.

(b) Integrate both sides twice in r, clearing the factors of r each time, to get a general solution for u_z.

(c) The boundary conditions we want here are that $u_z(R) = 0$, another “no-slip” condition, and that $u_z(0)$ is finite, because fluid moving infinitely fast is generally not a good thing. Using these conditions, find a specific solution for u_z.

(d) In order to calculate the flow rate Q, we need to integrate this velocity profile over the circular cross-section of the cylinder. Integrate u_z over the circle of radius R to find a formula for Q.

(e) Describe a procedure where you use this information to calculate the viscosity of a fluid. Hint: The fact that the fluid flow is being driven by gravity means you need to have the pipe be vertical. You will want to have the fluid flow through the pipe and measure something, just like in part 1 you measured the torque.

These calculations are the basis of the Capillary viscometer, which can also be used to find the viscosity of a fluid.

Project 2: Harvesting from a Population and Bifurcation Analysis

Matt Charnley

April 15, 2017

We recently investigated the logistic model for population growth. In this project you will look at how this is affected by harvesting from the population, whether this is constant-yield or constant-effort harvesting. We will see that there is a specific value of the harvesting parameter for which the type of solution changes. The name for this point (where the existence and stability of critical points changes) is a **bifurcation point**. Here, you will look at the bifurcation point in the case of a harvesting model, and then extend this analysis to a general first order ODE.

All of your answers for this project should be done two ways, both explicitly/by hand and using Maple to sketch graphs. The equations are simple enough that everything can be calculated by hand, but the pictures will help to illustrate your point. For instance, if a critical point is stable, you should show some solutions that converge towards it. If a point is unstable, you should show solutions that start near the solution, and then move away from it. You should also have multiple graphs for each parameter that is varying.

1 Single Species with Harvesting

(a) Consider a single species whose population grows under the logistic model, namely

$$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y$$

where K is the carrying capacity of the system and r is the normal growth rate of the population. Find and analyze the critical points of this system.

(b) Now, we want to implement harvesting. First we will use constant-effort harvesting. This means that there will be a constant effort put forth to harvesting, namely, the extra amount that the population is reduced is proportional to how many creatures there are. Let E be this proportionality constant. Write a new differential equation for the rate of change of the population. Analyze the critical points of this equation, which will now depend on E. What happens as E changes? Are there any important values of E where the equation changes significantly?

(c) Finally, we will look at constant yield harvesting. This means that the population will be depleted by a specific constant amount, call it H, each year. Write a new differential equation for this system. Analyze the critical points here as well, which will depend on H. What happens as H changes? Are there any specific values of H for which the equation changes significantly?

(d) What are your recommendations for what the values of H or E should be? This is much more of an opinion question. Interpret the results of the previous parts to discuss harvesting habits. Your answer should discuss some/all of the following.

1. If we have estimates for r and K (you can make up values if you would like), then what would you recommend for values of E and H?
2. How would you be able to check if the values for E or H are too high or low? How does the population over time tell you this?
3. If environmental factors change, which produces an effective change in r or K, how could/should E and H change to compensate? Make up a few example of this to fuel the discussion.
4. How sensitive is the situation to changes in H? For instance, if one person goes against fishing limits, and H jumps by 10 for a given time step, what could happen to the population? Why do limits on the number of fishing or hunting licenses exist?

2 General Bifurcation Analysis

In this section, you will analyze a different ODE that also exhibits bifurcation behavior. Between your in-class group, you should each choose a different one of the following four ODEs to analyze. If you have a group of three people, you should choose between the first three ODEs, if there are four of you, you should add in the fourth ODE as well. In each case, the capital letter is the parameter to be varied.

(a) $\frac{dy}{dt} = y^3 - Ay$

(b) $\frac{dy}{dt} = y^2 - By + y$

(c) $\frac{dy}{dt} = Cy - y^2$

(d) $\frac{dy}{dt} = Dy - y^3$

For your given ODE, you should perform the following analysis:

1. Analyze your problem (by hand) to find the critical points and their stability. Look at where specific values of the parameter cause changes in the stability of critical points.
2. Show Maple graphs in each of the situations that you described in the first part. They should clearly state the value of the parameter and the stability of each critical point.
3. Draw a bifurcation plot. This is a plot of the phase line (critical points) against the value of the parameter. For instance, if at $A = 1$, the ODE has a critical point at $y = 2$ and $y = 4$, the graph should have curves at $y = 2$ and $y = 4$ over the point $A = 1$ (your axes should be y vs. A).
4. Talk about what you see in these plots and what it means for the ODE or the physical system governed by it.

Problem Set 11: Sections 2.7 and 8.1-8.3

Matt Charnley

June 20, 2017

This worksheet is a little different because of what this section is. We're talking about numerical methods, and we will be spending most of the class in the library so that you all can practice this. Follow the problem set from start to finish, and you should hopefully be able to complete the coding part of the write-up before you leave class. What you turn in will be partially hand-written and partially computer code. I am looking to see your code for this assignment. There is a sample file on Canvas with some basic loops that you can use as a starting point.

1 Problem 1

Look at the ODE $y' = 3 - t + y$

(a) Solve the ODE explicitly (first order linear).

(b) Assume we want to start with the initial condition $y(0) = 2$. Write out the first 2 steps of Euler's method with $h = 0.5$. How much error is there at $t = 1$?

(c) Write out the general form of Euler's Method for this equation (how you get from y_n to y_{n+1}) with a step size h.

(d) Do the same for the backwards Euler's method, and solve it explicitly for y_{n+1}.

(e) Do this for the centered difference or Heun's method and solve out for y_{n+1}.

Now, we want to move all of this into the computer.

(a) Write a Maple code (or really, whatever language you want to use, I'm just providing maple) to do the normal Euler's Method for the above ODE. Get to the value at $t = 1$ with a step size of $h = 0.05$, $h = 0.025$ and $h = 0.0125$. Keep track of your errors in each case. You should see approximately the proper decrease in error as you decrease the step size.

(b) Do the same for the backwards Euler, Heun's method, and Runge-Kutta method. You should be able to reuse most of your code for each of the steps. Record the error here as well.

The rest of this problem set will be due tomorrow at the start of class.

2 Problem 2

Look at the ODE $y' = 4 + 2t - 5y$

(a) Solve the ODE explicitly (first order linear).

(b) Assume we want to start with the initial condition $y(0) = 2$. Write out the first 2 steps of Euler's method with $h = 0.5$. How much error is there at $t = 1$?

(c) Write out the general form of Euler's Method for this equation (how you get from y_n to y_{n+1}) with a step size h.

(d) Do the same for the backwards Euler's method, and solve it explicitly for y_{n+1}.

(e) Do this for the centered difference or Heun's method and solve out for y_{n+1}.

Now, we want to move all of this into the computer.

(a) Write a Maple code (or really, whatever language you want to use, I'm just providing maple) to do the normal Euler's Method for the above ODE. Get to the value at $t = 1$ with a step size of $h = 0.05$, $h = 0.025$ and $h = 0.0125$. Keep track of your errors in each case. You should see approximately the proper decrease in error as you decrease the step size.

(b) Do the same for the backwards Euler, Heun's method, and Runge-Kutta method. You should be able to reuse most of your code for each of the steps. Record the error here as well.

3 Problem 3

In this problem, we are looking at a non-linear ODE, so solving for the backwards Euler and Heun's method is more difficult. So you don't have to do that.
Consider the ODE

$$y' = (2x + 4)y^2 \qquad\qquad y(0) = 10$$

(a) Solve this separable equation explicitly.

(b) Use the Euler's method code you wrote earlier to approximate the solution at $x = 1$ for using $h = 0.05$, $h = 0.025$ and $h = 0.0125$. Write down the error in each case.

(c) Use the Runge-Kutta code you wrote earlier to do the same with the same values of h. Again, keep track of the error in each case.

4 Discussion

(a) For Problem 2, what do you notice about the errors as you decrease the size of h for all 4 of the methods you used?

(b) For Problem 3, do the same analysis for the two methods you used for the non-linear problem.

Problem Set 17: Sections 7.5 and 7.6

Matt Charnley

June 29, 2017

These problems are to be worked on in class. All groups should work on the first set of problems, then move on to the second and third sets. The second set will be done as presentations, and the third set will have a problem assigned as homework. This problem is due at the start of next class.

1 Warm-ups

(a) Problem 3 on page 405

(b) Problem 15 on page 405

(c) Problems 24-27 on page 406 (only need to do part (a) and take a guess at part (b))

(d) Problem 3 on page 417

(e) Problem 9 on page 417

(f) Problem 13 on pages 417-418

2 Exercises/Presentations

The first two problems here I think are really nice and tie these new calculations back to the second order systems. The other ones are more computation.

(a) Problem 29 on page 406

(b) Problem 28 on page 419

(c) Problem 1 on page 405

(d) Problem 6 on page 405

(e) Problems 1 and 6 on page 417

(f) Problems 15 and 18 on pages 417-418

3 Problems

Write up the solution to the following problems. In both cases, solve the given initial value problem and describe the behavior of the solution as $t \to \infty$.

1. Problem 16 on page 405:

$$\vec{x}\,' = \begin{bmatrix} -2 & 1 \\ -5 & 4 \end{bmatrix} \vec{x} \qquad \vec{x}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

2. Problem 10 on page 417:

$$\vec{x}\,' = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} \vec{x} \qquad \vec{x}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Worksheet 18: Sections 7.5 and 7.6

June 29, 2017

Name: ______________________________

Assignment: Read Sections 7.5 and 7.6 and watch the 'Day 18 Videos' posted to the Canvas site. The last few questions for each section will come from the ends of the videos.

1 Section 7.5

1. What name do we give to the drawings of a sample of trajectories for a system of the form $\vec{x}' = A\vec{x}$, particularly when there are two functions in $\vec{x}$?

2. If we know that ξ is an eigenvector of A with eigenvalue r, then what function do I know solves $\vec{x}' = A\vec{x}$?

3. What are the three different options for the type of equilibrium point at 0 for a system $\vec{x}' = A\vec{x}$ where A has two distinct real eigenvalues?

4. What are the other two options for the eigenvalues of a 2×2 matrix (other than real and distinct roots)?

2 Section 7.6

1. If the eigenvalues of A are complex conjugates, then what do we know about the corresponding eigenvectors?

2. If $r = \lambda + i\mu$, what does the **complex valued** solution look like?

3. How do we get from the above solution to two independent solutions? How is this more complicated than the similar process for the second order ODE case?

4. What kind of equilibrium points can we get for complex eigenvalues?

3 Video Questions

1. Section 7.5 Video 1 Question

2. Section 7.5 Video 2 Question

Worksheet 18 Quiz

June 29, 2017

Name: ____

Find the general solution of the following ODE system

$$\vec{x}\,' = \begin{bmatrix} 2 & 2 \\ 0 & 3 \end{bmatrix} \vec{x}$$

D.4 Summer 2018 - Math 252

This section contains the following documents from my Summer section of Math 252:

1. The course syllabus
2. A sample Mini-Quiz with its solution
3. A sample Readiness Assessment with its solution
4. A fairly standard example of a problem set that would be worked on in class
5. A sample practice problem with its solution
6. Another problem set that had more of an activity component to it
7. The set of review problems that made up the Gallery Walk activity on the first day of class
8. Assignment sheet for the bifurcation jigsaw activity
9. Assignment sheet for the oscillators workshop activity
10. Assignment packet for the SIR modeling activity
11. Assignment sheet for the end-of-semester student generated test questions

MATH 252 - Section B6 - Summer 2018

Contact Information

Name: Matt Charnley
Email: charnley@math.rutgers.edu
Office: Hill 606

Office Hours

Mondays, 3:30 - 5:00 PM
Tuesdays, 5:00 - 7:00 PM
or by appointment.

Class Meetings

MWF 6:00-8:45 PM, SEC 220

Exam Schedule

- Midterm 1: Friday, June 1 - First half of class
- Midterm 2: Wednesday, June 13 - First half of class
- Final Exam: Friday, July 6 - 6:00 - 9:00 PM

Textbook

This course will use *Differential Equations*, **4th edition**, by Paul Blanchard, Robert Devaney, and Glen Hall. ISBN-13: 978-1-133-10903-7.

Course Information

The information for this course can be found on Sakai. All announcements and assignments will be posted to this site. If you have any issues accessing the Sakai site, let me know as soon as possible. My personal website will also likely have some of these materials posted to it, but the most up-to-date resource will be Sakai.

Learning Goals

During this course, students will

1. Gain a familiarity with differential equations, which will show up in a variety of places after this class.
2. Understand how qualitative, quantitative, and numerical techniques can be applied to a problem, and when each one should be used.
3. Improve skills and confidence in talking about and presenting mathematics to their peers.
4. Become exposed to some of the ideas from higher-level mathematics, which will be expanded upon in future classes.

Class Overview

MATH 252 is an introduction to differential equations, generally directed at math majors. The course takes a three-pronged approach to studying differential equations: Qualitative methods (general behavior of solutions), Quantitative methods (analytical solutions), and Numerical methods (approximating solutions on a computer). We will take a look at all three of these over the course of the summer. The topics this course will cover are:

- First Order Differential Equations
- Systems of Differential Equations
- Linear Systems and Higher-order Linear Differential Equations
- Numerical Methods for ODEs
- Non-linear Systems of ODEs

NOTE: Linear Algebra (Math 250) is a prerequisite for this class. You will be expected to know the basics of linear algebra and matrix manipulation for this class. See the Midterm 1 topic outline for more information.

Grade Breakdown

Final grades for the class will be decided according to the following breakdown:

In-Class Assignments	10%
Quizzes	10%
MATLAB	10%
Midterm Exams	$10\% + 25\%$
Final Exam	35%

Class Structure

This class will be run in a mix between lecture and workshop formats. A lot of research has been done fairly recently on the implementation of Active Learning practices in math classrooms, and an article to this end has been posted to Sakai. I personally feel like these types of activities are very helpful in learning math, as you learn math best by doing problems, not by sitting around listening to lectures. Therefore, my plan is to implement several different activities in the classroom to move it more towards Active Learning. These may be things that you haven't seen in a math class before, but both I and the current research in the field believe that they are helpful in developing a better understanding of the course material. It only really works if you buy into it though, so I'm hoping you can give it a shot with me. If you have any questions, let me know and I'd be happy to talk about it.

The general plan for the class is as follows. Monday and Friday classes will generally start with a mini quiz and a Readiness Assessment, the first of which will test your knowledge of the homework problems assigned in the previous class, and the second will cover the readings assigned for the current class. You will be expected to have read the appropriate sections of the book before coming to class, and the Readiness Assessment will test your basic knowledge of these sections. These will be done individually, but there may be a group component to them afterwards. The rest of the class will consist of a mixture of workshop time, various activities, and lecture on the topics of the day. Over the course of each day, there will be two Practice Problems which will be completed, allowing you to practice the topics we are going over in class and show me that you know what is going on in class. Wednesday classes will be slightly different in that they will start with a quiz, testing material from the previous week of class. The rest of class will more or less be the same, consisting of lectures, activities, and workshops to help deepen or expand your knowledge of the material being covered.

Academic Integrity Policy

All students in this course are expected to be familiar with and abide by the academic integrity policy (`http://academicintegrity.rutgers.edu/academic-integrity-at-rutgers`). Violations of this policy are taken very seriously. In short, dont cheat, and dont plagiarize. In terms of exams, its fairly easy to understand what cheating/plagiarism is. For homework, you are definitely allowed to work with other students, but everything you turn in should be your own work. In particular, this means that you should NOT just write down and turn in a solution that you got from a friend, classmate, or the Internet. You should also be able to explain every step of what you turn in to me if asked. I would much prefer that you turn in a half-finished assignment than one that you looked up online or took from a classmate. In the first case at least both you and I know what you need to improve on and can work towards it. If you have any questions about this policy, please let me know. I am more than happy to talk about it.

Attendance Policy

Attendance is mandatory at every class. Each day in the classroom during the summer corresponds to an entire week of class during the semester. Therefore, missing a single class can be very detrimental to your learning and development in this course. Attendance will be taken in the form of practice problems and Readiness Assessments given out every day in class. If you need to miss a class, come talk to me about it as soon as possible.

Homework Assignments

There will be no traditional homework assignments for this course. There will be homework assigned each night, but the assignments will not be collected or graded. It is up to you to determine how many of the problems you want to do and how completely you want to work them out. You will get credit for this homework in terms of the quizzes and Readiness Assessments that will take place at the start of each class. These will be taken from problems very similar to those on the homework, so doing these problems will directly help you to do well on the mini-quizzes, as well as on the exams. I am also planning to post answers (not solutions) to the homework if the answer is not already in the book, so that you can check your work after doing the problems.

You will also be expected to read sections of the textbook and/or watch videos online before class. These will be announced at the end of the preceding class, and your knowledge of these sections will be tested with the Readiness Assessments at the start of each class. These assessments will cover basic knowledge of these sections. With the active learning component of the class, there will be less time for lecturing, so you all having a base level understanding of the material before you show up to class will help things to run smoothly. I also taught MATH 244 as a flipped classroom last summer, which means I made video lectures for the entire course. I may, at points throughout the summer, send you links to the videos for the appropriate sections for you to watch before class.

Projects / MATLAB Assignments

There will be 5 MATLAB assignments over the course of the summer. Each of these will involve taking pre-written code and modifying it to run some experiments that will illustrate concepts from class. These are assignments that have been given for the last several years, and I think they are well-made assignments. The assignments and sample code will be posted to Sakai, and you can download MATLAB by following the link here: `https://software.rutgers.edu/product/3437`.

In-Class Assignments

In-class assignments will take 3 forms. Monday and Friday classes will start with a mini-quiz about the homework from the previous class. This will be very similar to the homework problems assigned from that class. These mini-quizzes will be followed by a Readiness Assessment, covering basic knowledge on the reading assignments for the current class. Finally, there will be Practice Problems assigned each day in class, giving samples of the types of problems that could be seen on quizzes or tests related to the material being discussed in class.

Quizzes

On each Wednesday class that does not have a midterm, the class will start with a quiz. This quiz will cover the material that has been discussed over the last week. The quiz will last approximately 40 minutes and will be of similar difficulty to problems from the homework. These will be done individually and will be closed-book, closed-note.

Exams

This course will consist of two midterm exams and a final exam. The first midterm will happen this Friday, June 1. This exam is a prerequisite exam, covering material that you should know coming into this class, and is worth 10% of the final grade. There is an file on Sakai that contains an outline of what will be covered on this exam. This exam will take an hour. The second midterm will happen near the middle of the course and the tentative date is listed at the top of the syllabus. This exam will last 80 minutes. Due to the length of each class period, this will only take up half of the class, and we will continue with lecture/activities after the exam. These will be closed-book, closed-note exams, which will be taken individually. Calculators and electronic devices will not be permitted on exams.

Exam Rewrites

For each of the midterm exams, you will be allowed to rewrite the problems that you do not get full points on. You will receive exams with scores on the problems, but no marks on the pages, and can rewrite problems to get back half of the points that you missed. However, in order to get any points back for the rewrite, the rewritten version of the problem needs to be completely correct. More details about this process will be provided when the first midterm is returned.

Final Exam

The final exam will take place on Friday, July 6, from 6:00 - 9:00 pm in the normal classroom, and will be cumulative. Calculators and electronic devices will not be permitted on the final exam.

Make-Up Policies

There will be no make-ups for quizzes, practices problems, or exams. There is a decent chance that I will drop some number of practices problems or Readiness Assessments at the end of the course, but that will depend on how things go throughout the summer. If you have a legitimate reason for missing a midterm exam or quiz, then we can discuss potential options for your grade at that point, but try not to miss them. If you will not be in class on a day when an assignment is due, you need to send me a scanned version of the document before the end of class on that day. Pictures of the assignment will not be accepted, and anything received after the end of class will not be graded.

Disability Accommodations

Rutgers University welcomes students with disabilities into all of the University's educational programs. In order to receive consideration for reasonable accommodations, a student with a disability must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: `https://ods.rutgers.edu/students/documentation-guidelines`. If the documentation supports your request for reasonable accommodations, your campuss disability services office will provide you with a Letter of Accommodations. Please share this letter with your instructors and discuss the accommodations with them as early in your courses as possible. To begin this process, please complete the Registration form on the ODS web site at: `https://ods.rutgers.edu/students/registration-form`.

Adjustments

All information in this syllabus is subject to change at any time. Any changes will be announced on Sakai, changed on this document, and announced in class.

Tentative Course Schedule

Date	Section(s)	Topics	Due Dates
W 5/30	1.1, 1.2, Notes	Introduction, Solutions, Modeling	
F 6/1	1.2, 1.5	MIDTERM 1, Separation of Variables	
M 6/4	1.3, 1.4, 1.6	Geometric and Numerical Methods	
W 6/6	1.7	Bifurcations	MATLAB 1 Due, Quiz 1
F 6/8	1.8, 1.9	Theoretic and Analytic Methods	
M 6/11	2.1, 2.2, 2.5	Introduction to Systems	
W 6/13	2.7	MIDTERM 2, SIR Model	MATLAB 2 Due
F 6/15	3.1, 3.2, Notes	Linear Systems	
M 6/18	3.3, 3.4, 3.5	Phase Plane Analysis	
W 6/20	3.7	Trace-Determinant Plane	MATLAB 3 Due, Quiz 2
F 6/22	1.8, 2.3, Notes	Analytic Solution Methods	
M 6/25	3.6, 4.1, 4.2	Second Order Equations	
W 6/27	4.3, 4.4	Resonance, Steady State	MATLAB 4 Due, Quiz 3
F 6/29	5.1, 5.2	Non-Linear Systems	
M 7/2	5.3, 3.8, 2.8	Other Topics, Review	MATLAB 5 Due
F 7/6		FINAL EXAM	

MATH 252 - Mini-Quiz 4

June 18, 2018

Name: Key

Find the general solution to the system

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} \vec{x}$$

and determine the particular solution with initial condition $\vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Find eigenvalues: $\det \begin{bmatrix} 3-\lambda & -1 \\ -2 & 4-\lambda \end{bmatrix} = (3-\lambda)(4-\lambda) - 2$

$\lambda^2 - 7\lambda + 12 - 2$

$\lambda^2 - 7\lambda + 10 = \underline{(\lambda-5)}\underline{(\lambda-2)}$

$\lambda = 5$ $\quad \begin{bmatrix} 3-5 & -1 \\ -2 & 4-5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \vec{0}$

$-2v_1 - v_2 = 0$
$-2v_1 - v_2 = 0$ $\Rightarrow$ $v_1 = 1$, $v_2 = -2$ $\quad \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\lambda = 2$ $\quad \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \vec{0}$

$v_1 - v_2 = 0$ $\Rightarrow$ $v_1 = 1$, $v_2 = 1$ $\quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

So, the general solution is

$$\vec{x}(t) = k_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{5t} + k_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$$

To match the initial condition, we need

$k_1 + k_2 = 1$
$-2k_1 + k_2 = 2$

$3k_1 = -1 \quad k_1 = -1/3 \quad k_2 = 4/3$

$$\Rightarrow \vec{x}(t) = \begin{bmatrix} -1/3 e^{5t} + 4/3 e^{2t} \\ 2/3 e^{5t} + 4/3 e^{2t} \end{bmatrix}$$

MATH 252 - Readiness Assessment 7

June 25, 2018

Name: Key

1. Consider mass on a spring system whose model is given by the second order equation

$$\alpha y'' + \gamma y' + \beta y = 0$$

In terms of the physical problem, what is the meaning of:

(a) α

Mass

(b) γ

Drag Coefficient

(c) β

Spring Constant

(d) y

Position of the object

(e) $\frac{dy}{dt}$

Velocity of the object

2. Assume that a mass on a spring system has general solution given by

$$y(t) = c_1 e^{-t}\sin(2t) + c_2 e^{-t}\cos(2t) + \frac{1}{4}\sin(3t) - \frac{3}{4}\cos(3t)$$

(a) What is the natural response of the system? What is the natural frequency?

Natural response: $c_1 e^{-t} \sin(2t) + c_2 e^{-t} \cos(2t)$

Natural frequency is $\frac{1}{\pi}$

(b) What is the forced response of the system?

Forced response: $1/4 \sin(3t) - \frac{3}{4} \cos(3t)$

(c) What is the long-term behavior (as $t \to \infty$)?

As $t \to \infty$, the solution goes to the forced response

$1/4 \sin(3t) - 3/4 \cos(3t)$

MATH 252 - Problem Set 10

Matt Charnley

June 25, 2018

Part 1

For this set, we will be looking at section 4.1. In addition to the problems presented, you should also identify what kind of an equation it is. That is, is the system undamped, underdamped, critically damped, or overdamped, or does it not model a mass on a spring (this is when any of the coefficients are negative).

1. Section 4.1, Problems 1-11 odd.
2. Section 4.1, Problems 13, 15, 17.
3. Section 4.1, Problems 20, 21, 23.
4. Section 4.1, Problems 25, 27, 29.

Part 2

For this set, the same additions hold as for part 1. The only thing different here is going to be we have periodic forcing instead of exponentials or constants. You should begin to connect the amount of dampening in the system with what the solution formula and graphs look like. For all of these, I'm expecting you to solve them by Undetermined Coefficients, not the complex method they describe in the book. You should also be able to get the amplitude of the answer and know how to get the phase angle. I obviously won't expect you to be able to find the phase angle by hand, because that is generally impossible without a calculator.

1. Section 4.2, Problems 1-13 odd.
2. Section 4.2, Problem 17. This is a very good problem for checking your understanding of how these equations work.

MATH 252 - Practice 16

June 25, 2018

Name: Key

For the following second order equation, determine if it is undamped, underdamped, critically damped, or overdamped, find the general solution, and compute the particular solution if, at $t = 0$, the mass starts at the equilibrium point and is given an initial velocity of 2.

$$y'' + 5y' + 6y = \sin 2t$$

$r^2 + 5r + 6 = 0 = (r+2)(r+3) \longrightarrow$ Overdamped

Homogeneous Solution: $y_h(t) = C_1 e^{-2t} + C_2 e^{-3t}$

Undetermined Coefficients: Guess $y_p(t) = A \sin 2t + B \cos 2t$

$y' = 2A \cos 2t - 2B \sin 2t$

$y'' = -4A \sin 2t - 4B \cos 2t$

$y'' + 5y' + 6y = -4A \sin 2t - 4B \cos 2t + 10A \cos 2t - 10B \sin 2t$
$+ 6A \sin 2t + 6B \cos 2t$

$= (2A - 10B) \sin 2t + (2B + 10A) \cos 2t$

So $2A - 10B = 1$

$5(2B + 10A = 0)$

$52A = 1 \qquad A = 1/52$

$B = -5/52$

General Solution: $y(t) = C_1 e^{-2t} + C_2 e^{-3t} + \frac{1}{52} \sin 2t - \frac{5}{52} \cos 2t$

$$y'(t) = -2C_1 e^{-2t} - 3C_2 e^{-3t} + \frac{1}{26}\cos 2t + \frac{5}{26}\sin 2t$$

$$y(0) = C_1 + C_2 - 5/52 = 0$$

$$y'(0) = -2C_1 - 3C_2 + \frac{1}{26} = 2$$

$$C_1 + C_2 = 5/52 \rightarrow 2C_1 + 2C_2 = 5/26$$

$$2C_1 + 3C_2 = \frac{-51}{26}$$

$$-C_2 = \frac{56}{26} \qquad C_2 = -\frac{28}{13}$$

$$C_1 = \frac{5}{52} - C_2 = \frac{5}{52} + \frac{112}{52} = \frac{117}{52}$$

Particular Solution

$$\boxed{y(t) = \frac{117}{52} e^{-2t} - \frac{28}{13} e^{-3t} + \frac{1}{52}\sin 2t - \frac{5}{52}\cos 2t.}$$

MATH 252 - Problem Set 8

Matt Charnley

June 20, 2018

Part 1

For each of the following matrices, use the Trace-Determinant Plane to determine what kind of equilibrium solution the origin is if this matrix was used as the coefficient matrix in

$$\frac{d\vec{x}}{dt} = A\vec{x}$$

After you get through them, find a few general solutions and draw phase portraits to confirm your answer.

1. $\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$ **2.** $\begin{bmatrix} -3 & 2 \\ 1 & -3 \end{bmatrix}$ **3.** $\begin{bmatrix} 4 & 0 \\ -3 & -1 \end{bmatrix}$ **4.** $\begin{bmatrix} 0 & -2 \\ 7 & 1 \end{bmatrix}$ **5.** $\begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$ **7.** $\begin{bmatrix} -2 & 4 \\ 9 & -1 \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix}$ **9.** $\begin{bmatrix} 2 & -5 \\ 1 & -1 \end{bmatrix}$ **10.** $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

Part 2

In this section, you will analyze one-parameter families of systems in groups. Each group will take one of the following problems, work out the details, and then present it to the class. The components of the answer that I want to see are as follows:

1. Calculation of the trace and determinant of the family.
2. A sketch of the curve traced out by this one-parameter system in the Trace-Determinant Plane and how you determined this curve.
3. An identification of where the type of critical point at the origin changes, as well as what it changes from and to at that point.
4. (Optional) A few phase portraits of the system at specific values of the parameter μ.

The problems are:

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 3-2\mu \\ -1 & \mu \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 0 & -1 \\ e^{\mu} & \mu \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & \mu \\ \mu+6 & 2 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & -1 \\ 2 & 1 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & -1 \\ 2 & -1 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 3-\mu^2 & \mu+1 \\ 2-\mu & 1 \end{bmatrix}$$

MATH 252 - Midterm 1 Review

Matt Charnley

May 24, 2018

1. Find the roots of
   (a) $x^2 - 4x - 32$
   (b) $x^2 + 3x + 18$
2. Find all roots of $x^3 + 3x^2 - 2x - 6$.
3. Find all roots of $x^3 + 3x^2 - 4$.
4. Logarithms:
   (a) Simplify $\ln(40) - 3\ln(6)\ln(10)$
   (b) Solve $\ln(x^2) = 10$
5. Exponentials:
   (a) Simplify $\left(e^{2x} + e^{x^3}\right)^2$
   (b) Solve $e^{2x-5} = 4$
6. Trig Identities: Simplify $(\sin(2t) + 2\cos(2t))^2 - \cos(4t)$
7. Find the derivative of $x^4 + 3x^2 - 2x + 1$
8. Find the derivative of $x^3 \sin(2x)$
9. Find the derivative of $\frac{x^2+3}{2x+e^x}$
10. Find the derivative of $\tan(\ln(x))$.
11. Find the derivative of $xe^{\cos(x)}$
12. Compute $\int x^3 - 4x + 5 \; dx$
13. Compute $\int 3x\cos(x^2) \; dx$
14. Compute $\int 2x(1 + 4x^2)^{10} \; dx$
15. Compute $\int x^2 \sin x \; dx$
16. Compute $\int \ln(x) \; dx$
17. Compute $\int_0^3 f'(t) \; dx$ if $f(0) = 3$ and $f(3) = 10$.
18. Compute $\frac{d}{dt}\left[t^2 \int_0^t f(s) \; ds\right]$ for a function $f(s)$.
19. Compute $A + B$ for

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 2 & 0 & -3 \\ 3 & 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 6 \\ 1 & 5 & 2 \\ -1 & -2 & 4 \end{bmatrix}$$

20. Compute AB for

$$A = \begin{bmatrix} 6 & 1 \\ 4 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 4 & 7 \end{bmatrix}$$

21. Write out

$$\begin{bmatrix} 1 & 0 & 4 \\ 2 & 4 & 1 \\ -3 & 1 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

as a linear system of equations.

22. Determine if

$$\left\{ \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

is a linearly independent set of vectors.

23. Determine if

$$A = \begin{bmatrix} 6 & 1 \\ 4 & -3 \end{bmatrix}$$

is invertible.

24. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

25. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

26. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$

MATH 252 - Bifurcation Jigsaw

Matt Charnley

June 6, 2018

Purpose

The purpose of this activity is to allow you, as groups, to explore bifurcations and the different types that exist by analyzing a problem on your own. It will also give you an opportunity to improve your skills at talking about math concepts with your peers, as you will be presenting your discoveries to the other students in the class.

Structure

This activity will have three parts. During the first part, you and your group will work on analyzing two different one-parameter families of ODEs to search for bifurcation points. Each group will be working on the same problems here, and after some time to figure it out, we will discuss the results, so you know what the answer to a problem like this looks like. During the second part of this activity, each group will analyze another bifurcation problem, but each group will have a different problem. As a group, you will be responsible for finding and analyzing the bifurcation points of your problem following the format below. Once I see that everyone has the proper results, we will move on to the last part of the activity. Finally, in the 'Jigsaw' part of the activity, the groups will be shuffled so that every new group has at least one person in it who did each of the problems. Then, everyone in the new group will take a turn sharing what they discovered about the problem they analyzed. That way, at the end of class, everyone will have heard about all of the problems.

Format of Answer

Given a one-parameter family of ODEs, your analysis of any bifurcation points should contain the following.

1. A determination of any/all bifurcation points of the family of equations. This may include sketches of the graph of the function $f_\mu(y)$.
2. For each of the bifurcation points, a phase line (with solution sketches) for a value slightly larger and slightly smaller than the bifurcation point.
3. A description of what happens at the bifurcation points. (How many equilibrium points are there before and after? What type are they?)
4. A sketch of the bifurcation diagram for this family.

In terms of turning in this assignment, each group will need to hand in one copy of their bifurcation analysis for the second part of this activity. It can be handed in today or at the start of class on Friday.

Problems

For the initial part of this activity, each group will analyze the families

$$\frac{dy}{dt} = y^2 - \mu y$$

and

$$\frac{dy}{dt} = \mu - y^3$$

For the second phase, the possible familes of ODEs are

1. $\dfrac{dy}{dt} = \mu y - y^3$
2. $\dfrac{dy}{dt} = 2y\left(1 - \dfrac{y}{10}\right) - \mu$
3. $\dfrac{dy}{dt} = y^2 - \mu y + 1$
4. $\dfrac{dy}{dt} = (y^2 - \mu)(y^2 - 9)$
5. $\dfrac{dy}{dt} = y^3 + \mu y^2$
6. $\dfrac{dy}{dt} = y^4 + \mu y^2$

where, in each case, μ is the parameter to be varied.

MATH 252 - Harmonic Oscillators

Matt Charnley

June 27, 2018

1 Beats and Resonance

Beats and resonance happen when an undamped oscillator is forced at specific frequencies. This worksheet here will help you explore how and why this happens. For this set here, we will be considering the mass on a spring system given by

$$y'' + 16y = f(t)$$

(a) What is the general solution of this equation if $f(t) = 0$?

(b) What is the particular solution with $y(0) = 2$, $y'(0) = 0$ in this case?

(c) Now, we want to add in a forcing term. Let $f(t) = \cos(8t)$. Find the general solution to this ODE, and the particular solution with $y(0) = y'(0) = 0$.

(d) Using the identity

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

write this solution as a product of sine functions.

(e) Sketch the graph of this solution. The way you should think about this is that one of the two sine functions oscillates slower than the other. Draw the graph of this function and the negative of this function, and then you're going to draw the actual solution between those two curves. The ratio of the two frequencies should tell you how many times the curve cycles in each of the larger bumps of the slowly oscillating function.

(f) Do the same analysis for $f(t) = \cos(5t)$, $f(t) = \cos(4.5t)$, $f(t) = \cos(4.1t)$.

(g) Consider the forcing function $f(t) = \cos(\omega t)$ for $\omega \neq 4$. What does the result look like here? What is the amplitude of the resulting solution, and how does it depend on ω?

(h) What is the beating frequency of this solution (the frequency of the slower oscillation)? How does this change with ω?

(i) What happens when $\omega = 4$? What is the general solution here? What is the particular solution with $y(0) = y'(0) = 0$? Sketch the graph of this solution.

2 Amplitude of Steady State

Here, we are looking at damped oscillators of the form

$$y'' + py' + 4y = f(t)$$

(a) If $p = 5$, what is the general solution to this equation with $f(t) = 0$?

(b) If $f(t) = \cos(3t)$, what is the solution to the non-homogeneous part of the equation? You may still assume $p = 5$ here. What is the amplitude of this solution?

(c) Try the same for $f(t) = \cos(1.5t)$. Try it for $f(t) = \cos(2t)$ and $p = 1$.

(d) Perform the same analysis as part b, but leave p unspecified. The main thing I care about here is the amplitude. How does this change as p varies?

(e) Now, consider a forcing term of the form $f(t) = \cos(\omega t)$. Do the same analysis with both p and ω unspecified. You should end up with the amplitude as a function of ω and p. For what value of ω does it achieve it's maximum? Why is this not surprising? Look at the above section.

(f) Our input force for this process was of amplitude 1, since it was just a cosine function. What kinds of amplitudes can we get for the output steady state? Is this surprising?

3 Wrap-Up

In some sense, part 2 of this worksheet is more physically relevant, because there aren't really physical situations with no damping, since air resistance is a thing. However, we see from analysis of the ODE that we can still get some interesting results, and even some drastic behavior if the situation is just right. In particular, both the damping coefficient and the forcing frequency play a role in how the system responds to the force. One could use this knowledge to make sure that, for a given range of frequencies, the amplitude is never larger than 1, which might cause a system to break. This type of problems extends more into the engineering field that relates to the system being studied. The mass (which we fixed at 4) also plays a role, but more so in where the maximum amplitude occurs instead of what it is.

The main things to take out of the first part of this exercise are the ideas of **beats** and **resonance**. Both of these can only occur in an undamped system, beats occuring when the forcing frequency is close to that of the natural frequency of the system, and resonance occurs when these frequencies are equal. Resonance can only really happen for so long, until the system grows so fast that the model is no longer reasonable, but beats occur all the time, particularly when tuning an instrument. The beats indicate that the frequency is off just a little bit, and so it needs to be tuned. You should be able to recognize when beats and resonance can occur, know how to choose a forcing function to make it occur, and sketch the graphs that correspond to each of these situations.

MATH 252 - SIR Model and Diseases

Matt Charnley

June 13, 2018

Purpose

The purpose of this activity is to give you a chance to experience the SIR model for the spread of diseases. Over the course of this activity, you will see how the different parameters in this model affect how the disease behaves, and how making changes to the model can help it to emulate different types of diseases. You'll also get more time playing around with MATLAB code and seeing how you can use this to approximate the solution to these systems of ODEs.

Introduction

The basic idea of a disease model is that there are three main groups of people with respect to a disease: people who are susceptible to the disease, people who are currently infected (and contagious), and people who have recovered from the disease and are immune to it. These populations are denoted S (susceptible), I (infected) and R (recovered), hence the SIR model for diseases. For our particular case, we will let S, I, and R be the proportion of the population that falls into each of these categories, so that all of them are always between 0 and 1. In general, we will have initial conditions where $R(0) = 0$, meaning that no one has yet recovered from the disease, and $I(0)$ is a very small fraction of the population. A disease can be characterized by its infection parameter (how often a contact between an infected and susceptible individual results in a spread of the infection) and its recovery parameter (how quickly an infected individual 'recovers' from the disease, which can mean either no longer being infected or through death, because then you can't spread the disease anymore). While these models aren't perfect, they can give us a feel for how diseases spread and what can be done to prevent it. The main assumption in this model is that people are 'well-mixed' in the sense that any two people could interact at any time. These models work better for small, isolated communities, but we can still gain information from these types of results.

The Basic Model section below will give an introduction to the simplest of these models and kind of show you how they work. In addition to this, there is a folder full of MATLAB files on Sakai that will run all of these models. These should run fairly similar to the MATLAB files that you have been seeing for the MATLAB projects so far. The basic plan will be that the code does its thing, you'll just need to change the parameters and run the code to see what it does.

Basic Model: SIS

The first model we will discuss is the SIS model. This is an ODE system that could to model the behavior of the common cold. The idea of this model is that once someone recovers from being infected, they are immediately susceptible to the disease again. Thus, the model looks like

$$\begin{aligned}\frac{dS}{dt} &= -aIS + bI\\ \frac{dI}{dt} &= aIS - bI\end{aligned}$$

where a is a parameter that describes how likely an infected person is to spread the infection to a susceptible individual, and b represents the rate at which infected individuals recover.

1. Find (by hand) the equilibrium points of the system. Looking at the equations, determine whether the solution will go towards or away from the different equilibrium points as t increases. Hint: Maybe there's a bifurcation in here somewhere.

2. Run the code for:

   (a) $a = 1$, $b = .5$
   (b) $a = 10$, $b = 1$
   (c) $a = 10$, $b = 5$
   (d) $a = 1$, $b = 4$

   or any other set of parameters you want to see how it behaves. You should hopefully get the result that you expected from the first part. You should also look at different initial conditions. The main one to start with is $S = 0.99$, $I = 0.01$

3. Explain what the 'equilibrium solution' means here and what this means for something like the cold.

Other Models

SIR

The SIR model applies to something more like measles, where once a person gets the disease, they are immune to it from that point forward. In this case, we get a system that looks like

$$\frac{dS}{dt} = -aIS$$
$$\frac{dI}{dt} = aIS - bI$$
$$\frac{dR}{dt} = bI$$

where a and b represent the same type of quantities as before. For this system, we can ignore R, and just focus on the relation between S and I.

1. Find all equilibrium points of this system.

2. When is $\frac{dI}{dt}$ positive and when is it negative? (For what values of the parameters)

3. When does an epidemic happen? An epidemic is defined as when the proportion of infected people is increasing.

SIRS

A modification of the SIR model can be used to model something like the flu. The idea with modeling the flu is that, after getting the disease, individuals are only immune to the disease for a period of time, until the disease mutates and everyone is susceptible again. In order to model this, we pick a parameter T to be the mutation time for the disease, and people are restored to the susceptible pool based on the number of recovered individuals T time units in the past. This gives rise to the *delay differential equation*

$$\frac{dS}{dt} = -aIS(t) + cR(t-T)$$
$$\frac{dI}{dt} = aIS(t) - bI(t)$$
$$\frac{dR}{dt} = bI(t) - cR(t-T)$$

where a and b are the same type of parameter as before, and c represents how fast recovered individuals are returned to the susceptible pool.

1. See how varying the parameters affects the results of the code. This includes varying a, b, c, and T.
2. How is this system substantially different than the previous few examples?

Pre-Vaccination

One way to limit the spread of a disease is by vaccinating against the disease. In terms of the ODEs, this is represented by having the R function not start at 0. Since R is the number of recovered individuals who are immune to the disease, having someone vaccinated means they start immune to the disease, which means they start in the R category and don't move out of it. In this case, we have the normal SIR equations

$$\begin{aligned}\frac{dS}{dt} &= -aIS \\ \frac{dI}{dt} &= aIS - bI \\ \frac{dR}{dt} &= bI\end{aligned}$$

but our initial conditions are $R(0) = p$, $S(0) = 0.99(1 - p)$ and $I(0) = 0.01(1 - p)$, i.e., 1 percent of the non-vaccinated people are infected with the disease.

1. Set $R(0) = 0$, and find a set of parameters a and b that give rise to an epidemic.
2. Increase the value of p until an epidemic no longer occurs.
3. For your values of a and b, compute $R_0 = a/b$. How does the p you found relate to this value?
4. To get an idea of comparison, the R_0 value for measles is between 12 and 18 and smallpox is between 5 and 7. Run the code for $a = 15$, $b = 1$ and figure out where the p needs to be to prevent an epidemic. This p is the critical percentage of population that needs to be vaccinated to prevent an epidemic. This is the idea of herd immunity.

Post-Vaccination

Another extension of this model is the idea of a vaccine to a disease being created once the epidemic is started. It is then a race to vaccinate the population before the disease spreads. This can be modeled by the following system of differential equation

$$\begin{aligned}\frac{dS}{dt} &= -aIS - cS \\ \frac{dI}{dt} &= aIS - bI \\ \frac{dR}{dt} &= bI + cS\end{aligned}$$

where c in this context denotes how much of the population is given the vaccine per time unit. Here, the initial conditions are $S(0) = 0.99$, $I(0) = 0.01$, $R(0) = 0$. Run some trials with an increasing value of c and see how this changes the results.

Zombies

A modified version of the SIR model can be applied to a zombie epidemic as well. The main difference here is that there is no recovery, but zombies still die off if they can not feed. Thus, we can start with the standard model

$$\frac{dS}{dt} = -aIS$$
$$\frac{dI}{dt} = aIS - bI$$

There are a few modifications that can be made to this to make it seem more like a zombie model. The first of those is that the zombies are organized, so the spread is more aggressive than that of a normal disease. This can be repesented by

$$\frac{dS}{dt} = -aS\sqrt{I}$$
$$\frac{dI}{dt} = aS\sqrt{I} - bI$$

Since $I < 1$, $\sqrt{I} > I$, so this is a faster rate. A second modification is that the zombies don't die off on their own, but can be killed by the survivors. This results in a system that looks like

$$\frac{dS}{dt} = -aSI$$
$$\frac{dI}{dt} = aSI - cS^2I$$

where c describes how quickly and efficiently survivors can kill off zombies. This last term represents the fact that if there are multiple survivors interacting with a zombie, they have a chance of killing it before they get converted. These could also be combined to give more complicated models. Run this for a variety of different parameters and see how they affect if the people survive or how long they last.

Assignment

By next Monday, each group needs to write up an exploration of one of the "Other Models." Your write-up should include some graphs of the system at a variety of parameters, an explanation of how changing the parameters affects the behavior of the system.

MATH 252 - Student Generated Test Questions

Matt Charnley

June 29, 2018

Purpose

The purpose of this assignment is to give you the opportunity to think about what we have learned in this class and formulate this into questions that could be seen on an exam. It will also result in you all having a collective list of problems that can be used as a review for the final.

Assignment

By Monday, July 2, everyone will be responsible for submitting at least 2 questions from a section or topic from this course, along with their solutions. When creating these problems, you should think about what types of problems you have seen on quizzes and exams up to this point, as well as the extension of those problems to what we have done more recently. You can also use the problems in the textbook as motivation for creating your problems. The plan will be to type up all of these problems and post them to Sakai so that you can use them for review. I will also post the solutions (once I check them over) so that you have those as well. You should use all of the resources at your disposal when writing the solution; these are (assuming you are ok with this) going to be posted for the rest of the class to view, and so should be correct.
Here's what I need you to turn in:

1. At least 2 questions written from your assigned section/topic.
2. The corresponding solutions. When writing the solutions, it would be very helpful if you could write them on printer paper. It'll make it easier for me to scan them in.

Ideally, all of this will be posted on Sakai during the morning of Tuesday, July 3, so you have plenty of time with them to review.

Topics

This is my initial outline of topics and how many people will be working on each. If you want to move these around, we can do that.

1. Verifying solutions (1 person)
2. Separable Equations (1 person)
3. Slope Fields + Existence and Uniqueness (1 person)
4. Euler's Method (1 person)
5. Autonomous Equation (1 person)
6. Bifurcations (1 person)
7. Linear Equations (1 person)
8. Linear Systems - Phase Portraits (4 people)
9. Trace-Determinant Plane and One-Parameter Systems (2 people)
10. Matrix Exponentials (1 person)
11. Second Order Equations - Undetermined Coefficients (2 people)
12. Mass on a Spring system - General Behavior of Solutions (2 people)
13. Mass on a Spring - Beats and Resonance, Amplitude of Steady State (2 people)
14. Non-Linear Systems (4 people)
15. Modeling with Equations (2 people)
16. Modeling with Systems (1 person)

D.5 Fall 2018 - Math 104

This section contains the following documents from my Fall 2018 section of Math 104:

1. The course syllabus
2. A worksheet I designed about counting problems, which was built from previous work with Pascal's Triangle
3. A worksheet I designed introducing the idea of continuous distributions and the normal distribution
4. A sample practice problem that students were given at the end of class
5. A sample "small quiz" that was given to students the class after turning in a homework set
6. A sample "big quiz" that was given to cover material from about two weeks worth of lectures
7. A sample homework solution that was given to the grader

MATH 104 - Section 02 - Fall 2018

Contact Information

Name: Matt Charnley
Email: charnley@math.rutgers.edu
Office: Hill 606

Office Hours

Mondays, 1:00 - 2:30 PM
Fridays, 10:30 AM - 12:00 Noon
or by appointment.

Prerequisites

Intermediate Algebra at the level of Rutgers Math 026 or 027, or equivalent. A real mastery of elementary algebra and other basic skills is crucial to success in this course.

Course Description

Math 104 gives a mathematical yet accessible and concrete introduction to probability. Most of the course is devoted to understanding how probability works, and how it is applied in a number of areas, including medical testing and financial decision making. The end of the course introduces some statistics and applications. You will never be left wondering, "What is this good for? What does this have to do with real life?"

Math 104 is not intended for majors in STEM fields. Credit will not be granted to those who have already taken, or are concurrently taking, a higher level probability course (including but not limited to 01:640:477, 01:198:206, 01:960:381, 14:332:226, and 14:540:210).

Learning Goals

Over the course of this semester, students will learn how to answer the following general questions:

1. What is probability and where in my life can it be used?
2. What is expected value and how does/should it affect what I decide to do?
3. How can I compute probabilities?
4. What is the normal distribution and what is it good for?

In particular, after this course, students will be able to

- understand the difference between theoretical and experimental probability
- understand expected value as it arises in a variety of real-life contexts, and understand its role in decision making
- use Venn diagrams and other related tools to compute probabilities in a variety of contexts
- recognize and avoid common fallacies in probability, including the gambler's fallacy and the base rate fallacy
- understand the relevance of and rationale behind probability simulations, and carry them out
- understand the difference between odds and probability, and convert between them

- understand independent and dependent events, recognize the difference between them, and carry out relevant computations
- understand conditional and unconditional probabilities, recognize the difference between them, and carry out relevant computations
- use the binomial probability model to compute probabilities in a range of settings
- have a concrete understanding of the rationale behind Pascal's triangle, the meaning of its entries, and the reasons why they appear throughout this subject
- work with z-scores and probability tables or the standard normal distribution, to compute probabilities of events arising from the normal distribution
- understand p-values as applied to experimental results and use this to carry out a hypothesis test using given data
- interpret a broad range of real life settings to which probability is relevant, and interpret what people mean when they make statements related to probability
- understand why they should resist the temptation to go to a casino, or gamble online, or even play the lottery

SAS Core Curriculum Learning Goals

Math 104 fulfills both the Quantitative Information (QQ) and Mathematical or Formal Reasoning (QR) learning goals of the SAS Core Curriculum:

QQ: Formulate, evaluate, and communicate conclusions and inferences from quantitative information.
QR: Apply effective and efficient mathematical or other formal processes to reason and to solve problems.

Textbook

Finite Mathematics: An Applied Approach, by Michael Sullivan, 11th Edition customized for Rutgers University, available at the bookstores. The customized edition contains only the chapters which are used in the course.

The 10th Edition is no longer used in this course; you really do need the 11th. WileyPlus is no longer used in this course. No solution manual is required. The custom edition of the textbook can also be bought as an e-book, at `https://store.vitalsource.com/show/9781119947172`.

Calculator

You will need a scientific calculator for homework. Calculators will be permitted on half of the exams; see the exam section for more information.

Class Meetings

Tuesday/Thursday, 1:40 - 3:00 PM, Tillett 207

Office Hours

I will be holding office hours in my office, Hill 606, on Busch campus. My current schedule for office hours is Mondays, 1:00 - 2:30 PM and Fridays, 10:30 AM - 12:00 Noon. Any changes to this schedule will be announced in class and on Sakai. I will also be holding informal office hours right after class both Tuesdays and Thursdays, and will have office hours by appointment. If the times I have scheduled do not work for you, feel free to send me an email and we can work out a different time to meet. Do not be shy about asking! All appointments should be scheduled at least 24 hours in advance. There will also be unofficial office hours in the Sakai Chat Room. You can post questions there for your fellow students or me to answer. I will check in the Chat Room around once a day (maybe more) to answer any questions there that are left unanswered.

Email and Contact

My email is charnley@math.rutgers.edu, and you should feel free to email me whenever you have questions or concerns about this class. I will respond to all emails within two business days, and if not, you are free to email me again to make sure I respond. However, I will not be checking my email constantly, and I make no guarantee that I will respond to emails after 6pm each evening. Since homework is due at 11pm, that means that if you have questions you want answered, you need to have worked on the homework in advance to ask them before 6pm on the day homework is due. I still might respond, but it is also possible that you will get a response the next morning. I will also try to respond to emails over the weekend, but there are some weekends where I may be travelling, and will be unable to respond.

In addition to teaching you probability, it is also part of my job to make sure you all are ready to enter the workforce, in whatever capacity that is, once you leave here. Part of that is being able to send professional emails that also get your point across and get the information that you want. A few pointers towards this end:

- All emails you send about course-related material should be sent from your Rutgers email (ScarletMail).
- All of your emails should have a subject line and some sort of salutation at the beginning.
- All emails should be written in complete sentences, and slang terms should be avoided.
- You should sign your name at the bottom of the email.

Online Resources

This course uses Sakai, accessible at `sakai.rutgers.edu`; login with your ordinary Rutgers NetID and password. Use Sakai to view announcements, submit homework via the Assignments feature, view solutions to homework problems via Resources, access the textbook, and participate in online discussions about the subject matter of the course in the Chat Room. You are also expected to check your Rutgers email account frequently, since that is the account to which Sakai sends all messages, and which your professors are expecting you to check. If you wish to set up email forwarding from that account to a different one, you can do so, but it is your responsibility to ensure that it works properly.

Attendance

You are expected to attend all class meetings, whose content will go far beyond a mere rehashing of the textbook. The classroom setting will involve a fair amount of active learning, and your full participation and engagement are necessary for you to learn effectively. **If you entertain the notion that you can succeed in the course by learning the material on your own, please think again.**

Grade Breakdown

Final grades for the class will be decided according to the following breakdown:

Homework and Quizzes	20%
Midterm Exams	40%
Final Exam	40%

Exam Schedule

- Midterm 1: October 9, 2018, 1:40 - 3:00 PM
- Midterm 2: November 13, 2018, 1:40 - 3:00 PM
- Final Exam: December 17, 2018, 12:00 noon - 3:00 PM

Homework

There will be typically be a homework assigned after each class. Due dates for each assignment will be indicated on Sakai. Typically, you should expect 1-2 assignments due each week. These will generally be due on Tuesday and Saturday nights at 11:00 PM. **All written work must be submitted online via the Assignments tab of Sakai; it may not be submitted on paper, nor by email. There will be NO exceptions!**

Since the solutions to each homework assignment will be posted in the Resources section of Sakai shortly after the assignment is due, **late homework will not be accepted!** Finally, doing the homework is crucial to learning the subject thoroughly, and the system of electronic submission makes it possible for you to get feedback quickly on whether you have done the homework correctly.

Important guidelines for submitting homework:

1. To access Sakai, be sure that you have an active email account and know your NetID and password.
2. The order of the homework problems in your submitted work should be the same as the order in which they are listed in the assignment on Sakai.
3. If you upload a file, it must be in one of the following formats: .doc, .docx, .pdf, or .jpg. Files which are not in this format, and which the instructor cannot open, will be given a grade of 0.
4. If you need to scan your homework assignment, the campus computer labs and libraries do have scanners, and there are technical assistants there if you need help. Note: Scanned documents should be saved as pdf files.

5. A popular technique for producing images of handwritten work is to photograph each page, or each half of each page, using a cell phone. This is acceptable.
6. It is entirely your responsibility to make certain that the file you upload has the appearance you intend. Please check this by opening the file after you have uploaded it, and see that it does open, that the image is right side up and generally readable, etc. If the file format is incorrect or the image is blurry, you will be given a grade of 0.
7. If you ever have technical difficulties with Sakai, especially in uploading homework, please contact the excellent and very responsive Sakai help desk at sakai@rutgers.edu or 848-445-8721.
8. Do not wait until the last hour to upload your homework, since the system may be unexpectedly busy.
9. Whether you upload a file or type your answers into the textbox, be sure to hit submit at the bottom of the screen to submit your assignment.
10. You will very quickly get a confirmation email from the Sakai system that your assignment was submitted. Make sure that you get this confirmation email, because it is your responsibility to make sure that your submission goes through.
11. If you do not receive the confirmation email, your work probably did not go through, in which case you should log back in to confirm that your work was posted, and if it wasn't posted then you should resubmit.

Unless specified otherwise, **you must write your answers in complete, grammatically correct English sentences**. Being able to do this is a crucial aspect of quantitative literacy, which goes beyond mere computational proficiency.

You are permitted, and in fact encouraged, to work together on homework problems, but all written work which you submit must ultimately be your own. It is also super important for you to stay on top of the homework, and ideally you should be working on the homework a little bit every day. This will allow you to better digest the material and get a chance to ask your questions well in advance of the due date for the homework.

Quizzes

Students should expect a quiz each class day. Thus, attendance is crucial. If you have completed and understood the homework (both the graded and ungraded homework), you should not find the corresponding quiz terribly difficult. These quizzes will take place at the start of each class (after about 5 minutes for questions) on the most recent homework that has been turned in, and will be done without calculators. Please note that **there are no make-up quizzes!** However, I do intend to drop at least one of the lowest quiz grades. There will also be practice problems at the end of each class that you will need to turn in before you leave. These will count towards a portion of your homework grade, but if you do not turn in a problem, your quiz grade for that day will be affected. Therefore, it is necessary for you to stay until the end of each class meeting. There will also be 5 larger quizzes over the course of the semester that will take place at the end of class, covering the previous two weeks of material. These will replace the quiz at the start of class, be announced in advance, and included on the course schedule.

Exams

There will be two "midterm" exams and a comprehensive, cumulative final. The exams will be closed book and student-prepared formula sheets will **not** be permitted. The exams will have two parts; the first of which will be done without calculators, and the second will allow the use of calculators. As the first exam approaches, there will be another handout with more details on how the exam will work. **If you dont show up to an exam, you will receive a zero!**

Missing an exam is a serious matter, and should only occur as a result of a genuine, verifiable emergency situation. "Verifiable" means that there should be a doctor's note, notice of court appearance, etc. indicating that you were unable to attend at the time of the exam. If circumstances beyond your control prevent you from attending an exam, it is important that you contact the instructor as quickly as possible.

Makeups

There are no makeup quizzes or exams. As noted above, under truly compelling circumstances an absence from a quiz or exam can be excused, but instead of having a makeup, the portion of the final exam pertaining to the content of the missed midterm exam will count correspondingly more heavily.

Classroom Setting

Using your cell phone or other device to send or view texts, or to surf the internet, or for any other purposes not directly related to your in-class work, is not acceptable. Doing so is very distracting to you and to others, and is truly inappropriate. This is excellent news for you, and everyone else, since you have all paid a significant amount of money to be here, and there are ample opportunities for conversation and texting and social media outside of class time.

Academic Integrity

All Rutgers students are expected to be familiar with and abide by the academic integrity policy (`http://academicintegrity.rutgers.edu/policy-on-academic-integrity`). Violations of this policy are taken very seriously. During exams, cell phones, tablets, laptops and any other WiFi or cellular capable devices must be turned off (not just silenced), and completely put away; **having a cell phone, tablet, or laptop visible during the exam will automatically be reported as an academic integrity violation, with a minimum penalty of receiving a 0 on the exam.** Moreover, during an exam, if you leave the room you must turn in your exam paper, and will not be able to return to continue working on it.

Extra Help

If you are having difficulty, please take advantage of the opportunity to visit office hours. Also, please do not hesitate to ask questions by email, or in the Sakai Chat Room. The Rutgers Learning Centers also provide drop-in, free tutoring for Math 104, according to a schedule accessible at `http://lrc.rutgers.edu/content/tutoring`.

A few friendly words of advice:

1. Never fall behind in a math course!!!!! The ideas we will discuss need time to sink in, and are very difficult to learn quickly right before an exam, so it is important to clear up your confusions sooner rather than later.

2. An excellent way to improve your understanding of the subject is to study together with classmates. Explaining mathematical ideas to others is often the most effective way to sort out your own confusions and clarify your understanding; you don't know just what it is that you don't know until you try explaining it to someone else.

3. You are also warmly invited to ask questions in class, which students are far too hesitant to do in math courses, or in office hours, or by email, or in the Sakai Chat Room! I very much want you to succeed in this course.

Disability Accommodations

Rutgers University welcomes students with disabilities into all of the University's educational programs. In order to receive consideration for reasonable accommodations, a student with a disability must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: `https://ods.rutgers.edu/students/documentation-guidelines`. If the documentation supports your request for reasonable accommodations, your campuss disability services office will provide you with a Letter of Accommodations. Please share this letter with your instructors and discuss the accommodations with them as early in your courses as possible. To begin this process, please complete the Registration form on the ODS web site at: `https://ods.rutgers.edu/students/registration-form`.

Adjustments

All information in this syllabus is subject to change at any time. Any changes will be announced on Sakai, changed on this document, and announced in class.

Student Wellness Services

Just In Case Web App
http://codu.co/cee05e
Access helpful mental health information and resources for yourself or a friend in a mental health crisis on your smartphone or tablet and easily contact CAPS or RUPD.

Counseling, ADAP & Psychiatric Services (CAPS)
(848) 932-7884 / 17 Senior Street, New Brunswick, NJ 08901/ www.rhscaps.rutgers.edu
CAPS is a University mental health support service that includes counseling, alcohol and other drug assistance, and psychiatric services staffed by a team of professional within Rutgers Health services to support students efforts to succeed at Rutgers University. CAPS offers a variety of services that include: individual therapy, group therapy and workshops, crisis intervention, referral to specialists in the community and consultation and collaboration with campus partners.

Violence Prevention & Victim Assistance (VPVA)
(848) 932-1181 / 3 Bartlett Street, New Brunswick, NJ 08901 / www.vpva.rutgers.edu
The Office for Violence Prevention and Victim Assistance provides confidential crisis intervention, counseling and advocacy for victims of sexual and relationship violence and stalking to students, staff and faculty. To reach staff during office hours when the university is open or to reach an advocate after hours, call 848-932-1181.

Scarlet Listeners
(732) 247-5555 / http://www.scarletlisteners.com
Free and confidential peer counseling and referral hotline, providing a comforting and supportive safe space.

Tentative Course Schedule

Date	Material	Textbook Sections	Deadlines
September 4	Intuitions, Probability, Sample Space, Events	Section 7.4	
September 6	Real-World Examples, Relative Frequency, Experimental vs. Theoretical Probability	Notes	
September 11	Expected Value, Events as subsets	Section 7.6	
September 13	Basic Set Operations and Probabilities	Section 7.1	
September 18	Inclusion/Exclusion	Section 7.2	Big Quiz 1
September 20	Multiplication Rule and The Birthday Problem	Section 7.3	
September 25	Odds, Expected Value and Decision Making	Section 7.5	
September 27	Pascal's Triangle	Notes	
October 2	Pascal's Triangle	Notes	Big Quiz 2
October 4	Catch up and Review		
October 9	Midterm 1		
October 11	Binomial Coefficients	Section 8.5	
October 16	Binomial Probability Model	Section 8.6	
October 18	Expected Value for Binomial	Section 8.5	
October 23	Conditional Probabilities and the Monty Hall Problem	Section 8.1	Big Quiz 3
October 25	Conditional Round 2, False Positives, Law of Total Probability	Section 8.3	
October 30	Conditional Round 3, Independent events	Section 8.2	
November 1	Conditional Round 4, Tree Diagrams and Contingency Tables	Sections 8.1-8.3	
November 6	Conditional Binomial Probabilities	Sections 8.1-8.3, 8.6	Big Quiz 4
November 8	Catch up and Review		
November 13	Midterm 2		
November 15	Simpson's Paradox, Mean, Standard Deviation	Section 9.4	
November 20	Binomial to Normal Approximation	Section 9.6	
November 27	Empirical Rule and Z-Scores	Section 9.5	
November 29	Using Z-scores and Z-tables	Section 9.5	
December 4	Hypothesis Testing and p-values	Notes	
December 6	Hypothesis Testing Round 2	Notes	Big Quiz 5
December 11	Catch up & Review		
December 17	Final Exam		

Worksheet 8 - Binomial Coefficients

October 11, 2018

Name: ____

So, it turns out that those $C(n,k)$ numbers that came out of Pascal's Triangle that I called 'binomial coefficients' are a lot more prevalent across math than I may have let on. Today, we're going to start an exploration into combinatorics to see what these numbers can do. A lot of this will pull back to the multiplication principle from before.

Example 0.1. 1. How many ways are there to get 5 heads in 12 flips of a coin?

2. How many ways are there to write a 12 letter word consisting of only H and T so that there are 5 Hs?

3. How many ways are there to determine and answer key to a 12 question true-false test with exactly 5 true answers?

4. How many ways are there to choose a committee of 5 people from 12 eligible members?

5. How many ways are there to pick 5 numbers out of 12 in a lottery drawing?

It turns out that these are all the same number! That's not a coincidence; they are all counting the same thing in different contexts. These last two give an indication as to why we read $C(n, k)$ as n choose k, as in we are picking k things out of a set of n. In terms of the coin flips, you are basically choosing which of the flips are going to be heads or tails.

Example 0.2.

1. How many ways are there to draw a 5 card hand out of a 52 card deck?

2. How many ways are there to give 6 identical prizes to different people in a group of 40?

3. How many ways are there to draw 20 numbers from a lottery pool of 80?

Let's consider that last one and look to develop a formula for these $C(n,k)$ that does not depend on using Pascal's Triangle. Let's reduce to 12 balls, choosing 5 of them, for simplicity.

Example 0.3.

1. How many ways are there to select the first ball? What about the second?

2. How many ways are there to pick 5 balls out of the group of 12 where the order of selection matters?

3. Now say that I want to forget about the order. How many 'ordered' selections are equivalent to the same 'unordered' selection?

4. How many ways are there to select the 5 lottery balls from a group of 12?

Using the notation that

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

we can write this as

$$C(12,5) = \qquad\qquad \text{or} \qquad\qquad C(n,k) =$$

Computationally, we can see that this is the same as the actual formula for $C(n, k)$ that is commonly used. Let's provide a full motivation for this formula.

Example 0.4.

1. Put all 12 balls in an order:

2. Draw a line between the chosen and unchosen ones:

3. Get rid of the ordering:

Now, let's look at one other application of thinking like what we have been doing so far today:

Example 0.5. 1. How many ways are there to order the letters in the word MUSIC?

2. What about HAPPY?

3. What about LIBRARY?

4. What about MISSISSIPPI?

These binomial coefficients can help count so many more things in a variety of different contexts. Here's another worked out example to illustrate this.

Example 0.6. Assume that you have a group of 15 people, 7 men and 8 women, from which to make a committee of 5 people.

1. How many ways are there to make the committee?

2. How many ways are there to make the committee if it only consists of women?

3. How many ways are there to make a committee that contains 2 men and 3 women?

4. How many ways are there to make a committee of 2 men and 3 women that does not contain Mr. Jones?

5. How many ways are there to make a committee of 2 men and 3 women if Mr. and Mrs. Jones will not serve on the committee together?

6. How many ways are there to make a committee containing either 2 men and 3 women, or 3 men and 2 women?

7. What is the probability of ending up with a committee of 3 men and 2 women if the committee is chosen at random?

In case we don't get far enough in class, here are the answers to the previous example.

1. This is the same as the problems we have considered so far in class. You are picking a committee of 5 people from 15, so the answer is C(15,5).

2. For this problem, instead of having the entire group of 15 to choose from, we need to choose only from the women, which gives us 8 options. Therefore, the number of ways to do this is C(8,5).

3. In this case, we need to pick 2 of the men to serve on the committee and 3 of the women to serve. This goes back to our multiplication principle from earlier: The number of ways to pick the full committee should be the number of ways to pick the men *times* the number of ways to pick the women. Since there are 7 men and we are picking 2 of them, there are C(7,2) ways to pick the men, and similarly there are C(8,3) ways to pick the women. Thus, the total number of ways to choose the committee under these conditions is
$$C(7,2) \cdot C(8,3)$$

4. If Mr. Jones will not be on the committee, we just have one less man taking part in the selection process. Therefore, we will be choosing 2 men from 6, and 3 women from 8. This gives a total number of ways of doing this as
$$C(6,2) \cdot C(8,3)$$

5. This problem can be solved in two ways. First we could add up the following three options: Mr. Jones is on the committee but Mrs. Jones is not, Mrs. Jones is on the committee but Mr. Jones is not, and neither of them is on the committee. If we want to force someone to be on the committee, we basically pick them first, and then choose the rest of the committee from the remaining people. For instance, if we want to force Mr. Jones to be on the committee, then the number of ways to pick the men on the committee is
$$C(1,1) \cdot C(6,1)$$
where we have first chosen Mr. Jones to be on the committee, and then picked one of the remaining 6 men to fill the other seat. In this case, we want Mrs. Jones to not be on the committee, so she needs to sit out, leaving C(7,3) ways to fill the women's seats on the committee. If we work this out for the other two cases, we get the total number of ways here is
$$C(1,1) \cdot C(6,1) \cdot C(7,3) + C(6,2) \cdot C(1,1) \cdot C(7,2) + C(6,2) \cdot C(7,3).$$
The first term is Mr. Jones on the committee, the second is Mrs. Jones on the committee, and the last is neither. We could also compute this solution by looking at the total number of ways to organize the committee, and subtracting off all committees where both Mr. and Mrs. Jones are on it. The total number of ways was part 2 above, and the number of ways with Mr. and Mrs. Jones both on it means that we need to pick each of them for their respective sexes, and then fill the rest of the committee. This can be done in
$$C(1,1) \cdot C(6,1) \cdot C(1,1) \cdot C(7,2) = C(6,1) \cdot C(7,2)$$
ways. Therefore, the total number of ways to not have both Mr. and Mrs. Jones on the committee is
$$C(7,2) \cdot C(8,3) - C(6,1) \cdot C(7,2)$$

6. For this part, we need to add the number of ways of having a committee with 2 men and 3 women and the number of ways to have a committee with 3 men and 2 women. This is given by
$$C(7,2) \cdot C(8,3) + C(7,3) \cdot C(8,2)$$

7. For this, we just need to take the number of ways to get this committee and divide it by the total number of ways the committee can be formed. This is
$$\frac{C(7,3) \cdot C(8,2)}{C(15,5)}$$

Worksheet 12 - The Normal Distribution and Z-Scores

November 27, 2018

Name: ____

1 Continuous Distributions

Last time we talked about binomial distributions and saw that we could represent them by graphs like this:

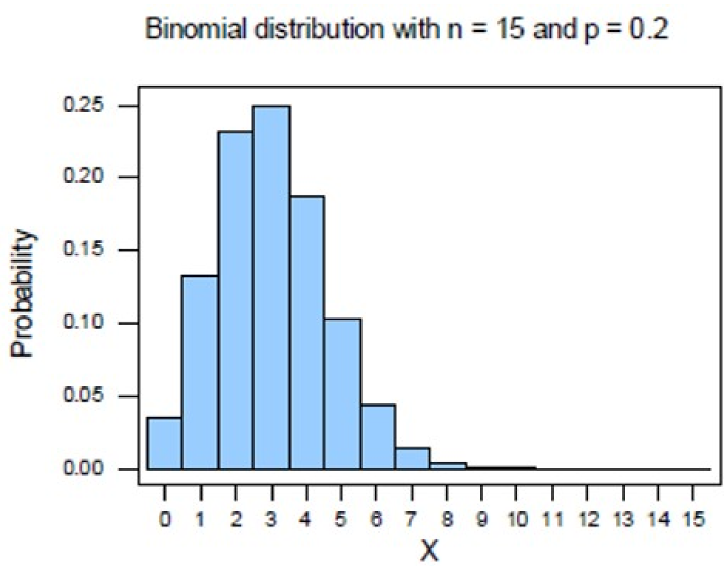


Figure 1: From: https://onlinecourses.science.psu.edu/stat414/node/70/

For this picture, we have

1. Heights of the bars:

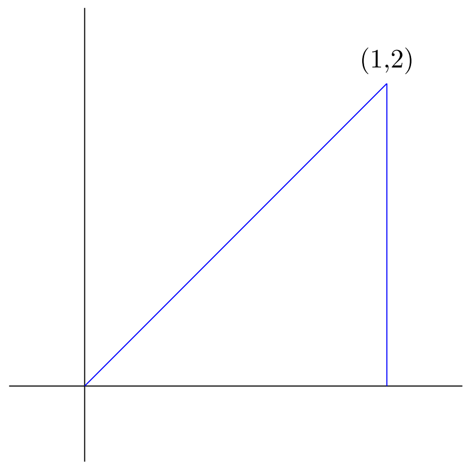
2. Finding the probability of some event:

3. Total area under the bars:

With this sort of picture, there's no reason why we need to have this be a histogram; we could do the same thing for any curve. The important thing we need is that the curve is always above the x-axis, and the total area under the curve is 1. We then find probabilities of events by computing the area under the curve on the appropriate range.

Example 1.1.

Example 1.2. Consider the distribution defined by the function $y = 2x$ on $0 \leq x \leq 1$.



1. Verify that the total area is 1.

2. What is the probability that an outcome here is less than 1/2?

3. What is the probability that an outcome here is greater than 2/3?

4. What is the probability that an outcome is between 1/3 and 3/4?

2 Normal Distribution

Now, we want to do the same as before with a different function:

$$y = \frac{1}{2\pi} e^{-\frac{x^2}{2}}$$

whose graph looks like

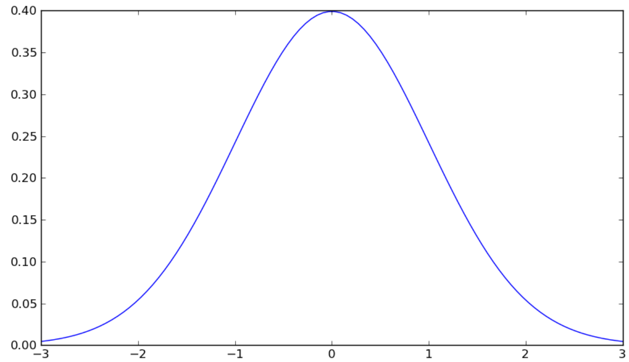


Figure 2: From https://stackoverflow.com/questions/10138085/python-pylab-plot-normal-distribution

Properties of this distribution:

With this, we can draw a diagram similar to the Empirical Rule for the normal distribution

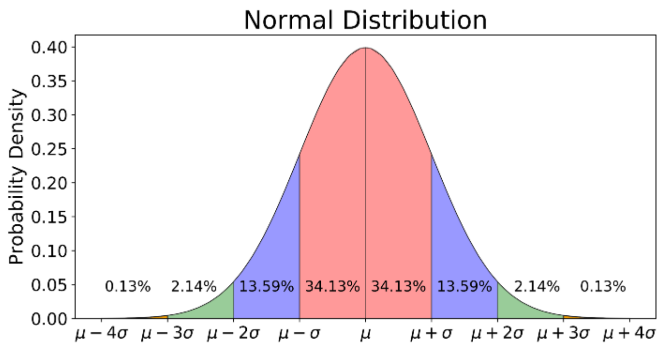


Figure 3: From https://towardsdatascience.com/understanding-the-68-95-99-7-rule-for-a-normal-distribution-b7b7cbf760c2

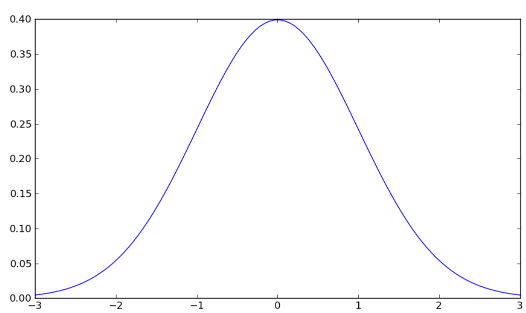
We can also think about a normal distribution with mean μ and standard deviation σ, which is defined by the graph of

$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

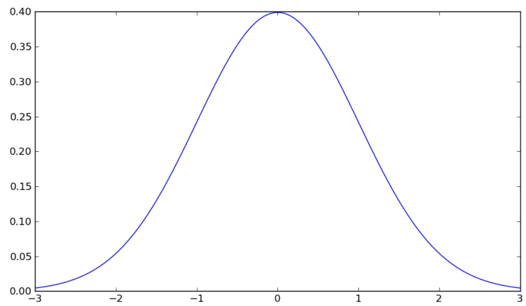
The empirical rule type numbers still apply, but now for a number of standard deviations away from the mean instead of the numbers from before.

Example 2.1. Assume that the mean height of a person is 70 inches with standard deviation 4 inches (this is probably not true), and that this is normally distributed (this is probably true).

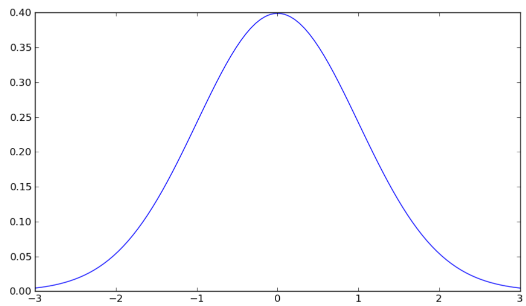
1. What proportion of people are between 66 and 74 inches tall?



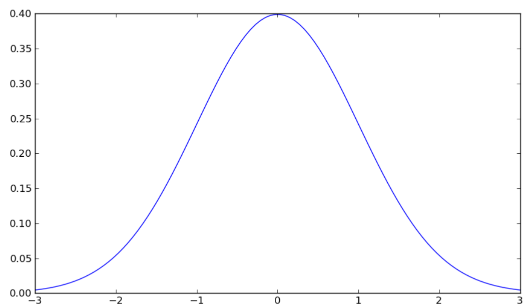
2. What proportion of people are between 62 and 70 inches tall?



3. If I pick a random person, what is the probability that they are between 66 and 78 inches tall?



4. If I pick a random person, what is the probability that they are more than 82 inches tall?



Now, to discuss the vague statement we made last time, about distributions being 'bell-shaped'. What we really mean is:

Example 2.2. Assume that you are going to look at a basketball player shooting free throws over the course of a season. Assume that this player shoots 300 free throws, and the probability of any one free throw being made is $\frac{3}{4}$, independent of all other shots.

1. How many free throws do you expect this player to make?

2. What is the standard deviation of the number of free throws made?

3. Write an expression for the exact probability that this player makes between 218 and 232, inclusive, free throws this season?

4. We know that this data is approximately normally distributed. Find the probability of this same quantity using the normal distribution.

3 Z-Scores

Why do we care about the normal distribution?

Why is the normal distribution nice?

Definition 3.1. Assume that we get a result of x from data that is normally distributed with mean μ and standard deviation σ. The **Z-score** associated to this value x is:

Example 3.1. Assume that scores are normally distributed with mean 70 and standard deviation 5. Find the Z score of the results 87, 68, 75.

Example 3.2. Under the same conditions, what is the probability that a student scores more than 75?

What about more than 78?

Z-tables:

Example 3.3. Assume that heights are normally distributed with mean 60 and standard deviation 7. What is the probability that someone is over 65 inches tall? What about that they are between 56 and 68 inches tall?

Example 3.4. Assume that a basketball player shoots 500 free throws over the course of a season, and the probability of them making each shot is 0.8, independent of each other attempt.

1. Find the mean and standard deviation of this distribution.
2. Use the normal approximation to compute the probability of the shooter making more than 420 free throws in the season.
3. Use the normal approximation to find the probability of the shooter making between 395 and 413 free throws over the course of the season.

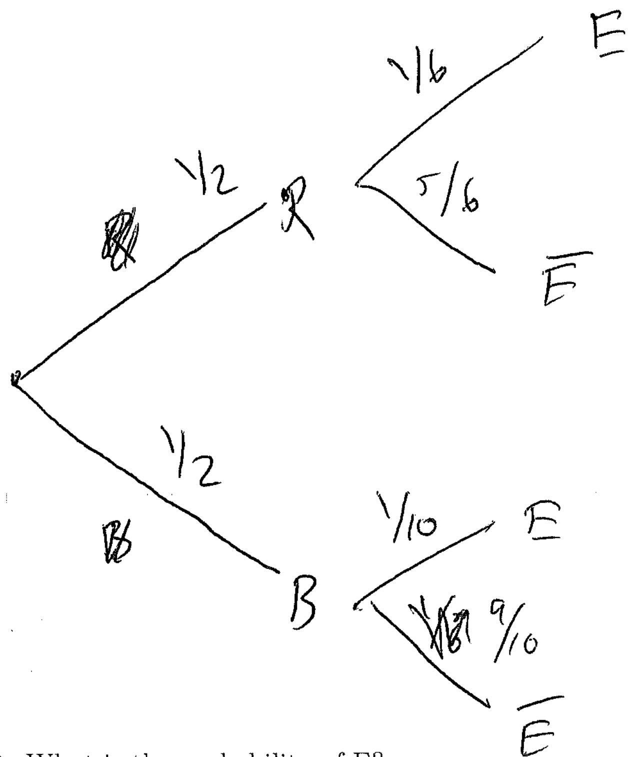
MATH 104 - Practice 11

October 25, 2018

Name: Key

Consider a two part experiment. In the first step, you draw a card out of a standard 52 card deck. If the card is red, you roll a fair 6-sided die, and if the card is black, you roll a fair 10-sided die. In terms of the die roll, we care about the event $E =$ "the die roll is a 1."

1. Draw a tree diagram for this experiment, writing all probabilities on the branches.



2. What is the probability of E?

$$\frac{1}{2}\cdot\frac{1}{6} + \frac{1}{2}\cdot\frac{1}{10} = \frac{1}{12} + \frac{1}{20}$$

$$= \frac{5}{60} + \frac{3}{60} = \frac{8}{60}$$

$$= \underline{\frac{2}{15}}$$

MATH 104 - Quiz 10

October 25, 2018

Name: Key

You are playing a game where you are going to roll a fair 6-sided die 10 times, and will win based on the number of 6's you see.

1. What is the expected number of times you will see a 6 in the 10 rolls?

$$10 \cdot \frac{1}{6} = \frac{10}{6} = 5/3$$

2. Suppose the game has the following payouts:

Number of 6's	0-4	5	6-9	10
Winnings	\$0	-\$2	\$1	\$15

Write an expression for your expected winnings from this game.

$$(-2)\left(\binom{10}{5}(1/6)^5(5/6)^5\right) + 1\left(\binom{10}{6}(1/6)^6(5/6)^4 + \binom{10}{7}(1/6)^7(5/6)^3\right.$$

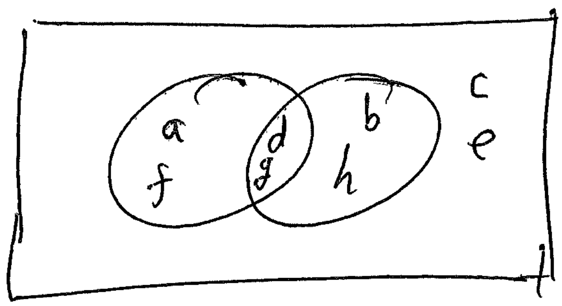
$$\left. + \binom{10}{8}(1/6)^8(5/6)^2 + \binom{10}{9}(1/6)^9(5/6)^1\right) + 15 \cdot \binom{10}{10}(1/6)^{10}(5/6)^0$$

MATH 104 - Quiz 7

October 2, 2018

Name: Key

1. Let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, d, f, g\}$ and $B = \{b, d, g, h\}$. Find each of the following sets. Hint: Constructing a Venn Diagram may be helpful.



(a) $A \cap B$

$\{d, g\}$

(b) $\bar{B}$

$\{a, c, e, f\}$

(c) $\bar{A} \cup B$

$\{b, c, d, e, g, h\}$

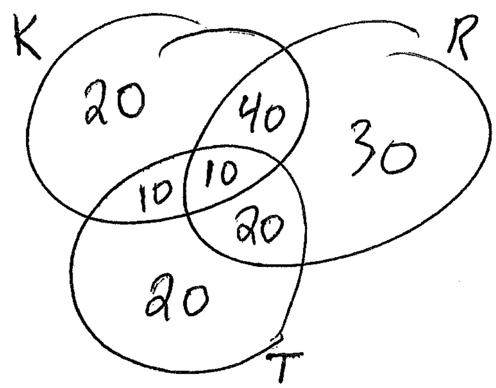
(d) $\overline{A \cup B}$

$\{c, e\}$

2. You survey 200 students as to what types of candy they like. You find that:

- 80 like Kit-Kat bars
- 100 like Reese's
- 60 like Twix
- 50 like both Kit-Kat and Reese's
- 30 like both Reese's and Twix
- 20 like both Kit-Kat and Twix
- 10 like all three.

(a) Construct a Venn Diagram for this situation.



(b) How many of the students surveyed liked none of the three candy bars?

$20+40+30+10+10+20+20 = 150$

$200 - 150 = \boxed{50}$

(c) How many students liked exactly 2 of the options?

70

(d) How many students liked only Kit-Kat bars out of the three options?

20

3. Consider developing a 6 character username that is only made up of lowercase letters.

(a) How many such usernames are possible?

26^6

(b) How many are possible that do not repeat any letters?

$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21$

(c) How many are possible so that adjacent letters are not repeated? (that is, you can not have two consecutive letters that are the same)

$26 \cdot 25 \cdot 25 \cdot 25 \cdot 25 \cdot 25$

4. Let U be the set of all students at Rutgers, A the set of all sophomores, and B the set of engineering majors. Using proper set builder terminology, describe the sets $A \cup \bar{B}$ and $A \cap B$.

$A \cup \bar{B}$ is the set of all students who are either sophomores or not engineering majors.

$A \cap B$ is the set of all students who are both sophomores and engineering majors (sophomore engineering majors)

5. (a) Assume that the odds of rolling a 4 on an unfair die are 4:1 against. What is the probability of rolling a 4?

$$\frac{1}{4+1} = \frac{1}{5}$$

(b) If the probability of rolling a 3 on this die is $\frac{2}{7}$, what are the odds for rolling a 3?

$$\frac{P(3)}{P(\bar{3})} = \frac{2/7}{5/7} = \frac{2}{5}$$

So the odds for rolling a 3 are $\boxed{2:5.}$

MATH 104 - Homework Set 8

Assigned: September 27, 2018

Due: October 2, 2018, 11 PM

Assigned Problems

1. Section 7.6, Problem 24. (10 points)

   The idea of this problem is to figure out how many ski sets to be stored will give the highest expected value. We need to compute the expected value for each number individually. The point is, if we do not have enough sets for the customers that show up, we lose out on the profit. That is, if we only store 92 sets of skis and 94 customers show up, we can only rent 92 skis and lose out on the potential extra profit for the last two customers. For example, if we store 93 skis, we compute the expected value as follows:

   - If 90 customers show up, our profit is $90 \times \$20 = \$1800$, and this occurs with probability 0.01.
   - If 91 customers show up, our profit is $91 \times \$20 = \$1820$, and this occurs with probability 0.10.
   - If 92 customers show up, our profit is $92 \times \$20 = \$1840$, and this occurs with probability 0.20.
   - If 93 or more customers show up, our profit is $93 \times \$20 = \$1860$, and this occurs with probability 0.69, because we can only rent up to 93 sets of skis, because that's all we have in stock.

   In all of these cases, our expenses are $93 \times 6 = \$558$. Thus, our expected value is

   $$E_{93} = \$1800 \cdot 0.01 + \$1820 \cdot 0.10 + \$1840 \cdot 0.20 + \$1860 \cdot 0.69 - \$558 = \$1293.40$$

   Computing this for all possible numbers of skis, we find that

Number of Skis	90	91	92	93	94	95	96	97
Expected Value	\$1260	\$1273.80	\$1285.60	\$1293.40	\$1297.20	\$1295	\$1290.80	\$1285.60

   Therefore, the optimal number of skis to store is 94, because that has the largest expected value.

   Grader Comments: 4 points for setting up the expected value calculations correctly. 4 points for calculating all of the expected values in the table. 2 points for the final answer.

2. Section 7.6, Problem 27. (6 points)

(a) If the probability of success at each location is $\frac{1}{2}$, then the expected value for the first location is

$$\frac{1}{2}(\$15000) + \frac{1}{2}(-\$3000) = \$6000$$

and the expected value for the second location is

$$\frac{1}{2}(\$20000) + \frac{1}{2}(-\$6000) = \$7000$$

so the company should choose the second location.

(b) If the probability of success at the first location is $\frac{2}{3}$ and it is $\frac{1}{3}$ at the second, the expected value of the first location is

$$\frac{2}{3}(\$15000) + \frac{1}{3}(-\$3000) = \$9000$$

and the expected value for the second location is

$$\frac{1}{3}(\$20000) + \frac{2}{3}(-\$6000) = \$2666.67$$

and so the company should choose the first location.

Grader Comments: Each expected value computation gets 1 point. The final answer for each (a) and (b) gets another point.

3. There are three different games you could play involving rolling two fair 6-sided dice. The games are as follows:

A. Pay \$6, and win the amount of money equal to the sum of the number of the dice.

B. Pay \$8, and win twice the number on the larger of the two dice.

C. Pay \$2, and win an amount of money equal to the smaller of the values of the two dice.

For both B and C, if the die rolls are the same, you win the amount of money corresponding to that number. That is if you roll two fours, you would win 8 dollars in B and 4 dollars in C. Which game should you play? Hint: Start by writing out the sample space of this experiment in a table. (12 points)

The sample space for this experiment is

	1	**2**	**3**	**4**	**5**	**6**
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Using this table, we can build tables for each of the three games, computing the amount we win for each roll. For game A,

$	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

for game B

$	**1**	**2**	**3**	**4**	**5**	**6**
1	2	4	6	8	10	12
2	4	4	6	8	10	12
3	6	6	6	8	10	12
4	8	8	8	8	10	12
5	10	10	10	10	10	12
6	12	12	12	12	12	12

and for game C

$	**1**	**2**	**3**	**4**	**5**	**6**
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

Note that these tables do not account for the amount of money you need to pay for the game. The remainder of this calculation can be done in two ways: You can either compute the expected value of the game without worrying about how much you need to pay and subtract the cost off at the end, or you can subtract the cost from every entry in the table and then compute the expected value. Either way will get you to the same final answer. To find the expected value here, since every outcome in the table above has probability $\frac{1}{36}$, we can just add up all of the entries in the table and divide by 36 (like finding the mean of an ideal data set). This will give the expected value for game A as

$$\frac{2(1)+3(2)+4(3)+5(4)+6(5)+7(6)+8(5)+9(4)+10(3)+11(2)+12(1)}{36} - 6 = \$1$$

while for game B, we get

$$\frac{2(1)+4(3)+6(5)+8(7)+10(9)+12(11)}{36}-8=\frac{322}{36}-8=8.94-8=\$0.94$$

and for game C

$$\frac{1(11)+2(9)+3(7)+4(5)+5(3)+6(1)}{36}-2=\frac{91}{36}-2=2.53-2=\$0.53$$

So, while all of these games are positive for the player, game A has the highest expected value, and is the one you should choose to play.

Grader Comments: 2 points for the sample space (this doesn't have to be written explicitly), 3 points for getting the expected value of each game, 1 point for the final conclusion.

D.6 Fall 2014 - Math 135

This section contains the following documents from my teaching assistant assignment for Math 135:

1. My recitation syllabus handed out during the first week of class
2. A set of practice problems that were given out at the first recitation to make sure students knew the prerequisite material
3. A detailed written up solution to one of the quizzes given in recitation
4. A review sheet handed out to the students to help them prepare for the final exam

Math 135: Recitation Syllabus

Teaching Assistant: Matt Charnley
Email: charnley@math.rutgers.edu
Office: Hill 606, Busch Campus
Office Hours:

Class Times:	Section 70: Tuesdays, 3:35-4:30, LSH-A121
	Section 71: Tuesdays, 5:15-6:10, LSH-A121
	Section 72: Tuesdays, 6:55-7:50, LSH-B110

Goal: The goal of these recitation sessions is to review and answer questions about the material discussed in lecture. They will deal with both general topic-related questions and specific questions from the homework. The recitation will generally go as follows:

1. Topic overview of what was covered in the past week of lecture.
2. Help/Solutions to questions on the homework/topics.
3. Quiz on the lecture topics from the previous week.

Quizzes: Quizzes will be given at every recitation session. Any student who is late to the recitation session will not be allowed to take the quiz that day. Quizzes will be graded and returned to you at the next recitation session. Solutions to the quizzes will be posted on Sakai after lecture on the same day the quiz is given out, therefore, quizzes are not allowed to be made up, for any reason. Out of all of the quiz grades earned throughout the semester, only the top 10 will be counted towards the final grade, allowing for quizzes to be missed in emergency circumstances without affecting the final grade.

Course Details: For more information about the course as a whole, please refer to the official syllabus given out in lecture.

Other Help: Calculus is a difficult class. You will need to put a lot of work into understanding the material to succeed in this class. It is important never to fall behind because the material builds on itself very rapidly; if you are behind one day, you will be completely lost on the next. Prof. Lieberman and I are both here to help you understand and learn the material. There are also several resources available from the university; please check out the Sakai site and the official syllabus for more information.

Recitation 1: Extra Practice

September 2, 2014

1. Factor these quadratic expressions.

a) $x^2 - 25$

b) $x^2 + 6x + 9$

c) $x^2 - 8x + 15$

d) $3x^2 + 11x - 4$

e) $x^2 + x - 21$

f) $2x^2 + 4x - 1$

2. Simplify the following rational and radical expressions.

a) $\sqrt{450}$

b) $\frac{225}{600}$

c) $\sqrt{28(x-4)^2(x-5)}$

d) $\frac{x^2-4x+4}{x^2+x-6}$

e) $\sqrt{72(x^2-4)(x^2+5x+6)}$

f) $\frac{x^2-9}{x^2+6x+9}$

3. Graph the following piecewise-defined functions.

a) $f(x) = \begin{cases} -1 & x < -1 \\ 2x - 1 & -1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$

b) $f(x) = \begin{cases} -x - 2 & x < -2 \\ x^2 & -2 \leq x < 1 \\ x - 4 & x \geq 1 \end{cases}$

c) $f(x) = \begin{cases} 2x & x < -1 \\ -x & -1 \leq x < 3 \\ (x-2)^2 & x \geq 3 \end{cases}$

d) $f(x) = \begin{cases} 1 & x < -1 \\ x + 1 & -1 \leq x < 2 \\ 2 - (x-2)^2 & x \geq 2 \end{cases}$

e) $f(x) = \begin{cases} x^2 & x < 0 \\ 2x & 0 \leq x < 2 \\ 4 & x \geq 2 \end{cases}$

f) $f(x) = \begin{cases} -(x+1)^2 & x < -1 \\ x^2 + 2 & -1 \leq x < 2 \\ -x^2 + 5 & x \geq 2 \end{cases}$

4. Identify the following functions as even, odd, or neither.

a) $f(x) = x^4 + 2$

b) $f(x) = x^2 + 2x$

c) $f(x) = x^5 + 4x^3 - x$

d) $f(x) = \sin(x)$

e) $f(x) = \cos(x)$

f) $f(x) = |x|$

5. Identify the center and radius of the given circles.

a) $(x-2)^2 + (y+5)^2 = 16$

b) $x^2 + y^2 + 4y + 4 = 25$

c) $x^4 - 6x + y^2 + 2x - 39 = 0$

d) $x^2 + 4x + y^2 = 0$

6. For each pair, P, Q, give the distance between P and Q, as well as the midpoint of the segment $\overline{PQ}$.

a) $P = (0, 3)$, $Q = (1, -4)$.

b) $P = (2, 5)$, $Q = (10, 13)$.

c) $P = (1, 4)$, $Q = (10, 4)$.

d) $P = (0, 0)$, $Q = (3, 4)$.

Hint: All critical and inflection points are Integers.

MATH 135: Quiz 9

November 4, 2014

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Name: Solutions   Sec: ____________

Fill in the following table and use it to sketch the graph of the function $f(x)$ below.

$$f(x) = \frac{(x+2)(x-2)^2}{x} \qquad f'(x) = \frac{2(x-2)\overbrace{(x^2+x+2)}^{A}}{x^2} \qquad f''(x) = \frac{2(x+2)\overbrace{(x^2-2x+4)}^{B}}{x^3}$$

For each row in the table, list the interval(s) or point(s) where f has the given property. If none exist, write "none". The axes for the sketch are on the back of this page.

Roots ($f(x)=0$)	$x=2,\ x=-2$	(1)
Increasing	$(2,\infty)$	(1)
Decreasing	$(-\infty,0)\cup(0,2)$	
Concave Up	$(-\infty,-2)\cup(0,\infty)$	(1)
Concave Down	$(-2,0)$	
Critical Points	$x=2$ (Minimum)	(1)
Inflection Points	$x=-2$	
Horizontal Asmyptotes	None	(1)
Vertical Asymptotes	$x=0$	

A, B do not factor, so by the hint they are never 0.

By plugging in 0, we see that $A>0$, $B>0$ for all x

→ You can also plug in points to check this in each case (1)

f: $\frac{(-)(-)^2}{(-)} = +$ | -2 | $\frac{(+)(-)^2}{(-)} = -$ | 0 | $+$ | 2 | $+$

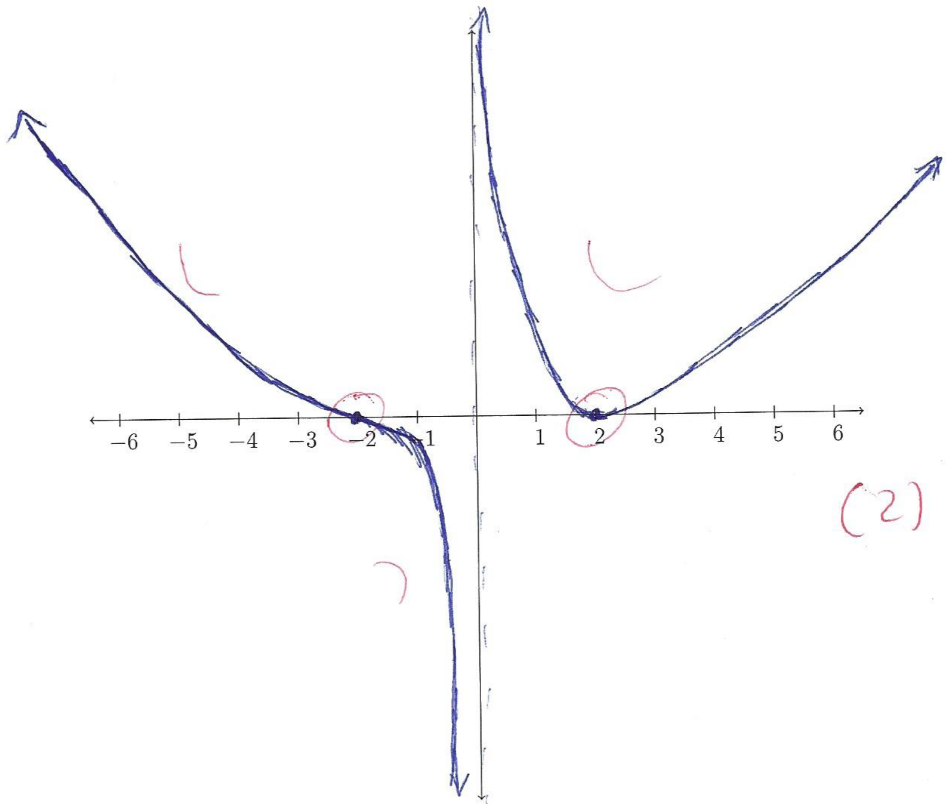
or $\lim_{x\to 0^-} \frac{(x+2)(x-2)^2}{x} = -\infty$

$\lim_{x\to 0^+} \frac{(x+2)(x-2)^2}{x} = +\infty$.

f': $-$ | 0 | $-$ | 2 | $+$ (1)

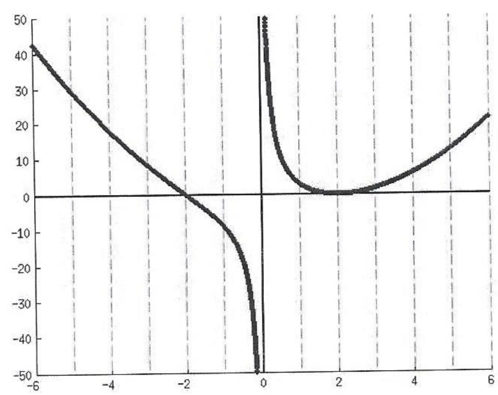
f'': $+$ | -2 | $-$ | 0 | $+$ (1)

1



Asymptote

Actual Graph:



Quiz 9 Detailed Write-up

$f(x) = \dfrac{(x+2)(x-2)^2}{x}$ $\quad f'(x) = \dfrac{2(x-2)(x^2+x+2)}{x^2}$ $\quad f''(x) = \dfrac{2(x+2)(x^2-2x+4)}{x^3}$

Hints : 1. Roots = x - intercepts

2. All critical points are integers

3. $x^2+x+2>0 \qquad x^2-2x+4>0$ all x

and they don't factor.

- Roots / x - intercepts : Set $f(x)=0$ and solve.

$$0 = \frac{(x+2)(x-2)^2}{x}$$

$$0 = (x+2)(x-2)^2 \quad \leftarrow$$ Numerator must be zero.

$$\boxed{x = -2, \quad x = +2}$$

So these are the roots.

- Vertical Asymptotes - Look where f is undefined.
  - The only place f is undefined is at $x=0$.
  - As $x \to 0$, the numerator goes to $(0-2)(0+2)^2 = 8$ and the denominator goes to zero.
  - Therefore, this is a vertical asymptote, and the only one since f is defined everywhere else.

$$\boxed{\text{Vertical Asymptote: } x = 0}$$

- <u>Horizontal Asymptotes</u> – $\lim_{x\to\infty} f(x)$

Method 1) The numerator of f goes like x^3, while the denominator is just x.

Therefore, there is no asymptote.

Method 2) Calculate the limit.

$$\lim_{x\to\infty} \frac{(x+2)(x-2)^2}{x} = \lim_{x\to\infty} \frac{x^3-2x^2-4x+8}{x} \cdot \frac{(1/x^3)}{(1/x^3)}$$

$$= \lim_{x\to 0} \frac{1-2/x-4/x^2+8/x^3}{1/x^2}$$

Taking the limit of each term $\text{“}=\text{”} \dfrac{1-0-0+0}{0} = \infty$ or undef.

Since this limit is undefined / ∞, there is no horizontal Asymptote.

$$\boxed{\text{Horizontal Asymptote: None}}$$

- Critical Points

  - 1^{st} order: $f'(x) = 0$ and solve

$$0 = \frac{2(x-2)(x^2+x+2)}{x^2}$$

$$0 = 2(x-2)(x^2+x+2)$$ Numerator must be zero

$x - 2 = 0$ or $x^2+x+2 = 0$

$x = 2$ NO SOLUTIONS

By the hints given in class

$$\begin{cases} x^2+x+2 > 0 \quad \text{all } x \text{ ; or} \\ \text{All critical points are integers + } x^2+x+2 \text{ does not factor} \end{cases}$$

We get that x^2+x+2 is Never 0.

→ Quadratic Formula can also check that.

Therefore, the only 1^{st} order critical point is at $x=2$ (and the asymptote at $x=0$)

$$\boxed{\text{Critical Points} \quad x = 2}$$

- 2nd Order : $f''(x) = 0$

$$0 = \frac{2(x+2)(x^2-2x+4)}{x^3}$$

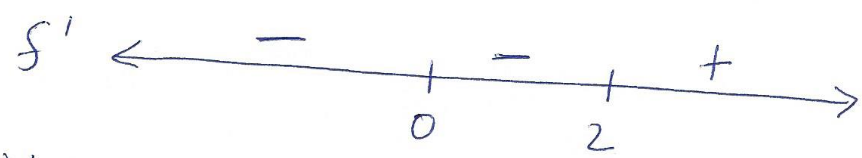
$$0 = 2(x+2)(x^2-2x+4)$$

$x = -2$

NO SOLUTIONS by the Same argument for 1st order.

$\boxed{\text{2nd Order (possible inflection point) } x = -2}$

- Increasing/Decreasing : Check f' in intervals
  - We need to mark the asymptote and critical points.



- You can either do this by positive/negative arguments, or plugging in points.

$$f'(-1) = \frac{2(-1-2)((-1)^2-1+2)}{(-1)^2} = \frac{2(-3)(1-1+2)}{1} = -12$$

$$f'(1) = \frac{2(1-2)(1^2+1+2)}{1^2} = -8$$

$$f'(3) = \frac{2(3-2)(3^2+3+2)}{3^2} = \frac{2 \cdot 1 \cdot 14}{9} = 28/9$$

and: $x = 2$ is a minimum

$\boxed{\text{Increasing: } (2, \infty) \qquad \text{Decreasing: } (-\infty, 0) \cup (0, 2)}$

- Concave Up / Down : Check f'' on intervals
  - Again mark the 2^{nd} order critical point(s) and asymptote.

f'': $+$ on $(-\infty,-2)$, $-$ on $(-2,0)$, $+$ on $(0,\infty)$; marks at -2 and 0.

$$f''(-3) = \frac{2(-3+2)((-3)^2-2(-3)+4)}{(-3)^3} = \frac{2\cdot(-1)\cdot(9+6+4)}{-27} = \frac{38}{27}$$

$$f''(-1) = \frac{2(-1+2)((-1)^2-2(-1)+4)}{(-1)^3} = \frac{2(1)(1+2+4)}{-1} = -14$$

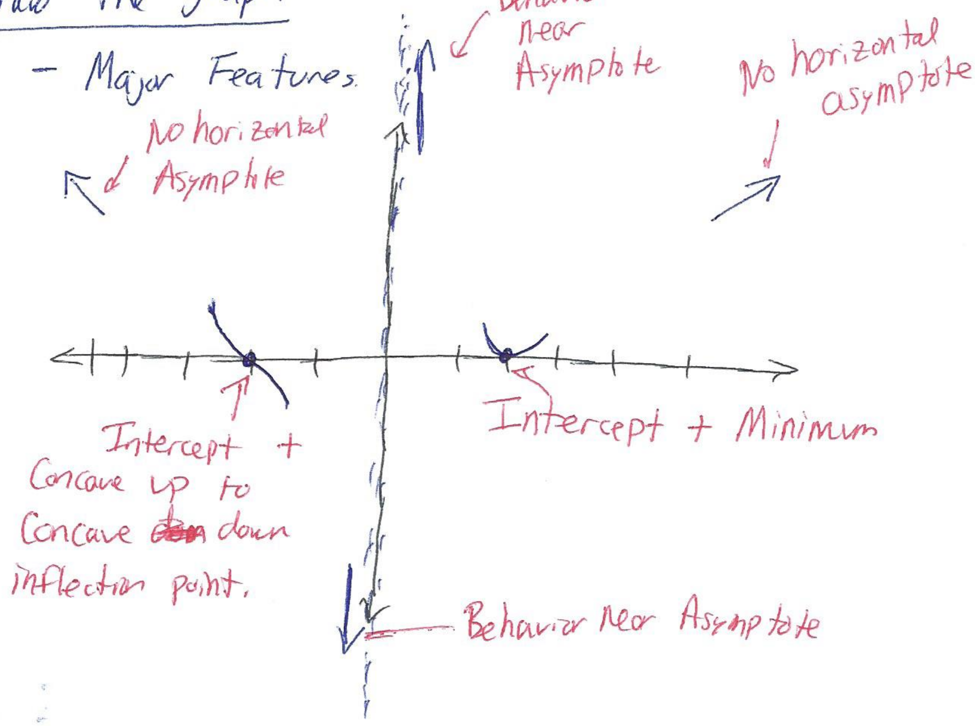
$$f''(1) = \frac{2(1+2)(1^2-2+4)}{1^3} = \frac{2\cdot3\cdot3}{1} = 18$$

$\boxed{\begin{array}{l}\text{Concave up : } (-\infty,-2)\cup(0,\infty)\\ \text{Concave down : } (-2,0)\end{array}}$

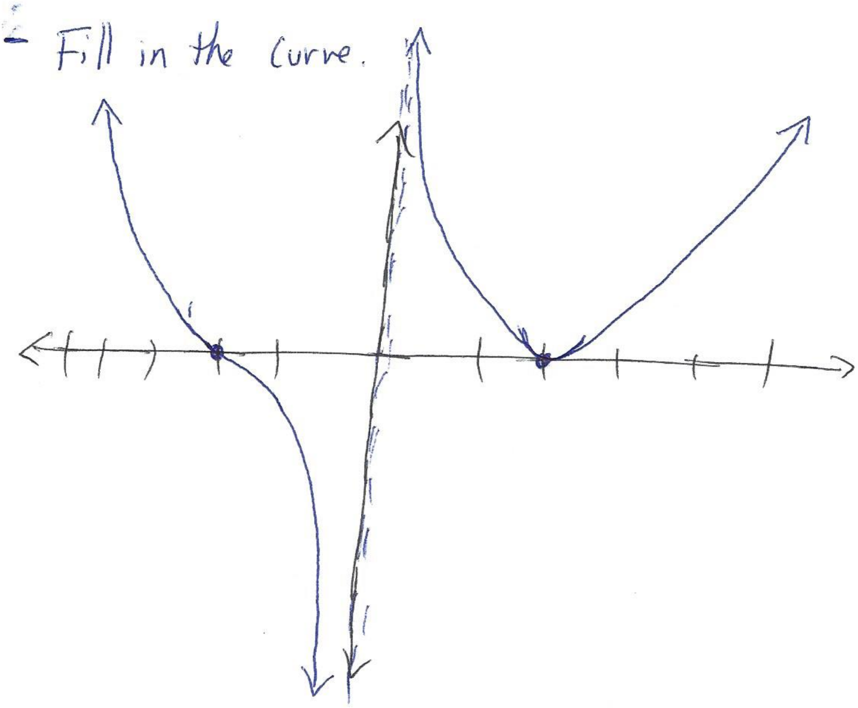
and $\boxed{x=-2 \text{ is an inflection point}}$

- Behavior near the asymptote
  - As $x\to0^-$, $f'(x)<0$, so the graph must go down to $-\infty$ on the left side
  - As $x\to0^+$, $f'(x)<0$, so the graph must come down from $+\infty$ on the right.

- Draw the graph
  - Major Features.



  - Fill in the curve.



MATH 135: Final Exam Concept Review

Matt Charnley

December 8, 2014

Chapter 1

(a) Section 1.2
   - (i) Absolute value equalities and inequalities.
   - (ii) Distance and midpoint. Circles.
   - (iii) Trigonometric equations.

(b) Section 1.3
   - (i) Slope and equation of lines.
   - (ii) Parallel and perpendicular. Intercepts.

(c) Section 1.4
   - (i) Function notation. Composition of functions.
   - (ii) Basic graphs. Intercepts and Symmetry.
   - (iii) Polynomial and rational functions.

Chapter 2

(a) Section 2.1
   - (i) Definition and notation of limit.
   - (ii) One-sided limits.
   - (iii) Reading limits from graph.
   - (iv) Infinite limits.

(b) Section 2.2
   - (i) Algebraic rules for limits.
   - (ii) Factoring and canceling.
   - (iii) Piecewise defined functions.
   - (iv) Special Trig limits.

$$\lim_{h\to 0}\frac{\sin h}{h} = 1 \qquad \lim_{h\to 0}\frac{\cos h - 1}{h} = 0$$

   - (v) Trig Limits.
   - (vi) Squeeze Theorem

(c) Section 2.3
   - (i) Definition of continuity
   - (ii) Continuity of polynomials and rational functions.

(iii) Intermediate Value Theorem.

(d) Section 2.4

(i) Exponential Function
(ii) Logarithm function.
(iii) Solving logarithmic equations.
(iv) Compounding interest / Biological growth.

Chapter 3

(a) Section 3.1

(i) Lines tangent to graph.
(ii) Definition of derivative.
(iii) Graphs of f and f'.
(iv) Existence of derivatives.

(b) Section 3.2

(i) Algebraic rules for derivatives.
(ii) Product and Quotient rules.
(iii) Higher order derivatives.

(c) Section 3.3

(i) Trig derivatives.
(ii) Derivatives of exponential and logs.

(d) Section 3.4

(i) Average, instantaneous, relative rates of change.
(ii) Rectilinear Motion
(iii) Falling Body Problems

(e) Section 3.5

(i) Chain Rule
(ii) "Extended Derivative" Rules

(f) Section 3.6

(i) Implicit Differentiation
(ii) Logarithmic differentiation
(iii) Finding tangent lines this way.

(g) Section 3.7

(i) Related Rates
(ii) All the different types of problems.

(h) Section 3.8

(i) Tangent Line Approximations
(ii) Differentials
(iii) Error propagation

Chapter 4

(a) Section 4.1

   (i) Absolute and relative maxima and minima.
   (ii) Critical points/numbers
   (iii) Max/min or optimization on a closed interval.

(b) Section 4.2

   (i) Mean value theorem.
   (ii) How to use it.

(c) Section 4.3

   (i) Graph Sketching - Increasing/Decreasing
   (ii) Concave up/down.
   (iii) First and second derivative test.
   (iv) Inflection points.
   (v) Using this to draw graphs.

(d) Section 4.4

   (i) Limits at infinity - Horizontal Asmyptotes
   (ii) Infinite limits - Vertical Asmyptotes
   (iii) Vertical tangents.

(e) Section 4.5

   (i) L'Hôpital's Rule
   (ii) When you can and can't apply it.
   (iii) Manipulating to get things in the right form.
   (iv) Limits involving e and ln.

(f) Section 4.6

   (i) Optimization for physical problems.
   (ii) Draw a picture.
   (iii) Derive and equation.
   (iv) Maximize or minimize.

(g) Section 4.7

   (i) More optimization. Same things, different terms.
   (ii) Economics (revenue, profit, costs, marginal...)
   (iii) Physiology.

Chapter 5

(a) Section 5.1

   (i) Antiderivatives
   (ii) Indefinite integrals.
   (iii) Rules for antiderivatives.
   (iv) Applications.

(b) Section 5.2

   (i) Area as the limit of a sum. Add up little rectangles.
   (ii) Sigma notation.

(c) Section 5.3

   (i) Riemann Sums
   (ii) Definite integral as the limit of Riemann Sums.
   (iii) Integral at a point and changing order of limits.
   (iv) Area and distance as integrals.

(d) Section 5.4

   (i) FTC - version 1. Integral is the difference of anti-derivative.
   (ii) FTC - version 2. Derivative of the integral gives the function back.

(e) Section 5.5

   (i) u substitution.
   (ii) Make the inside u. Don't forget to find du.
   (iii) Change limits of integration for a definite integral.

D.7 Spring 2015 - Math 251

This section contains the following documents from my teaching assistant assignment for Math 251 in Spring 2015:

1. My recitation syllabus handed out during the first week of class
2. Solutions to a sample quiz given during recitation
3. An example document on Clairaut's theorem that was distributed to the class to illustrate that the conditions of this theorem are necessary

Math 251: Recitation Syllabus

Teaching Assistant: Matt Charnley
Email: charnley@math.rutgers.edu
Website: http://math.rutgers.edu/~mpc163/Courses/SP15_MATH251.html
Office: Hill 606, Busch Campus
Office Hours:
- Mondays, 12:00 noon – 1:30 pm
- Wednesdays, 3:15 pm – 4:45 pm
- By appointment

Class Times: Section 22: Thursdays, 4:30-5:50, SC-205
Section 23: Thursdays, 6:10-7:30, SC-120
Section 24: Thursdays, 7:40-9:00, SC-120

Goal: The goal of these recitation sessions is to give extra help with solving problems from the textbook. An important part of Calculus 3 is learning how to attack the large variety of problems and feel comfortable with the concepts involved. With recitation on Thursdays, and homework due on Mondays, the discussion in recitation will cover the material from Monday and Wednesday's lectures of the same week. I would recommend looking over the material from Wednesday's lecture before recitation so that you can have questions answered about those topics. Recitation will consist of a mixture of topic reviews and problem solving at the board, depending on the topics and interest of the class.

Quizzes: Quizzes will be given approximately every other week, but this may vary slightly depending on the test schedule and other factors. Quizzes will be about the material discussed in that day's recitation. Make-up quizzes will not be offered. As of now, two quizzes will be dropped over the course of the semester, so missing a week or two will not affect your final grade for the class. Quiz solutions will be posted both on Sakai and on my website.

Maple: Over the course of the semester, there will be 5 Maple Labs that will need to be turned in, in addition to Lab 0, which you have right now. They will be assigned over the course of the semester, and you will have two weeks to complete each one once they are assigned. Outside of the first recitation in the computer lab, all of the work on the Maple Labs will take place outside of class. I will be happy to answer questions about them via email or in office hours, but will not dedicate recitation time to it.

Course Details: Information for the course will be primarily posted on Sakai. There will also be information on my website above, but it will all be duplicated from Sakai. For more information about the course, please see the syllabus or other information posted on Sakai

MATH 251: Quiz 8

April 30, 2015

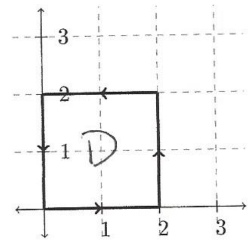
Name: Solutions Sec: ____________

1. Use Green's Theorem to calculate the integral $\oint_C \vec{F} \cdot d\vec{s}$ for the vector field

$$\vec{F} = \langle 2xy + x^4, 3xy^2 - \sin(y) \rangle$$

/5

and the curve



By Green's Thm. $\oint_C \vec{F} \cdot d\vec{s} = \iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dA$ (2)

$\frac{\partial F_2}{\partial x} = 3y^2 \qquad \frac{\partial F_1}{\partial y} = 2x$ (1)

So

$$\oint_C \vec{F} \cdot d\vec{s} = \int_0^2 \int_0^2 3y^2 - 2x \, dx\, dy$$

(1)

$$= \int_0^2 3xy^2 - x^2 \Big|_0^2 dy$$

$$= \int_0^2 6y^2 - 4 \, dy = 2y^3 - 4y \Big|_0^2$$

$$= 16 - 8 = \boxed{8}$$ (1)

2. Use Stokes' Theorem to evaluate the integral

$$\iint_{\mathcal{S}} \operatorname{curl}(\vec{F}) \cdot d\vec{S}$$

for the surface $\mathcal{S}$ with outward normal vector and vector field $\vec{F}$ below, where the boundary of $\mathcal{S}$ is the ellipse $4x^2 + y^2 = 16$ in the xy-plane. This boundary can be parametrized as $c(t) = \langle 2\cos(t), 4\sin(t), 0\rangle$. $0 \le t \le 2\pi$

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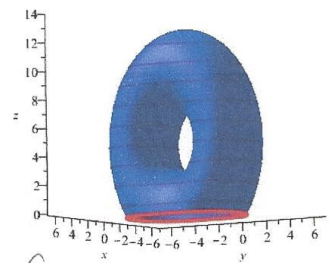
$$\vec{F} = \langle 3x + 4zx^2, x + y + z, x^2 + y^2 + z^2\rangle$$



By Stokes' thm: $\displaystyle\iint_S \operatorname{curl}(\vec{F})\cdot d\vec{S} = \oint_{\partial S} \vec{F}\cdot d\vec{s}$ (1)

$c(t) = \langle 2\cos(t), 4\sin(t), 0\rangle$

$c'(t) = \langle -2\sin(t), 4\cos(t), 0\rangle$ (1)

$F(c(t)) = \langle 6\cos t + 0, 2\cos t + 4\sin t + 0, 4\cos^2 t + 16\sin^2 t + 0\rangle$

$F(c(t))\cdot c'(t) = -12\cos(t)\sin t + 8\cos^2 t + 16\sin t\cos t + 0$

$= 4\cos t\sin t + 8\cos^2 t$

$= 4\cos t\sin t + 4 + 4\cos 2t$ (1)

So $\displaystyle\iint_S \operatorname{curl}(\vec{F})\cdot d\vec{S} = \oint_{\partial S}\vec{F}\cdot d\vec{s} = \int_0^{2\pi} \vec{F}(c(t))\cdot c'(t)\,dt = \int_0^{2\pi} 4\cos t\sin t + 4 + 4\cos 2t\,dt$ (1)

$= 2\sin^2 t + 4t + 2\sin 2t \Big|_0^{2\pi} = 8\pi$ (1)

Clairaut's Theorem

Matt Charnley

February 20, 2015

In this document, we describe a function that does not satisfy the hypotheses of Clairaut's theorem, and therefore, the mixed second partial derivatives are not equal.

Let

$$f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}$$

with $f(0,0) = 0$. We compute the first partial derivatives

$$f_x(x,y) = \begin{cases} \frac{y(x^2-y^2)+2x^2y}{(x^2+y^2)} - \frac{2x^2y(x^2-y^2)}{(x^2+y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

and

$$f_y(x,y) = \begin{cases} \frac{x(x^2-y^2)+2xy^2}{(x^2+y^2)} - \frac{2xy^2(x^2-y^2)}{(x^2+y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

where we needed to look at the actual function f and take limits to get the value of these derivatives at $(0,0)$. To compute the second derivatives at $(0,0)$ we follow this same approach.

$$f_{xy}(0,0) = \lim_{h\to 0} \frac{f_x(0,h) - f_x(0,0)}{h} = \lim_{h\to 0} \frac{(-h+0)-0}{h} = -1$$

since, plugging $x = 0$ into the equation for f_x gives

$$f_x(0,y) = \frac{y(0-y^2)+0}{0+y^2} - 0 = -y.$$

Doing the same thing for f_{yx} we see that

$$f_{yx}(0,0) = \lim_{h\to 0} \frac{f_y(h,0) - f_y(0,0)}{h} = \lim_{h\to 0} \frac{(h+0)-0}{h} = 1.$$

Thus, we have that $f_{xy} \neq f_{yx}$ at the point $(0,0)$. This is because neither of these derivatives are continuous in a disk containing $(0,0)$.

D.8 Fall 2015 - Math 251

This section contains the following documents from my teaching assistant assignment for Math 251 in Fall 2015:

1. My recitation syllabus handed out the first week of class
2. My written-up solution to part of the midterm that was given during my recitation session
3. Images of the lecture notes for the first section of extra video lectures that I recorded to make up for missed material in class

Math 251: Recitation Syllabus

Teaching Assistant: Matt Charnley
Email: charnley@math.rutgers.edu
Website: http://math.rutgers.edu/~mpc163/Courses/FA15_MATH251.html
Office: Hill 606, Busch Campus

Office Hours:

- Tuesdays, 3:15 pm – 4:45 pm
- Wednesdays, 12:00 noon – 1:30 pm
- By appointment

Class Times: Section H1: Thursdays, 3:20-4:40, BE-251
Section H2: Thursdays, 5:00-6:20, BE-013
Section H3: Thursdays, 6:40-8:00, BE-013

Goal: The goal of these recitation sessions is to give extra help with solving problems from the textbook. An important part of Calculus 3 is learning how to attack the large variety of problems and feel comfortable with the concepts involved. The discussion in recitation will cover the material from Monday and Wednesday's lectures of the same week. I would recommend looking over the material from Wednesday's lecture before recitation so that you can have questions answered about those topics. Recitation will consist of a mixture of topic reviews and problem solving at the board, depending on the topics and interest of the class.

Quizzes: Quizzes will be given approximately every other week, but this may vary slightly depending on the test schedule and other factors. Quizzes will cover all material up until the Monday before recitation. Make-up quizzes will not be offered. Two quizzes will be dropped over the course of the semester, so missing a week or two will not affect your final grade for the class. Quiz solutions will be posted on my website.

Maple: Over the course of the semester, there will be 5 Maple Labs that will need to be turned in, in addition to Lab 0, which you have right now. They will be assigned over the course of the semester, and you will have two weeks to complete each one once they are assigned. Outside of the first recitation in the computer lab, all of the work on the Maple Labs will take place outside of class. I will be happy to answer questions about them via email or in office hours, but will not dedicate recitation time to it.

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MATH 251: In-Class Midterm

December 3, 2015

Name: Solutions   Sec: ____________

1. Let $\mathcal{D}$ be the region between the curves $y = 5 - x^2$ and $y = x^2 - 3$. Sketch the region and compute the integral of $f(x, y) = x^2$ over this region.

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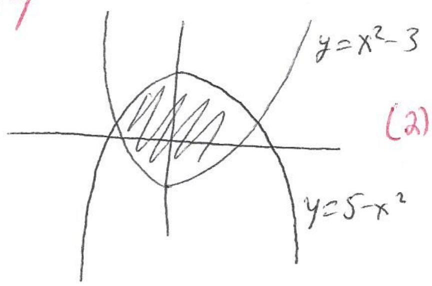


(2)

$$\int_{-2}^{2}\int_{x^2-3}^{5-x^2} x^2 \, dy \, dx$$

$$= \int_{-2}^{2} x^2 y \Big|_{x^2-3}^{5-x^2} dx$$

(2)

$$= \int_{-2}^{2} 5x^2 - x^4 - x^4 + 3x^2 \, dx$$

$$= \int_{-2}^{2} 8x^2 - 2x^4$$

$$= \frac{8}{3}x^3 - \frac{2}{5}x^5 \Big|_{-2}^{2}$$

(2)

$$= \frac{8}{3}(2)^3 - \frac{2}{5}(2)^5 + \frac{8}{3}(+2)^3 \mp \frac{2}{5}(+2)^5$$

(1)

$$= \frac{16 \cdot 8}{3} - \frac{4 \cdot 32}{5} = 128\left(\frac{1}{3} - \frac{1}{5}\right) = \boxed{\frac{256}{15}}$$

Intersections:

$5 - x^2 = x^2 - 3$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

2. Compute the volume of the region $\mathcal{R}$ sitting above the triangle bounded by $x = 0$, $y = 0$ and $y = 1 - x$ in the xy-plane, and between the planes, $x + y + z = 5$ and $2x + y + 3z = 6$.

/6

$z = 5 - x - y \qquad z = \frac{1}{3}(6 - 2x - y)$

$$\int_0^1 \int_0^{1-x} \int_{\frac{1}{3}(6-2x-y)}^{5-x-y} 1 \, dz \, dy \, dx \quad (2)$$

$$= \int_0^1 \int_0^{1-x} 5 - x - y - \frac{1}{3}(6 - 2x - y) \, dy \, dx$$

$$= \int_0^1 \int_0^{1-x} 3 - \frac{1}{3}x - \frac{2}{3}y \, dy \, dx \quad (2)$$

$$= \int_0^1 3(\cancel{1-x}\, y) - \frac{1}{3}xy - \frac{1}{3}y^2 \Big|_0^{1-x} dx$$

$$= \int_0^1 3(1-x) - \frac{1}{3}x + \frac{1}{3}x^2 - \frac{1}{3}(1-x)^2 \, dx \quad (1)$$

$$= -\frac{3}{2}(1-x)^2 - \frac{1}{6}x^2 + \frac{1}{9}x^3 + \frac{1}{9}(1-x)^3 \Big|_0^1$$

$$= \frac{1}{9} - \frac{1}{6} + \frac{3}{2} - \frac{1}{9} = 8/6 = \boxed{4/3} \quad (1)$$

$\rho = 4$

3. Find the integral of $f(x, y, z) = x + z$ over the region inside the hemisphere of radius 4 where $y \geq 0$, and above the plane $z = 2$.

7

$0 \leq \theta \leq \pi$

$z = 2 \rightarrow \rho \cos\varphi = 2$

$\rho = 2/\cos\varphi$

$4 = \rho = 2/\cos\varphi$

$\cos\varphi = 1/2$ (2)

$\varphi = \pi/3$

$$\rightarrow \int_0^{\pi/3}\int_0^{\pi}\int_{2/\cos\varphi}^{4} (\rho\cos\theta\sin\varphi + \rho\cos\varphi)\rho^2\sin\varphi \, d\rho\, d\theta\, d\varphi$$ (2)

$$= \int_0^{\pi/3}\int_0^{\pi}\int_{2/\cos\varphi}^{4} \rho^3\cos\theta\sin^2\varphi + \rho^3\cos\varphi\sin\varphi \, d\rho\, d\theta\, d\varphi$$

$$= \int_0^{\pi/3}\int_0^{\pi} \frac{\rho^4}{4}\left(\cos\theta\sin^2\varphi + \cos\varphi\sin\varphi\right)\Big|_{2/\cos\varphi}^{4} d\theta\, d\varphi$$

$$= \int_0^{\pi/3}\int_0^{\pi}\left(64 - \frac{4}{\cos^4\varphi}\right)\left(\cos\theta\sin^2\varphi + \cos\varphi\sin\varphi\right) d\theta\, d\varphi$$ (1)

$$= \int_0^{\pi/3}\left(64 - \frac{4}{\cos^4\varphi}\right)\left(\sin\theta\sin^2\varphi + \theta\cos\varphi\sin\varphi\right)\Big|_0^{\pi} d\varphi$$ (1)

$$= \pi\int_0^{\pi/3} 64\cos\varphi\sin\varphi - 4\frac{\sin\varphi}{\cos^3\varphi}\, d\varphi$$

$u = \cos\varphi$

$du = -\sin\varphi\, d\varphi$

$$= \pi\left[-32\cos^2\varphi - 2\cos^{-2}\varphi\right]_0^{\pi/3} = \pi\left[-\frac{32}{4} - 2\cdot 4 + 32 + 2\right]$$

$$= \boxed{18\pi}$$ (1)

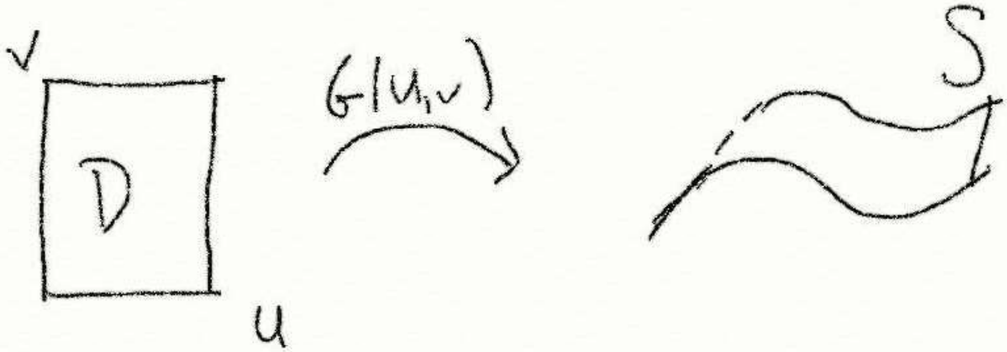
Section 16.4: Parameterized Surfaces and Surface Integrals

16.2: Parametrized curve

$$c(t) = \langle x(t), y(t), z(t) \rangle$$

<u>Parametrized Surface</u> $S = G(u,v)$

$$G(u,v) = \langle \underline{x(u,v)}, \underline{y(u,v)}, \underline{z(u,v)} \rangle$$



<u>Ex</u> (r, θ, z) fix $r = R$

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$$\begin{array}{l} \qquad\qquad 2\cos u \qquad 2\sin u \quad v \\ G(u,v) = \langle R\cos u,\ R\sin u,\ v \rangle \end{array}$$

$$D = [0, 2\pi] \times \mathbb{R} \quad [1,5]$$

$(\rho, \theta, \varphi) \qquad \rho = R$

$x = \rho\cos\theta\sin\varphi$

$y = \rho\sin\theta\sin\varphi$

$z = \rho\cos\varphi$

$$G(u,v) = \langle R\cos u\sin v,\ R\sin u\sin v,\ R\cos v \rangle$$

$$D = [0, 2\pi] \times [0, \pi]$$

<u>Cone</u> $\quad z^2 = x^2 + y^2 = r^2 \rightarrow z = \pm r$

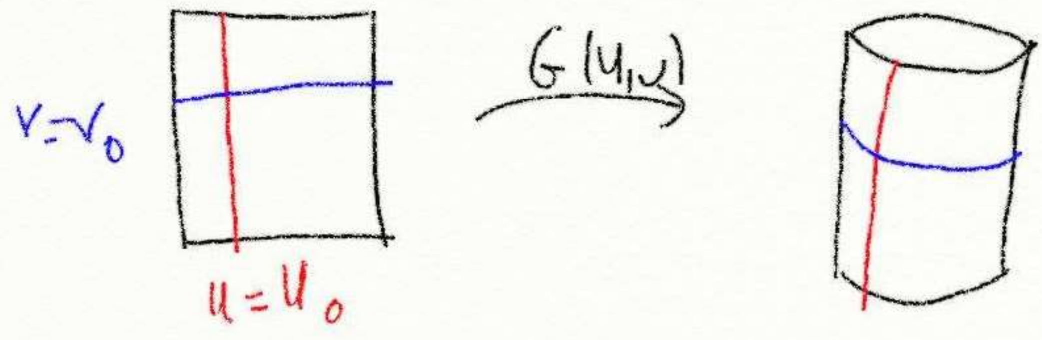
$x = r\cos\theta$

$y = r\sin\theta$

$$G(u,v) = \langle \underline{u\cos v},\ \underline{u\sin v},\ \underline{u} \rangle$$

Graph of $\quad z = f(x,y)$

$$G(u,v) = \langle u,\ v,\ f(u,v) \rangle$$



curve $G(u_0, v) = \langle x(u_0,v),\ y(u_0,v),\ z(u_0,v) \rangle$

Tangent Vector $T_v(u_0,v) = \frac{\partial G}{\partial v}(u_0,v)$

$$= \left\langle \frac{\partial x}{\partial v}(u_0,v),\ \frac{\partial y}{\partial v}(u_0,v),\ \frac{\partial z}{\partial v}(u_0,v) \right\rangle$$

$T_u(u,v) = \frac{\partial G}{\partial u}(u,v) = \left\langle \frac{\partial x}{\partial u}(u,v),\ \frac{\partial y}{\partial u}(u,v),\ \frac{\partial z}{\partial u}(u,v) \right\rangle$

$$\vec{n}(u,v) = \vec{T}_u(u,v) \times \vec{T}_v(u,v)$$

Tangent Plane: $\vec{n} \cdot \langle x - x_0,\ y - y_0,\ z - z_0 \rangle = 0$

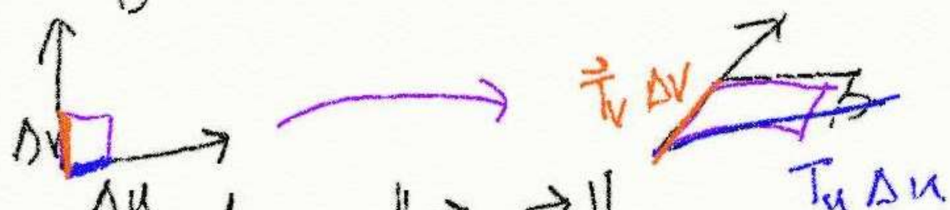
Ex $G(u,v) = \langle u,\ uv,\ v^2+1 \rangle$

$T_u = \langle 1,\ v,\ 0 \rangle$

$T_v = \langle 0, u, 2v \rangle$

$\vec{n} = \langle 2v^2, -2v, u \rangle = T_u \times T_v$

Surface Area



$\text{Area} = \| \vec{v}_1 \times \vec{v}_2 \|$

$\Delta u \longrightarrow \vec{T}_u \Delta u$

$\Delta v \longrightarrow T_v \Delta v$

$$\text{Area} = \| \vec{T}_u \Delta u \times \vec{T}_v \Delta v \|$$

$$= \| \vec{n} \| \Delta u \Delta v$$

Surface S, Domain D, $G(u, v)$

$$\boxed{\iint_S f\,dS = \iint_D f(G(u,v))\,\|\vec{n}(u,v)\|\,du\,dv}$$

$$\text{Area}(S) = \iint_D \|\vec{n}(u,v)\|\,du\,dv$$

$$G(u,v) = \langle x(u,v),\, y(u,v),\, 0\rangle$$

$$\vec{T}_u = \left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, 0\right\rangle$$

$$T_v = \left\langle \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, 0\right\rangle$$

$$\vec{n} = \left\langle 0, 0, \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}\right\rangle$$

$$\|\vec{n}\| = \left|\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}\right| = |\text{Jac}(G)|$$

Ex 1 Find the surface area of the upper hemisphere of radius R, and integrate z over this surface

$$G(u,v) = \langle R\cos u \sin v,\ R\sin u \sin v,\ R\cos v\rangle$$

$$D = [0, 2\pi] \times [0, \pi/2]$$

$$\vec{T}_u = \langle -R \sin u \sin v,\ R \cos u \sin v,\ 0 \rangle$$

$$T_v = \langle R \cos u \cos v,\ R \sin u \cos v,\ -R \sin v \rangle$$

$$\vec{n} = \langle -R^2 \cos u \sin^2 v,\ -R^2 \sin u \sin^2 v,\ -R^2 \sin^2 u \sin v \cos v$$
$$- R^2 \cos^2 u \sin v \cos$$

$$= -R^2 \langle \cos u \sin^2 v,\ \sin u \sin^2 v,\ \cos v \sin v \rangle$$

$$= -R^2 \sin v \langle \cos u \sin v,\ \sin u \sin v,\ \cos v \rangle$$

$$\|\vec{n}\| = R^2 \sin v$$

$$\text{Area} = \int_0^{2\pi} \int_0^{\pi/2} R^2 \sin v \, dv \, du$$

$$= 2\pi \cdot R^2 \cdot -\cos v \Big|_0^{\pi/2} = \underline{2\pi R^2}$$

$$\iint_S z \, dS = \int_0^{\pi} \int_0^{\pi/2} R \cos v \cdot R^2 \sin v \, dv \, du$$

$$= 2\pi R^3 \cdot \frac{\sin^2 v}{2}\Big|_0^{\pi/2}$$

$$= \boxed{\pi R^3}$$

Given the surface

$$G(u,v) = \langle u, u+v, v^2 \rangle \quad [0,1]\times[0,1]$$

$\iint_S y - x \, dS$. Find the tangent plane at $u=1, v=1$

$$T_u = \langle 1, 1, 0 \rangle$$

$$T_v = \langle 0, 1, 2v \rangle$$

$$\vec{n} = \langle 2v, -2v, 1 \rangle$$

$$\|\vec{n}\| = \sqrt{4v^2 + 4v^2 + 1} = \sqrt{8v^2+1}$$

$$\iint_S y - x \, dS = \int_0^1 \int_0^1 v + u - u \sqrt{8v^2+1} \, du \, dv$$

$$= \int_0^1 \int_0^1 v \sqrt{8v^2+1} \, du \, dv$$

$w = 8v^2+1$

$dw = 16v\,dv$

$$= \int_0^1 v\sqrt{8v^2+1}\,dv$$

$$= \frac{1}{16}\int_1^9 \sqrt{w}\,dw$$

$$= \frac{1}{16} w^{3/2} \cdot \frac{2}{3} \Big|_1^9$$

$$= \frac{1}{24}\left[27-1\right] = \frac{26}{24} = \boxed{\frac{13}{12}}$$

$\vec{n} = \langle 2v, -2v, 1\rangle$

at $u=1, v=1$ $\quad \vec{n} = \langle 2, -2, 1\rangle$

$G(1,1) = \langle 1, 2, 1\rangle$

$$2(x-1) - 2(y-2) + (z-1) = 0$$

D.9 Spring 2016 - Math 575

This section contains the following documents from my teaching assistant assignment for Math 575:

1. A sample homework solution done in MATLAB
2. A sample homework solution with more conceptual type problems
3. A sample piece of Python code from another homework set
4. A document outlining some properties of variational formulations, which I noticed the students were having trouble with

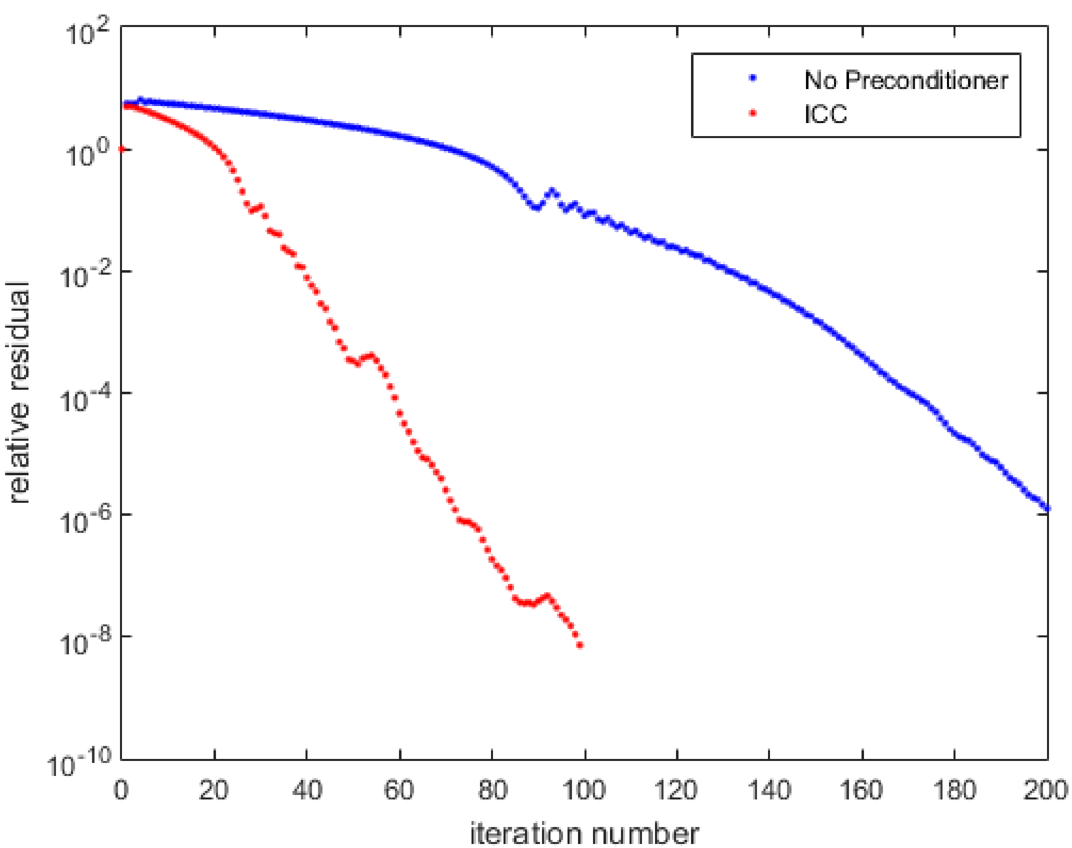
Math575HW2.m

Matt Charnley

Computes the solution to a given finite difference scheme. Builds the appropriate matrix and solves the linear system on a square.

```
f = @(x,y) 1;
g = @(x,y) 0;

min_coord = 0;
max_coord = 1;

N = 128;
x_val = linspace(min_coord, max_coord, N+1);
y_val = x_val;

h = (max_coord - min_coord)/N;

% The order for the coordinates will start from the bottom left, move
% over in x, then move up to the first y-coordinate, back on the far left
% in x, repeating until we get to the top right.

coord_x = zeros(1, (N+1)^2);
coord_y = zeros(1, (N+1)^2);

for i=1:(N+1)^2
   coord_x(i) = x_val(mod(i-1, N+1)+1);
   coord_y(i) = y_val(floor((i-1)/(N+1))+1);
end

A1 = zeros((N+1)^2, (N+1)^2);
b1 = zeros(1, (N+1)^2);

for i=1:(N+1)^2
   if (i <= N+1) || (i > N*(N+1)) || (mod(i, N+1) == 0) || (mod(i, N+1) == 1)
       A1(i,i) = 1;
       b1(i) = g(coord_x(i), coord_y(i));
   else
       A1(i,i) = 4 + h^2;
       A1(i,i-1) = -1;
       A1(i,i+1) = -1;
       A1(i, i+N+1) = -1;
       A1(i, i-N-1) = -1;
       b1(i) = f(coord_x(i), coord_y(i))*h^(2);
   end
end

% Try to cut out the unnecessary rows from the matrix. (i.e., the boundary
% terms
indSet = [];
for i = 1:(N+1)^2
    if A1(i,i) == 1
        indSet = [indSet, i];
```

```
        for j = 1:(N+1)^2
            if A1(j,i) == -1
                b1(j) = b1(j) + g(coord_x(i), coord_y(i));
            end
        end
    end
end

leftover = setdiff(1:(N+1)^2, indSet);

A_new = A1(leftover, leftover);
b_new = b1(leftover);

A2 = sparse(A_new);
[x0m, fl0m, rr0m, it0m, rv0m] = pcg(A2, b_new', 1e-8, 200);

Lm = ichol(A2);
[x1m, fl1m, rr1m, it1m, rv1m] = pcg(A2,b_new',1e-8, 200, Lm, Lm');

figure;
semilogy(0:it0m,rv0m/norm(b1),'b.');
hold on;
semilogy(0:it1m,rv1m/norm(b1),'r.');
legend('No Preconditioner','ICC');
xlabel('iteration number');
ylabel('relative residual');
hold off;
```



Published with MATLAB® R2015a

Math 575: HW 5

Matt Charnley

April 5, 2016

Problem 1. For this problem, we let ϕ_i be the basis functions for our given triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) and $\hat{\phi}_i$ be those for the reference triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$. Also, (x, y) will be coordinates in the original triangle, and $(\hat{x}, \hat{y})$ will be coordinates in the reference triangle. We will first establish some relationships between these two sets of functions and coordinates.

Define the matrix

$$A = \begin{pmatrix} x_2 - x_1 & x_3 - x_1 \\ y_2 - y_1 & y_3 - y_1 \end{pmatrix} \qquad A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{pmatrix}$$

Then, we know that

$$\begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \qquad \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = A^{-1} \begin{pmatrix} x - x_1 \\ y - y_i \end{pmatrix}$$

We can also see that (by shifting to the other coordinates) that

$$\begin{pmatrix} x - x_i \\ y - y_i \end{pmatrix} = A \begin{pmatrix} \hat{x} - \hat{x}_i \\ \hat{y} - \hat{y}_i \end{pmatrix}$$

or, by expanding this out, we see that

$$x - x_i = A_{11}(\hat{x} - \hat{x}_i) + A_{12}(\hat{y} - \hat{y}_i) \qquad y - y_i = A_{21}(\hat{x} - \hat{x}_i) + A_{22}(\hat{y} - \hat{y}_i)$$

Similarly, we have that

$$\nabla \phi_i = (A^{-1})^T \nabla \hat{\phi}_i$$

or

$$\partial_x \phi_i = (A^{-1})_{11} \partial_x \hat{\phi}_i + (A^{-1})_{21} \partial_y \hat{\phi}_i \qquad \partial_y \phi_i = (A^{-1})_{12} \partial_x \hat{\phi}_i + (A^{-1})_{22} \partial_y \hat{\phi}_i$$

or, using how A^{-1} is defined

$$\partial_x \phi_i = \frac{1}{\det(A)} [A_{22} \partial_x \hat{\phi}_i - A_{21} \partial_y \hat{\phi}_i] \qquad \partial_y \phi_i = \frac{1}{\det(A)} [-A_{12} \partial_x \hat{\phi}_i + A_{11} \partial_y \hat{\phi}_i]$$

The important thing to note is that all of these relations are linear. By carrying out the computations, it can be seen from these relations that if the set of formulas are satisfied for the reference triangle, then they are satisfied for the normal triangle. We can see that

$$
\begin{aligned}
\sum_{i=1}^{3} \phi_i(x,y) &= \sum_{i=1}^{3} \hat{\phi}_i(x,y) = 1 \\
\sum_{i=1}^{3} (x_i - x)\phi_i(x,y) &= \sum_{i=1}^{3} [A_{11}(\hat{x} - \hat{x}_i) + A_{12}(\hat{y} - \hat{y}_i)]\hat{\phi}_i(x,y) = A_{11}\sum_{i=1}^{3}(\hat{x} - \hat{x}_i)\hat{\phi}_i(x,y) + A_{12}\sum_{i=1}^{3}(\hat{y} - \hat{y}_i)\hat{\phi}_i(x,y) = 0 + 0 \\
\sum_{i=1}^{3} (y_i - y)\phi_i(x,y) &= \sum_{i=1}^{3} [A_{21}(\hat{x} - \hat{x}_i) + A_{22}(\hat{y} - \hat{y}_i)]\hat{\phi}_i(x,y) = A_{21}\sum_{i=1}^{3}(\hat{x} - \hat{x}_i)\hat{\phi}_i(x,y) + A_{22}\sum_{i=1}^{3}(\hat{y} - \hat{y}_i)\hat{\phi}_i(x,y) = 0 + 0 \\
\sum_{i=1}^{3} \partial_x \phi_i(x,y) &= \sum_{i=1}^{3} \frac{1}{\det(A)}[A_{22}\partial_x\hat{\phi}_i - A_{21}\partial_y\hat{\phi}_i] = \frac{A_{22}}{\det(A)}\sum_{i=1}^{3}\partial_x\hat{\phi}_i - \frac{A_{21}}{\det(A)}\sum_{i=1}^{3}\partial_y\hat{\phi}_i = 0 - 0 = 0 \\
\sum_{i=1}^{3} \partial_y \phi_i(x,y) &= \sum_{i=1}^{3} \frac{1}{\det(A)}[-A_{12}\partial_x\hat{\phi}_i + A_{11}\partial_y\hat{\phi}_i] = -\frac{A_{12}}{\det(A)}\sum_{i=1}^{3}\partial_x\hat{\phi}_i - \frac{A_{11}}{\det(A)}\sum_{i=1}^{3}\partial_y\hat{\phi}_i = -0 + 0 = 0 \\
\sum_{i=1}^{3} (x_i - x)\partial_x\phi_i(x,y) &= \sum_{i=1}^{3} [A_{11}(\hat{x} - \hat{x}_i) + A_{12}(\hat{y} - \hat{y}_i)]\frac{1}{\det(A)}[A_{22}\partial_x\hat{\phi}_i - A_{21}\partial_y\hat{\phi}_i] \\
&= \frac{1}{\det(A)}\left[\sum_{i=1}^{3} A_{11}A_{22}(\hat{x} - \hat{x}_i)\partial_x\hat{\phi}_i + \cdots\right] = \frac{1}{\det(A)}\left[A_{11}A_{22} - A_{21}A_{12}\right] = 1 \\
\sum_{i=1}^{3} (y_i - y)\partial_x\phi_i(x,y) &= \frac{1}{\det(A)}\sum_{i=1}^{3} [A_{21}(\hat{x} - \hat{x}_i) + A_{22}(\hat{y} - \hat{y}_i)][A_{22}\partial_x\hat{\phi}_i - A_{21}\partial_y\hat{\phi}_i] \\
&= \frac{1}{\det(A)}\left[A_{21}A_{22}\sum_{i=1}^{3}(\hat{x} - \hat{x}_i)\partial_x\hat{\phi}_i + \cdots\right] = \frac{1}{\det(A)}\left[A_{21}A_{22} - A_{21}A_{22}\right] = 0 \\
\sum_{i=1}^{3} (x_i - x)\partial_y\phi_i(x,y) &= \frac{1}{\det(A)}\sum_{i=1}^{3} [A_{11}(\hat{x} - \hat{x}_i) + A_{12}(\hat{y} - \hat{y}_i)][-A_{12}\partial_x\hat{\phi}_i + A_{11}\partial_y\hat{\phi}_i] \\
&= \frac{1}{\det(A)}\left[-A_{11}A_{12}\sum_{i=1}^{3}(\hat{x} - \hat{x}_i)\partial_x\hat{\phi}_i + \cdots\right] = \frac{1}{\det(A)}\left[-A_{11}A_{12} + A_{11}A_{12}\right] = 0 \\
\sum_{i=1}^{3} (y_i - y)\partial_y\phi_i(x,y) &= \frac{1}{\det(A)}\sum_{i=1}^{3} [A_{21}(\hat{x} - \hat{x}_i) + A_{22}(\hat{y} - \hat{y}_i)][-A_{12}\partial_x\hat{\phi}_i + A_{11}\partial_y\hat{\phi}_i] \\
&= \frac{1}{\det(A)}\left[-A_{21}A_{12}\sum_{i=1}^{3}(\hat{x} - \hat{x}_i)\partial_x\hat{\phi}_i + \cdots\right] = \frac{1}{\det(A)}\left[-A_{21}A_{12} + A_{11}A_{22}\right] = 1
\end{aligned}
$$

Thus, we just need to show that all of the formulas hold for the reference triangle.

$$
\begin{aligned}
\sum_{i=1}^{3} \hat{\phi}_i(x,y) &= 1 - x - y + x + y = 1 \\
\sum_{i=1}^{3} (x_i - x)\hat{\phi}_i(x,y) &= -x(1-x-y) + (1-x)x + (-x)(y) = -x + x^2 + xy + x - x^2 - xy = 0 \\
\sum_{i=1}^{3} (y_i - y)\hat{\phi}_i(x,y) &= -y(1-x-y) + (-y)x + (1-y)(y) = -y + xy + y^2 + -xy + y - y^2 = 0 \\
\sum_{i=1}^{3} \partial_x \hat{\phi}_i(x,y) &= -1 + 1 + 0 = 0 \\
\sum_{i=1}^{3} \partial_y \hat{\phi}_i(x,y) &= -1 + 0 + 1 = 0 \\
\sum_{i=1}^{3} (x_i - x)\partial_x \hat{\phi}_i(x,y) &= -x(-1) + (1-x)1 + (-x)0 = x + 1 - x = 1 \\
\sum_{i=1}^{3} (y_i - y)\partial_x \hat{\phi}_i(x,y) &= -y(-1) + (-y)1 + (1-y)(0) = y - y = 0 \\
\sum_{i=1}^{3} (x_i - x)\partial_y \hat{\phi}_i(x,y) &= -x(-1) + (1-x)(0) + (-x)(1) = x - x = 0 \\
\sum_{i=1}^{3} (y_i - y)\partial_y \hat{\phi}_i(x,y) &= -y(-1) + (-y)(0) + (1-y)(1) = y + 1 - y = 1
\end{aligned}
$$

So we're good and everything works.

```
from dolfin import *
from math import *
import numpy as np
from operator import truediv

# Define source term
class Source(Expression):
        def eval(self, values, x):
                values[0] = 0.0

# Define exact solution
class Exact(Expression):
        def eval(self, values, x):
                r = hypot(x[0],x[1])
                theta = atan2(x[1],x[0])
                if theta < 0:
                        theta = theta + 2*pi
                values[0] = (r ** (2./3))*sin(2./3*theta)

# Dirichlet boundary condition
class DirichletBoundary(SubDomain):
        def inside(self, x, on_boundary):
                return on_boundary

marking = input('Marking Strategy(Uniform(1), Local(2), Dorfler(3)) ')
f = Source()
uexact = Exact()

max_num_element = 5000
num_element = 0

# degree of the elements
deg = 1

tol = 0.15
theta = 0.8
errors = []

# initial mesh
mesh = Mesh('l-shape-mesh.xml')

while num_element < max_num_element:
        num_element = mesh.num_cells()

        # Define function space
        Vh = FunctionSpace(mesh, 'Lagrange', deg)
        bc = DirichletBC(Vh, Exact(), DirichletBoundary())

        # Define variational problem
        v = TestFunction(Vh)
        u = TrialFunction(Vh)
        a = inner(grad(u),grad(v))*dx
        F = f*v*dx

        # Compute solution
        uh = Function(Vh)
        solve(a == F, uh, bc)

        # Compare with exact solution interpolated into high degree space
        Vex = FunctionSpace(mesh, 'Lagrange', deg+2)
        ui = interpolate(uexact, Vex)
        err = ui - interpolate(uh, Vex)

        # Compute error indicator
        h = CellSize(mesh)
        n = FacetNormal(mesh)
        DG0 = FunctionSpace(mesh, 'DG', 0)
```

```
        w = TestFunction(DG0)
        eta2 = assemble((h ** 2) * w * ((div(grad(uh)) + f) ** 2) * dx \
        + avg(w) * avg(h) * (jump(grad(uh),n) ** 2) / 2. * dS)
        eta2 = eta2.array()

        max_eta2 = max(eta2)
        sum_eta2 = sum(eta2)

        # Compute norms of the error
        L2err = sqrt(assemble(err*err*dx))
        H1err = sqrt(assemble(inner(grad(err),grad(err))*dx))
        EI = sqrt(sum(eta2))
        errors.append([num_element, L2err, H1err, EI])

        if sum_eta2 < tol ** 2:
                break

        # Adaptive mesh refinement
        marked = np.zeros(eta2.size, dtype='bool')

        if marking == 1:
                marked = marked + 1
        elif marking == 2:
                marked = marked + (eta2 >= theta**2*max_eta2)
        elif marking == 3:
                while eta2.dot(marked) < (theta**2)*(sum_eta2):
                        # print " " + str(eta2.dot(marked))
                        # print eta2.dot(marked) - (theta**2)*(sum_eta2)
                        tempMax = max(eta2*(1-marked))
                        ind = np.where(eta2 == tempMax)
                        marked[ind] = 1
                # print " " + str(eta2.dot(marked))
                # print eta2.dot(marked) - (theta**2)*(sum_eta2)

        cells_marked = CellFunction('bool', mesh)
        cells_marked.array()[:] = marked

        mesh = refine(mesh, cells_marked);
        fig = plot(mesh);
        fig.write_pdf('MeshRefine' + str(marking) + 'NumElt' + str(num_element));

print "\n Num_element   L2 error        H1 error        EI              L2 rate H1 rate\n"
print "  {:7d}  {:4.2e} {:4.2e} ".format( \
        errors[0][0], errors[0][1], errors[0][2], errors[0][3])
for i in range(1,len(errors)):
        element_ratio = np.log(truediv(errors[i][0],errors[i-1][0]))/2
        print "  {:7d}  {:4.2e} {:4.2e} {:4.2e} {:5.2f}   {:5.2f}".format( \
        errors[i][0], errors[i][1], errors[i][2], errors[i][3],\
        np.log(errors[i-1][1]/errors[i][1])/element_ratio, \
        np.log(errors[i-1][2]/errors[i][2])/element_ratio)
```

Boundary Conditions from Variational Formulation

Matt Charnley

April 4, 2016

We begin with the variational problem: Find $u \in V$ so that for all $v \in V$,

$$\int_0^1 u'(x)v'(x)\ dx + \int_0^1 u(x)v(x)\ dx = \int_0^1 f(x)v(x)\ dx - 2v(0)$$

where

$$V = \{v : ||v||_{L^2} + ||v'||_{L^2} < \infty, \quad v(1) = 0\}.$$

After integrating by parts and using the conditions on the space V, we reduce to the equation

$$\int_0^1 (-u''(x) + u(x))v(x)\ dx - u'(0)v(0) = \int_0^1 f(x)v(x)\ dx - 2v(0)$$

and we want to extract the PDE and boundary conditions from this. We can start by choosing

$$v \in H_0^1 = V \cap \{v(0) = 0\} \subset V$$

which are functions v in V that vanish at both endpoints.

If we plug in such a function, we then see that the second term vanishes, giving that

$$\int_0^1 (-u''(x) + u(x))v(x)\ dx = \int_0^1 f(x)v(x)\ dx$$

which holds for every such $v \in H_0^1$. However, the H_0^1 space is enough to say that if such a relation holds for all $v \in H_0^1$, then we have that

$$-u''(x) + u(x) = f(x)$$

in the entire domain $(0, 1)$, which holds whether we integrate against a function in H_0^1, or a function in V. Our original saying that, for all $v \in V$, we have that

$$\int_0^1 (-u''(x) + u(x))v(x)\ dx - u'(0)v(0) = \int_0^1 f(x)v(x)\ dx - 2v(0)$$

reduces, since we know $-u'' + u = f$ to just say that

$$u'(0)v(0) = 2v(0)$$

Using $v(x) = 1 - x$ then gives that $u'(0) = 2$.

Finally, the other boundary condition comes from the definition of the space V, i.e., that $u(1) = 0$. Therefore we have the ODE

$$-u'' + u = f \qquad u(1) = 0 \qquad u'(0) = 2$$

D.10 Fall 2016 - Math 501 and 503

This section contains the following documents from my time running the Math 501 and 503 problem sessions:

1. A problem set from the middle of the semester, containing written qualifying exam problems

2. The last problem set of the semester, containing a topic review to help them prepare for the finals

Week 5 Qual Problems

Matt Charnley

November 1, 2016

1 Complex Analysis

1. Fall 2016: Complex #1. Use a contour integral to evaluate
$$\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2}.$$

2. Spring 2015: Complex #2. Use contour integration to evaluate the integral
$$\int_0^{2\pi} \frac{d\theta}{a+\sin\theta} \qquad a > 1$$

3. Fall 2014: Complex #1. Use contour integration to show that for all $a > 0$,
$$\int_{-\infty}^{\infty} \frac{\cos ax}{1+x^2}\,dx = \pi e^{-a}.$$

4. Spring 2014: Day 2 #3. Calculate the indefinite integral
$$\int_{-\infty}^{\infty} \frac{\cos(x)}{(x^2+1)(x^2+9)}\,dx$$
using residues and contour integration. Be sure to justify all limits that you take in the calculation.

5. Spring 2014: Day 1 #9. Suppose that $f(z)$ is a meromorphic function on the extended complex plane (including $z = \infty$) with only two poles: $z = -1$ is a pole of order 1 with principal part $\frac{1}{z+1}$, and $z = 2$ is a pole of order 3 with principal part $\frac{2}{z-2} + \frac{4}{(z-2)^3}$. Suppose further that $f(0) = 1$.
   (a) Determine $\int_{|z|=4} f(z)\,dz$.
   (b) Determine the number of solutions to $f(z) = 1$ in the extended complex plane.
   (c) Determine $f(z)$ explicitly.

6. Fall 2016: Complex #5. Suppose f and g are holomorphic in a region containing the closed unit disc $\bar{D}$. Suppose f has a simple zero at $z = 0$ and vanishes nowhere else in $\bar{D}$. Let
$$f_t(z) = f(z) + tg(z).$$
Show that, if $t > 0$ is sufficiently small, then
   (a) $f_t(z)$ has a unique zero in $\bar{D}$.
   (b) If z_t is that zero, then the mapping $t \mapsto z_t$ is continuous.

7. Spring 2014: Day 1 #3. Let $f(z)$ be an analytic function defined in some open neighborhood of the closed disc $|z| \le r$. Assume there exists an $a > 0$ such that $|f(0)| < a$ and $|f(z)| > a$ for all z with $|z| = r$. Prove that $f(z)$ must have a zero in the open disc $|z| < r$.

2 Real Analysis

1. Spring 2016: Real #2. Let $[a, b]$ be a bounded interval, and let m be Lebesgue measure. Let M be a positive real number, and $\{f_n\}$ a sequence of measurable functions on $[a, b]$ for which $\int_a^b |f_n| \, dm \leq M$ for all n. Assume that $f_n \to f$ for m almost every x.

   (a) State Fatou's Lemma.

   (b) Show that $\int_a^b |f| \, dm \leq M$.

   (c) Suppose that $||f_n - f||_1 \to 0$. Prove that for every $\epsilon > 0$, there exists $\delta > 0$ such that if $A \subset [a, b]$ is m-measurable with $m(A) \leq \delta$, then $\int_A |f_n| \leq \epsilon$ for all n.

2. Spring 2016: Real #5. Let m denote Lebesgue measure on $\mathbb{R}$ and m^2 Lebesgue measure on $\mathbb{R}^2$. Let $f \in L^1(\mathbb{R})$.

   (a) Show that $h(x, y) = f(x)f(x + y) \in L^1(\mathbb{R}^2, m^2)$.

   (b) Show that for almost every y, $x \mapsto f(x)f(x + y)$ defines a function in $L^1(\mathbb{R})$.

   (c) Give an example of a function $g \in L^1(\mathbb{R})$ for which $x \mapsto g(x)g(x + y)$ is not in $L^1(\mathbb{R})$ for at least one $y \in \mathbb{R}$.

3. Fall 2015: Real #1.

   (a) Let m denote Lebesgue measure on $\mathbb{R}$. Prove that the subset A of $L^1(m)$ defined by
   $$A := \{f \in L^1(m) : \int_{\mathbb{R}} |f| \, dm \leq 1\}$$
   is closed under pointwise convergence.

   (b) Prove that
   $$B := \{f \in L^1(m) : \int_{\mathbb{R}} |f| \, dm \geq 1\}$$
   is not closed under pointwise convergence.

4. Fall 2015: Real #2.

   (a) For α a real number and $\alpha > -1$, prove that $\int_0^\infty x^\alpha e^{-x} \, dm < \infty$ where dm is Lebesgue measure.

   (b) For $\alpha > -1$ and k a positive integer, prove that
   $$\lim_{k\to\infty} \int_0^k x^\alpha \left(1 - \frac{x}{k}\right)^k \, dm = \int_0^\infty x^\alpha e^{-x} \, dm.$$

5. Spring 2015: Real #1. In this problem $(X, \mathcal{M}, \mu)$ is an arbitrary measure space.

   (a) State the monotone convergence theorem.

   (b) Prove that if $f_n : X \to [0, \infty)$ is measurable and non-negative for each positive integer n, then
   $$\int_X \liminf_{n\to\infty} f_n \, d\mu \leq \liminf_{n\to\infty} \int_X f_n \, d\mu$$

   (c) Give an example showing that the above can be false if the functions f_n can take negative values.

6. Spring 2015: Real #5. Let $(X, \mathcal{M}, \mu)$ be a measure space. Suppose that $f_n,\ g_n,\ h_n \in L^1(X, \mathcal{M}, \mu)$, $n \geq 1$ satisfying the inequalities

$$f_n(x) \leq g_n(x) \leq h_n(x) \qquad \forall\, n \in \mathbb{N},\ x \in X.$$

Let $f(x)$, $g(x)$, $h(x)$ be functions so that

$$\lim_{n\to\infty} f_n(x) = f(x) \qquad \lim_{n\to\infty} g_n(x) = g(x) \qquad \lim_{n\to\infty} h_n(x) = h(x)$$

for almost all $x \in X$. Furthermore, assume that $f(x),\ h(x) \in L^1(X, \mathcal{M}, \mu)$ with

$$\lim_{n\to\infty} \int_X f_n(x)\ d\mu = \int_X f(x)\ d\mu \qquad \text{and} \qquad \lim_{n\to\infty} \int_X h_n(x)\ d\mu = \int_X h(x)\ d\mu.$$

Show that

$$\lim_{n\to\infty} \int_X g_n(x)\ d\mu = \int_X g(x)\ d\mu.$$

Hint: Look at $h_n - g_n$ and $g_n - f_n$.

7. Spring 2014: Day 2 #1. Let m denote Lebesgue measure on $[0,1]$. Let $\{f_n\}$ be a sequence of Lebesgue Measurable functions on $[0,1]$ with values in $[0,\infty]$ so that

$$\lim_{n\to\infty} f_n(x) = 0$$

for a.e. x and

$$\int_0^1 f_n(x)\ dm(x) = 1 \qquad \forall\, n.$$

Define $g(x) = \sup_n\{f_n(x)\}$. Show that

$$\int_0^1 g(x)\ dm(x) = \infty.$$

More Qual Problems, Again

Matt Charnley

December 9, 2016

Real Topics

1. Convergence Theorems: Fatou, Monotone, Dominated.
2. Types of convergence: Pointwise, Uniformly, in L^1, in measure.
3. Product measures: Definition and construction.
4. Fubini-Tonelli: Statement, how to use them together.
5. Integration on $\mathbb{R}^n$ and Polar Coordinates: It works like Calc 3.
6. Change of Coordinates: The determinant/Jacobian thing. If he did this.
7. Signed measures: What are they?
8. Mutually Singular, Absolutely continuous measures: How do measures relate to each other? How to prove each of these?
9. Jordan and Hahn Decompositions.
10. Lebesgue-Radon-Nikodym Theorem. Set up, necessary assumptions, uses.
11. Lebesgue Differentiation Theorem, Hardy-Littlewood maximal function. If he gets here/it's on the exam.
12. And then everything from the first midterm.

Real Problems

1. Fall 2015: Real #4.
   (i) Let $[a, b]$ be a closed, bounded interval and $f : [a, b] \to \mathbb{R}$. Given an "epsilon-delta" definition of what it means for f to be "absolutely continuous on $[a, b]$."
   (ii) Assume now that $f : [0, 1] \to \mathbb{R}$ has the property that for all $0 < \epsilon < 1$, the restriction of f to $[\epsilon, 1]$ is absolutely continuous. Assume also that there exists some $p > 2$ so that
   $$\int_0^1 x|f'(x)|^p \, dm < \infty$$
   where m is Lebesgue measure. Prove that $\lim_{x\to 0^+} f(x)$ exists and is finite.
2. Fall 2013: Day 2, #1. Let $f \in L^1(-\infty, \infty)$ and let $h > 0$ be fixed. Prove that
   $$\frac{1}{2h}\int_{-\infty}^{\infty}\int_{x-h}^{x+h} f(y)\, dy\, dx = \int_{-\infty}^{\infty} f(x)\, dx$$
3. Fall 2013: Day 2, #8. Let $k(y) = c\exp(-y^2)$ with c chosen so that $\int_{-\infty}^{\infty} k(y)\, dm(y) = 1$. If $f \in L^1(-\infty, \infty)$, prove that, for every x in the Lebesgue set of f, there holds
   $$\lim_{t\to 0^+}\int_{-\infty}^{\infty} f(x-y)\frac{1}{t}k\left(\frac{y}{t}\right)\, dy = f(x)$$

Complex Topics

1. Conformal maps. Know the basic maps and tricks to putting them together.
2. Schwarz Lemma and applications. What do all automorphisms of the disk look like? How about the upper half plane?
3. Montel's Theorem.
4. Rouché's Theorem.
5. Meromorphic functions and Laurent Series.
6. Complex Logarithm and square roots. Branch cuts.
7. Doing contour integrals with these functions. Evaluating real integrals using contour integration of a complex functions. What contours can you use with a branch cut?
8. Residue Theorem.
9. Casorati-Weierstrass and the Removable Singularities Theorem, analyzing types of singularities.

Complex Problems

1. Spring 2016: Complex #2. Let $\mathbb{D}$ denote the open disc in $\mathbb{C}$ and suppose that $f : \mathbb{D} \to \mathbb{D}$ is holomorphic. Prove that
$$\frac{|f(0)| - |z|}{1 + |f(0)||z|} \leq |f(z)| \leq \frac{|f(0)| + |z|}{1 - |f(0)||z|}$$
2. Fall 2015: Complex #4. Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of analytic functions on the open set $U \subset \mathbb{C}$ with the properties:
   (i) $\{f_n\}_{n=1}^{\infty}$ is uniformly bounded on compact sets.
   (ii) The sequence $\{f_n\}_{n=1}^{\infty}$ converges pointwise to a function $f(x)$.

   Prove that f is analytic in U. (Hint: Apply the Lebesgue Dominated Convergence Theorem to a suitable contour integral.)
3. Fall 2013: Day 1, #6. Prove that if u is a bounded harmonic function in $\mathbb{H}$ with $\lim_{x \to x_0, y \to 0} u(x, y) = 0$ for all $x_0 \in \mathbb{R}$, then $u \equiv 0$.
4. Fall 2013: Day 2, #3. Prove that if U is an open connected subset of $\mathbb{C}$ and $\mathbf{f} = \{f_n\}_{n=1}^{\infty}$ is a sequence of holomorphic functions on U such that $Re(f_n(z)) > 0$ for all n and all $z \in U$, then $\mathbf{f}$ has a subsequence $\{f_{n_k}\}_{k=1}^{\infty}$ so that either
   (i) f_{n_k} converges uniformly on compact subsets of U to a holomorphic function $f(z)$.
   (ii) f_{n_k} converges uniformly to ∞ on compact subsets of U.

   Make sure your proof indicates clearly where and how you are using the assumption that U is connected.

D.11 Fall 2017 - Math 421

This section contains the following documents from my teaching assistant assignment for Math 421:

1. A worked out homework exercise that was posted to the students after they were having trouble with these types of problems
2. A description of Sturm-Liouville problems to help with something the students were not understanding correctly
3. A review sheet handed out for the second midterm
4. Images of the whiteboard that was projected to the students during one of my online office hours
5. The handout for a workshop that was run at the end of the semester to help review for the final.

Section 4.4: Example 4

Matt Charnley

October 12, 2017

This example computes the inverse Laplace transform

$$\mathscr{L}^{-1}\left\{\frac{1}{(s^2+k^2)^2}\right\}$$

1 Convolution Method

Let $F(s) = G(s) = \frac{1}{s^2+k^2}$. Then

$$\mathscr{L}^{-1}\left\{\frac{1}{(s^2+k^2)^2}\right\} = \mathscr{L}^{-1}\{F(s)G(s)\} = f * g$$

where f and g are the inverse Laplace transforms of F and G respectively, namely

$$f(t) = g(t) = \frac{1}{k}\sin(kt)$$

Then, we need to compute the convolution

$$f * g(t) = \frac{1}{k^2}\int_0^t \sin(kt - k\tau)\sin(k\tau)\ d\tau$$

1.1 Book Method

Use the product to sum formula

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

to get

$$\begin{aligned}
f * g(t) &= \frac{1}{k^2}\int_0^t \sin(kt - k\tau)\sin(k\tau)\ d\tau \\
&= \frac{1}{2k^2}\int_0^t \cos(kt - 2k\tau) - \cos(kt)\ d\tau \\
&= \frac{1}{2k^2}\left[\frac{-1}{2k}\sin(kt - 2k\tau) - \tau\cos(kt)\right]\Big|_0^t \\
&= \frac{1}{2k^2}\left[\frac{1}{2k}(\sin(kt) - \sin(-kt)) - t\cos(kt)\right] \\
&= \frac{\sin kt - kt\cos kt}{2k^3}
\end{aligned}$$

1.2 Direct Computation

We start with the same convolution integral, and want to use trigonometric identities to make the integral doable.

$$\begin{aligned}
f * g(t) &= \frac{1}{k^2}\int_0^t \sin(kt - k\tau)\sin(k\tau)\ d\tau \\
&= \frac{1}{k^2}\int_0^t [\sin(kt)\cos(k\tau) - \cos(kt)\sin(k\tau)]\sin(k\tau)\ d\tau \\
&= \frac{\sin kt}{k^2}\int_0^t \cos k\tau \sin k\tau\ d\tau - \frac{\cos kt}{k^2}\int_0^t \sin k\tau \sin k\tau\ d\tau \\
&= \frac{\sin kt}{2k^2}\int_0^t \sin 2k\tau\ d\tau - \frac{\cos kt}{2k^2}\int_0^t 1 - \cos 2k\tau\ d\tau \\
&= \frac{\sin kt}{2k^2}\frac{-1}{2k}\cos 2k\tau\ |_0^t - \frac{\cos kt}{2k^2}[\tau - \frac{1}{2k}\sin 2k\tau]\ |_0^t \\
&= \frac{\sin kt}{4k^3} - \frac{\sin kt \cos 2kt}{4k^3} - \frac{t\cos kt}{2k^2} + \frac{\cos kt \sin 2kt}{4k^3}
\end{aligned}$$

Now, if we rearrange terms in that last line

$$f*g(t) = \frac{\sin kt}{4k^3} - \frac{t\cos kt}{2k^2} + \frac{\cos kt\sin 2kt}{4k^3} - \frac{\sin kt\cos 2kt}{4k^3} = \frac{\sin kt}{4k^3} - \frac{t\cos kt}{2k^2} + \frac{1}{4k^3}[\cos kt\sin 2kt - \sin kt\cos 2kt]$$

We see that we get a term like $\cos A\sin B - \sin A\cos B = \sin B - A$. Therefore, we can rewrite this expression as

$$f * g(t) = \frac{\sin kt}{4k^3} - \frac{t\cos kt}{2k^2} + \frac{1}{4k^3}\sin kt = \frac{\sin kt}{2k^3} - \frac{t\cos kt}{2k^2}$$

which is the same as the answer the book got.

2 Derivative Method

We know that if we take a derivative in s of a function with $(s^2 + k^2)$ in the denominator, we get an $(s^2 + k^2)^2$, which matches the denominator that we are trying to get to. Using the formula for the Laplace transform of $tf(t)$, we can write out 4 Laplace transforms that we know.

$$\mathscr{L}\{\sin kt\} = \frac{k}{s^2 + k^2} \quad \mathscr{L}\{\cos kt\} = \frac{s}{s^2 + k^2} \quad \mathscr{L}\{t\sin kt\} = \frac{2sk}{(s^2 + k^2)^2} \quad \mathscr{L}\{t\cos kt\} = \frac{s^2 - k^2}{(s^2 + k^2)^2}$$

We can manipulate this last one into a slightly nicer form

$$\frac{s^2 - k^2}{(s^2 + k^2)^2} = \frac{s^2 + k^2 - 2k^2}{(s^2 + k^2)^2} = \frac{1}{s^2 + k^2} - \frac{2k^2}{(s^2 + k^2)^2}$$

Now, we can try to figure out how to solve the initial problem from here. We can write

$$
\begin{aligned}
\frac{1}{(s^2+k^2)^2} &= -\frac{1}{2k^2}\frac{-2k^2}{(s^2+k^2)^2} \\
&= -\frac{1}{2k^2}\left[\frac{-2k^2}{(s^2+k^2)^2}+\frac{1}{s^2+k^2}-\frac{1}{s^2+k^2}\right] \\
&= -\frac{1}{2k^2}\left[\frac{1}{s^2+k^2}-\frac{2k^2}{(s^2+k^2)^2}\right]+\frac{1}{2k^3}\frac{k}{s^2+k^2}
\end{aligned}
$$

Thus, the equality

$$
\frac{1}{(s^2+k^2)^2} = -\frac{1}{2k^2}\left[\frac{1}{s^2+k^2}-\frac{2k^2}{(s^2+k^2)^2}\right]+\frac{1}{2k^3}\frac{k}{s^2+k^2}
$$

tells us that

$$
\begin{aligned}
\mathscr{L}^{-1}\left\{\frac{1}{(s^2+k^2)^2}\right\} &= \mathscr{L}^{-1}\left\{-\frac{1}{2k^2}\left[\frac{1}{s^2+k^2}-\frac{2k^2}{(s^2+k^2)^2}\right]+\frac{1}{2k^3}\frac{k}{s^2+k^2}\right\} \\
&= -\frac{1}{2k^2}\mathscr{L}^{-1}\left\{\frac{1}{s^2+k^2}-\frac{2k^2}{(s^2+k^2)^2}\right\}+\frac{1}{2k^3}\mathscr{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} \\
&= -\frac{1}{2k^2}t\cos kt+\frac{1}{2k^3}\sin kt
\end{aligned}
$$

which is, again, the same formula as the book.

Sturm-Liouville Problems and Separation of Variables

Matt Charnley

November 4, 2017

In order to be able to talk about these problems, we need to review two different ODEs that you learned how to solve in 244 (which are also reviewed at the start of section 12.5)

$$y' + \alpha y = 0 \qquad \Rightarrow \qquad y = Ce^{-\alpha x}$$

and for $y'' + \lambda y = 0$, we will split into three cases based on whether λ is positive, negative or 0.

$$\begin{aligned} y'' + \alpha^2 y = 0 &\qquad \Rightarrow \qquad y = C_1 \cos(\alpha x) + C_2 \sin(\alpha x) &\qquad \lambda > 0 \\ y'' = 0 &\qquad \Rightarrow \qquad y = C_1 x + C_2 &\qquad \lambda = 0 \\ y'' - \alpha^2 y = 0 &\qquad \Rightarrow \qquad y = C_1 \cosh(\alpha x) + C_2 \sinh(\alpha x) &\qquad \lambda < 0 \end{aligned}$$

For pretty much everything that shows up in separation of variables, we need to deal exclusively with the second order equations. The first order on shows up in the heat equation, but we are generally using the α or λ from the second order problem to solve the first one, so the hard part is in the second order equation. The idea is the following: I am given an ODE of the form $y'' + \lambda y = 0$ along with boundary conditions at $x = 0$ and $x = L$. For a given λ, if we look at each case above, we have two constants to solve for, C_1 and C_2, and two boundary conditions at 0 and L, so it seems like this should always be doable. However, we want a non-trivial solution, so not setting both C_1 and C_2 equal to zero can put an extra constraint on what α can be. Therefore, we want to find all possible values of λ for which we have a non-zero solution to the problem. Let's look at a couple of examples and see what happens.

1 Example 1

Let's consider the following problem

$$y'' + \lambda y = 0 \qquad\qquad y(0) = 0 \quad y(L) = 0$$

In order to figure out all possible λ, we need to break this into three cases.

(a) Case 1: $\lambda = 0$. For this case, we know that we have the solution

$$y(x) = C_1 x + C_2$$

Now, if $y(0) = 0$, this implies that $C_2 = 0$. If, on top of that, we force $y(L) = 0$, this means that $y(L) = C_1 L + 0 = 0$, or $C_1 = 0$ because $L \neq 0$. Therefore, in this case, for $\lambda = 0$, the only solution we get is the zero solution.

(b) Case 2: $\lambda = -\alpha^2 < 0$. In this case, our solution takes the form

$$y(x) = C_1 \cosh(\alpha x) + C_2 \sinh(\alpha x)$$

If we plug $x = 0$ into this, we get that

$$y(0) = C_1 \cosh(0) + C_2 \sinh(0) = C_1$$

which we want to be zero by boundary conditions. Then, if we plug in $x = L$, we get that

$$y(L) = C_2 \sinh(\alpha L)$$

which we want to be zero to meet the boundary condition. We also do not want to pick $C_2 = 0$, because that would again give us the zero solution. Therefore, we need to have that $\sinh(\alpha L) = 0$. However, if you look at the graph or definition of the sinh function, it is only zero at 0, and nowhere else. Therefore, since $\alpha > 0$ and $L \neq 0$, we can never have that $\sinh(\alpha L) = 0$, which means the only way we can meet the second boundary condition is by setting $C_2 = 0$, giving us the trivial solution for $\lambda < 0$. Thus, we also get no interesting solutions in this case.

(c) Case 3: $\lambda = \alpha^2 > 0$. Well, we've failed twice so far, so hopefully this one works. In this case, our solution looks like

$$y(x) = C_1 \cos(\alpha x) + C_2 \sin(\alpha x)$$

If we plug in $x = 0$, we get

$$y(0) = C_1 \cos(0) + C_2 \sin(0) = C_1$$

which we want to be zero by the boundary condition. Then, if we plug in $x = L$, we get

$$y(L) = C_2 \sin(\alpha L) = 0$$

which puts us in a similar position as the previous case. However, this time, since we don't want $C_2 = 0$, we need $\sin(\alpha L) = 0$, and thankfully, the sine function is periodic, and hits zero more than just once. In order for $\sin(\alpha L) = 0$, we need $\alpha L = n\pi$, or $\alpha = \frac{n\pi}{L}$ for any integer n. Then, for each n, we get an eigenvalue $\lambda_n = \frac{n^2\pi^2}{L^2}$ and an eigenfunction $y_n = \sin\left(\frac{n\pi x}{L}\right)$ which satisfies

$$y_n'' + \lambda_n y_n = 0 \qquad\qquad y_n(0) = 0 \quad y_n(L) = 0$$

Therefore, for this problem, the full set of eigenvalues is $\lambda_n = \frac{n^2\pi^2}{L^2}$ for $n = 1, 2, ...$ and the corresponding eigenfunction are $y_n = \sin\left(\frac{n\pi x}{L}\right)$.

2 Example 2

Another problem

$$y'' + \lambda y = 0 \qquad\qquad y'(0) = 0 \quad y(L) = 0$$

In order to figure out all possible λ, we need to break this into three cases.

(a) Case 1: $\lambda = 0$. For this case, we know that we have the solution

$$y(x) = C_1 x + C_2$$

Now, if $y'(0) = 0$, this implies that $C_1 = 0$. If, on top of that, we force $y(L) = 0$, this means that $y(L) = 0 + C_2 = 0$, or $C_2 = 0$. Therefore, in this case, for $\lambda = 0$, the only solution we get is the zero solution.

(b) Case 2: $\lambda = -\alpha^2 < 0$. In this case, our solution takes the form

$$y(x) = C_1 \cosh(\alpha x) + C_2 \sinh(\alpha x)$$

If we plug $x = 0$ into the derivative, we get that

$$y'(0) = C_1\alpha \sinh(0) + C_2\alpha \cosh(0) = C_2\alpha$$

which we want to be zero by boundary conditions, so $C_2 = 0$. Then, if we plug in $x = L$, we get that

$$y(L) = C_1 \cosh(\alpha L)$$

which we want to be zero to meet the boundary condition. We also do not want to pick $C_1 = 0$, because that would again give us the zero solution. Therefore, we need to have that $\cosh(\alpha L) = 0$. However, if you look at the graph or definition of the cosh function, it is never equal to zero. Therefore, the only way we can meet the second boundary condition is by setting $C_1 = 0$, giving us the trivial solution for $\lambda < 0$. Thus, we also get no interesting solutions in this case.

(c) Case 3: $\lambda = \alpha^2 > 0$. As you probably guessed, this is the one that works here too. In this case, our solution looks like

$$y(x) = C_1 \cos(\alpha x) + C_2 \sin(\alpha x)$$

If we plug $x = 0$ into the derivative, we get

$$y'(0) = -C_1 \alpha \sin(0) + C_2 \alpha \cos(0) = C_2$$

which we want to be zero by the boundary condition. Then, if we plug in $x = L$, we get

$$y(L) = C_1 \cos(\alpha L) = 0$$

which puts us in a similar position as the previous case. However, this time, since we don't want $C_1 = 0$, we need $\cos(\alpha L) = 0$, which is again periodic. To get $\cos(\alpha L) = 0$, we need $\alpha L = (n + \frac{1}{2})\pi$, so that

$$\alpha_n = \frac{\left(n + \frac{1}{2}\right)\pi}{L}$$

Then, for each n, we get an eigenvalue $\lambda_n = \frac{(n+\frac{1}{2})^2\pi^2}{L^2}$ and an eigenfunction $y_n = \cos\left(\frac{(n+\frac{1}{2})\pi x}{L}\right)$ which satisfies

$$y_n'' + \lambda_n y_n = 0 \qquad y_n'(0) = 0 \quad y_n(L) = 0$$

Therefore, for this problem, the full set of eigenvalues is $\lambda_n = \frac{(n+\frac{1}{2})^2\pi^2}{L^2}$ for $n = 1, 2, \ldots$ and the corresponding eigenfunction are $y_n = \cos\left(\frac{(n+\frac{1}{2})\pi x}{L}\right)$.

3 Example 3

More problems:

$$y'' + \lambda y = 0 \qquad y(-1) = 0 \quad y'(1) - \frac{1}{2}y(1) = 0$$

(a) Case 1: $\lambda = 0$. For this case, we know that we have the solution

$$y(x) = C_1 x + C_2$$

Now, if $y(-1) = 0$, this implies that $-C_1 + C_2 = 0$. If, on top of that, we force $y'(1) - \frac{1}{2}y(1) = 0$, this means that $y'(1) - \frac{1}{2}y(1) = C_1 - \frac{1}{2}C_1 - \frac{1}{2}C_2$, or, again, $C_1 - C_2 = 0$. Thus, we can pick $C_1 = C_2 = 1$ to get a non-trivial solution, $y(x) = x + 1$. Therefore, in this case, for $\lambda = 0$, we get the eigenfunction $y(x) = x + 1$.

(b) Case 2: $\lambda = -\alpha^2 < 0$. In this case, our solution takes the form

$$y(x) = C_1 \cosh(\alpha x) + C_2 \sinh(\alpha x)$$

If we plug $x = -1$ into the derivative, we get that

$$y(-1) = C_1 \cosh(-\alpha) + C_2 \sinh(-\alpha) = 0$$

Next, we check the other boundary condition

$$y'(1) - \frac{1}{2}y(1) = C_1\alpha\sinh(\alpha) + C_2\alpha\cosh(\alpha) - \frac{1}{2}C_1\cosh(\alpha) - \frac{1}{2}C_2\sinh(\alpha) = 0$$

Then we are left to see for which values of α there is a solution to

$$\begin{aligned} C_1\cosh(\alpha) - C_2\sinh(\alpha) &= 0 \\ C_1\left(\alpha\sinh(\alpha) - \frac{1}{2}\cosh(\alpha)\right) + C_2\left(\alpha\cosh(\alpha) - \frac{1}{2}\sinh(\alpha)\right) &= 0 \end{aligned}$$

It takes some work to solve this out, but it turns out the only solution here occurs when $\alpha = 0$, so there are no non-trivial solutions in this case.

(c) Case 3: $\lambda = \alpha^2 > 0$. In this case, our solution looks like

$$y(x) = C_1\cos(\alpha x) + C_2\sin(\alpha x)$$

Plugging into the boundary conditions, we get that

$$y(-1) = C_1\cos(-\alpha) + C2\sin(-\alpha) = 0$$

and

$$y'(1) - \frac{1}{2}y(1) = -C_1\alpha\sin(\alpha) + C_2\alpha\cos(\alpha) - \frac{1}{2}C_1\cos(\alpha) - \frac{1}{2}C_2\sin(\alpha) = 0$$

which reduces to the system of equations

$$\begin{aligned} C_1\cos(\alpha) - C_2\sin(\alpha) &= 0 \\ C_1\left(-\frac{1}{2}\cos(\alpha) - \alpha\sin(\alpha)\right) + C_2\left(\alpha\cos(\alpha) - \frac{1}{2}\sin(\alpha)\right) &= 0 \end{aligned}$$

In order to have a non-zero solution, the determinant of the coefficient matrix (in terms of the constants C) must vanish, which means that

$$\alpha\cos(\alpha)^2 - \frac{1}{2}\cos(\alpha)(\sin(\alpha) - \alpha\sin(\alpha)^2 - \frac{1}{2}\sin(\alpha)(\cos(\alpha) = 0$$

which can be simplified to

$$\alpha(\cos^2(\alpha) - \sin^2(\alpha)) - \sin(\alpha)\cos(\alpha) = 0$$

Using double angle formulas, this becomes

$$\alpha cos(2\alpha) - \frac{1}{2}\sin(2\alpha) = 0$$

or

$$\alpha = \frac{1}{2}\tan(2\alpha)$$

As discussed in the textbook, this has an infinite sequence of solutions with $\alpha_n \to \infty$, and each of them gives rise to an eigenfunction. These are impossible to find by hand, but we know that they exist. These eigenfunctions and eigenvalues will all solve the same problem as in the above examples.

Thus for this problem, we get both an eigenvalue at 0 with corresponding linear solution $y_0(x) = x + 1$ and a sequence of positive eigenvalues with corresponding eigenfunctions that we can't find by hand.

4 Discussion

In each case before, we saw that the sine and cosine functions turned out to be the largest quantity of solutions we found. This is no coincidence. While it is possible to get some eigenfunctions with the linear solution or the sinh/cosh functions, you can not get infinitely many. This is basically what is stated in the theorem about Sturm Liouville problems in that the sequence of eigenvalues converges to $+\infty$, which means there can only be finitely many less than 0. Thus, for any of these reqular problems, you may see a few particular solutions with non-positive eigenvalues, but you will then always see an infinite family of solutions (with a sequence of positive eigenvalues) that involve sines and cosines.

Separation of Variables

The main way this is used (at least for this class) is in separation of variables problems. The idea is that once you separate variables, you are looking for solutions to certain Sturm-Liouville problems with boundary conditions that correspond to the boundary conditions of your initial problem. In the end, you will generally need to use generalized Fourier Series to find the final solution, because this sequence of functions you generate from the Sturm-Liouville problem will be an orthogonal set of functions, which you can then use in a Fourier-type expansion. The set from example 1 results in a standard sine series expansion, etc.

Exam 2 Review Problems

Matt Charnley

November 28, 2017

13. f is defined as the following function

$$f(x) = \begin{cases} 0 & -2 < x < 0 \\ x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$$

To compute the Fourier Series of f, we just need to calculate the coefficients based on the formulas in the book.

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\ dx \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\ dx \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\ dx$$

where in this case we have $L = 2$. We then calculate

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\ dx = \frac{1}{2}\int_{0}^{1} x\ dx = \frac{1}{2}\frac{x^2}{2}\ |_0^1 = \frac{1}{4}$$

$$\begin{aligned} a_n &= \frac{1}{2}\int_0^1 x\cos\left(\frac{n\pi x}{2}\right)\ dx \\ &= \frac{1}{2}\left[x\sin\left(\frac{n\pi x}{2}\right)\cdot\frac{2}{n\pi}\right]|_0^1 - \frac{1}{n\pi}\int_0^1 \sin\left(\frac{n\pi x}{2}\right)\ dx \\ &= \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n\pi}\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)|_0^1 \\ &= \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{2}{(n\pi)^2}\left(\cos\left(\frac{n\pi}{2}\right) - 1\right) \end{aligned}$$

$$\begin{aligned} b_n &= \frac{1}{2}\int_0^1 x\sin\left(\frac{n\pi x}{2}\right)\ dx \\ &= \frac{1}{2}\left[-x\cos\left(\frac{n\pi x}{2}\right)\cdot\frac{2}{n\pi}\right]|_0^1 + \frac{1}{n\pi}\int_0^1 \cos\left(\frac{n\pi x}{2}\right)\ dx \\ &= \frac{1}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{1}{n\pi}\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)|_0^1 \\ &= \frac{1}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{2}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right) \end{aligned}$$

Therefore the Fourier Series of f is

$$FS(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{2}\right) + b_n\sin\left(\frac{n\pi x}{2}\right)$$

which evaluates to

$$FS(x) = \frac{1}{8} + \sum_{n=1}^{\infty}\left(\left[\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{2}{(n\pi)^2}\left(\cos\left(\frac{n\pi}{2}\right) - 1\right)\right]\cos\left(\frac{n\pi x}{2}\right) + \left[\frac{1}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{2}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)\right]\sin\left(\frac{n\pi x}{2}\right)\right)$$

The graphs are shown on the back pages. This function is periodic with period 4.

14. For this problem, we consider the same f restricted to just $(0,2)$. We want to compute the Fourier Sine Series. We do that by

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$$

with $L = 2$ for this particular f. This gives

$$b_n = \frac{2}{2}\int_0^1 x\sin\left(\frac{n\pi x}{L}\right)\,dx$$

and, up to the multiple in front, this is the same thing we calculated for the b_n in problem 13. Thus, we have

$$b_n = \frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)$$

and the sine series expansion is

$$FS_S(x) = \sum_{n=1}^{\infty}\left[\frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)\right]\sin\left(\frac{n\pi x}{2}\right)$$

The graph is at the end. This function is also periodic with period 4.

15. Now, we want to do the cosine series for the same function. We calculate this by

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx$$

and, exactly as we saw before, since f is only non-zero on $(0,1)$, these will turn out to be exactly double the coefficients that we calculated in problem 13. Thus we get

$$a_0 = \frac{1}{2} \qquad a_n = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{4}{(n\pi)^2}\left(\cos\left(\frac{n\pi}{2}\right) - 1\right)$$

and the Fourier Cosine Series expansion is

$$FS_C(x) = \frac{1}{4} + \sum_{n=1}^{\infty}\left[\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{4}{(n\pi)^2}\left(\cos\left(\frac{n\pi}{2}\right) - 1\right)\right]\cos\left(\frac{n\pi x}{2}\right)$$

Graphs are again at the end. This function is periodic with period 4.

16. Now, something a little different. We want to do the periodic extension. These are calculated by the formulas

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2n\pi x}{L}\right)\,dx \qquad b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2n\pi x}{L}\right)\,dx$$

with $L = 2$. We calculate all of these coefficients in the same way. Except these turn out a little nicer.

$$a_0 = \frac{2}{2}\int_0^2 f(x)\,dx = \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$
\begin{aligned}
a_n &= \frac{2}{2}\int_0^1 x\cos\left(\frac{2n\pi x}{2}\right)\ dx = \int_0^1 x\cos(n\pi x)\ dx \\
&= x\frac{1}{n\pi}\sin(n\pi x)\ |_0^1 - \int_0^1 \frac{1}{n\pi}\sin(n\pi x)\ dx \\
&= 0 + \frac{1}{(n\pi)^2}\cos(n\pi x)\ |_0^1 \\
&= \frac{1}{(n\pi)^2}((-1)^n - 1)
\end{aligned}
$$

$$
\begin{aligned}
b_n &= \frac{2}{2}\int_0^1 x\sin\left(\frac{2n\pi x}{2}\right)\ dx = \int_0^1 x\sin(n\pi x)\ dx \\
&= -x\cos(n\pi x)\frac{1}{n\pi}\ |_0^1 + \int_0^1 \frac{1}{n\pi}\cos(n\pi x)\ dx \\
&= \frac{(-1)^{n+1}}{n\pi} + \frac{1}{n\pi}^2\sin(n\pi x)\ |_0^1 \\
&= \frac{(-1)^{n+1}}{n\pi}
\end{aligned}
$$

So then, the Fourier Series is

$$FS_P(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{(n\pi)^2}\cos(n\pi x) + \frac{(-1)^{n+1}}{n\pi}\sin(n\pi x)$$

This is a periodic function of period 2. See the graphs at the end.

17. For complex Fourier Series, we use the formula

$$c_n = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-in\pi x/L}\ dx$$

For this function in particular, we compute

$$
\begin{aligned}
c_n &= \frac{1}{4}\int_0^1 xe^{-in\pi x/2}\ dx && (1) \\
&= \frac{-2}{in\pi}xe^{-in\pi x/2}\ |_0^1 + \frac{2}{in\pi}\int_0^1 e^{-in\pi x/2}\ dx && (2) \\
&= \frac{-2}{in\pi}e^{-in\pi/2} + \frac{4}{(n\pi)^2}e^{-in\pi x/2}\ |_0^1 && (3) \\
c_n &= \frac{-2}{in\pi}e^{-in\pi/2} + \frac{4}{(n\pi)^2}(e^{-in\pi/2} - 1) && (4)
\end{aligned}
$$

However, this formula doesn't work for c_0, so we individually compute

$$c_0 = \frac{1}{4}\int_0^1 x\ dx = \frac{1}{8}$$

With this definition of c_n, we then have that the complex fourier series of f is

$$FS_{\mathbb{C}}(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/2}$$

18. Check the Sturm-Liouville document already on Sakai for details on this problem. That should give you enough to go on. You should get eigenvalues of 0 and n^2 for $n = 1, 2, ...$ and corresponding eigenfunctions of constants and $\cos(nx)$.

19. For characterizing these types of PDE's, all we care about are the second order terms, i.e., those with two derivatives. The other terms do not matter. Thus, we compute $B^2 - 4AC$ for this problem and get $1 - 4 \cdot 4 \cdot 3 < 0$, so this is an elliptic equation.

20. In order to do separation of variables, we make the assumption that $u(x,t) = X(x)T(t)$. Plugging this into the equation gives us

$$XT' - X'T = XT$$

Dividing both sides by XT, we get

$$\frac{T'}{T} - \frac{X'}{X} = 1$$

and rearranging the terms, we see

$$fracT'T = \frac{X'}{X} + 1 = -\lambda$$

where we can now use the separation constant because the left side only depends on t, and the right side only depends on x. This gives us two separate ODEs to solve

$$\frac{T'}{T} = -\lambda \qquad \rightarrow \qquad T' + \lambda T = 0$$

$$\frac{X'}{X} + 1 = -\lambda \qquad \rightarrow \qquad X' + (\lambda + 1)X = 0$$

These two ODEs have easy solutions, namely

$$T(t) = Ce^{-\lambda t} \qquad\qquad X(x) = Ce^{-(\lambda+1)x}$$

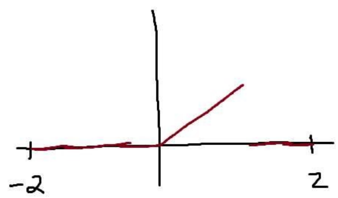
Therefore, the separated solutions look like

$$u(x,t) = Ce^{-\lambda t}e^{-(\lambda+1)x}$$

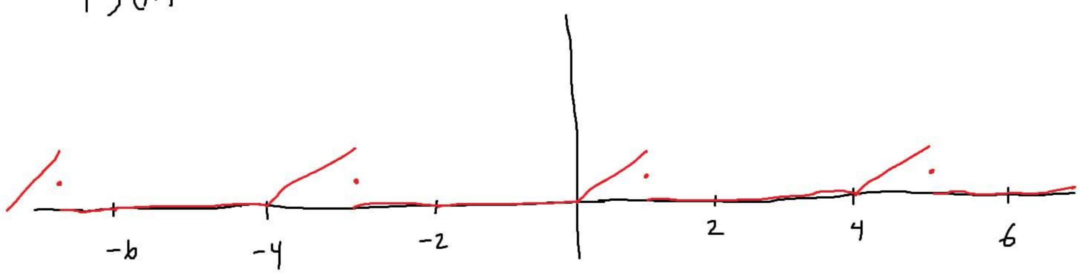
Graphs

Note: All lines should be straight.

13| f(x)

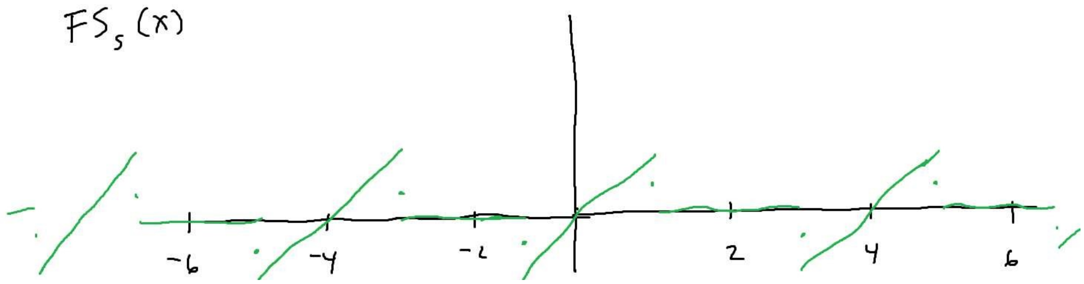


FS(x)



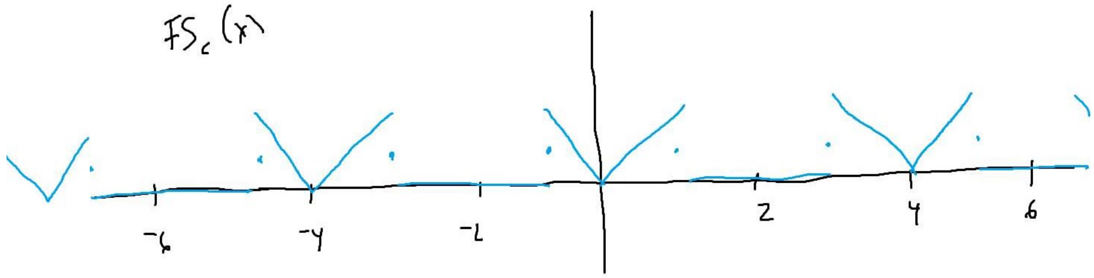
14| Sine Series

$FS_s(x)$



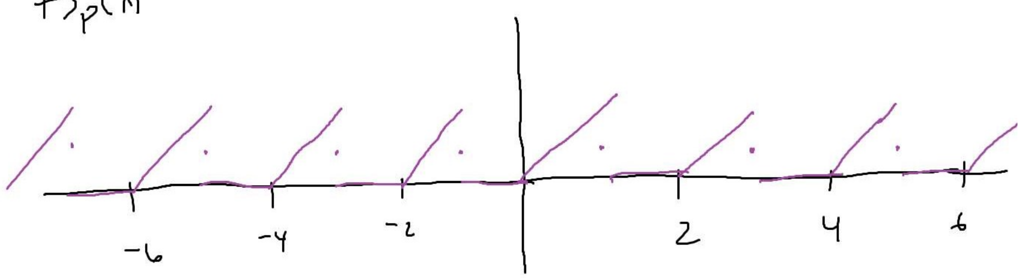
5 Cosine Series

$FS_c(x)$



6 Periodic Extension

$FS_p(x)$

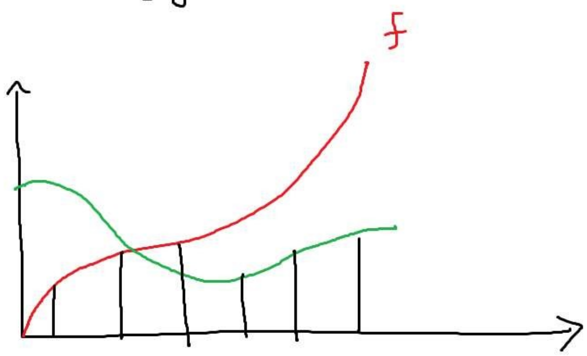


Convolutions

f, g defined on $[0, \infty)$ then

$$f * g(t) = \int_0^t f(t-\tau)\, g(\tau)\, d\tau$$

$$f * g(1) = \int_0^1 f(1-\tau)\, g(\tau)\, d\tau$$



$f(t) = e^t$ $\qquad g(t) = t^2$

$$f*g(t) = \int_0 e^{t-\tau}\tau^2 \, d\tau$$

4.4 #26

$$\mathcal{L}\left\{\int_0^t \tau \sin\tau \, d\tau\right\}$$

$f(t) = t \quad g(t) = \sin t$

$$f*g(t) = \int_0^t (t-\tau)\sin\tau \, d\tau$$

$$\mathcal{L}\{f*g(t)\} = \mathcal{L}\{f\} \cdot \mathcal{L}\{g\}$$

$\mathcal{L}\{f\} \; \mathcal{L}\{g\}$

$f*g$

$$\mathcal{L}\left\{\underbrace{\int_0^t (t-\tau)\sin\tau \, d\tau}_{\shortparallel}\right\} = \mathcal{L}\{t\} \cdot \mathcal{L}\{\sin t\}$$

$$\frac{1}{s^2} \cdot \frac{1}{s^2+1}$$

$$\mathcal{L}\left\{\int_0^t t\sin\tau \, d\tau\right\} - \underbrace{\mathcal{L}\left\{\int_0^t \tau\sin\tau \, d\tau\right\}}_{\text{Answer}}$$

$$\mathcal{L}\left\{t\int_0^t \sin\tau \, d\tau\right\}$$

$$= \mathcal{L}\{-t\cos t + t\}$$

$$= \frac{d}{ds}\mathcal{L}\{\cos t\} + \frac{1}{s^2}$$

$$\boxed{\mathcal{L}\{t\,f(t)\} = -\frac{d}{ds}F(s)}$$

$$= \frac{d}{ds}\left(\frac{s}{s^2+1}\right) + \frac{1}{s^2}$$

$$= \frac{s^2+1-s(2s)}{(s^2+1)^2} + \frac{1}{s^2}$$

$$= \frac{1-s^2}{(s^2+1)^2} + \frac{1}{s^2}$$

$$\frac{1}{s^2(s^2+1)} = \frac{1-s^2}{(s^2+1)^2} + \frac{1}{s^2} - A$$

Direct way

$$\mathcal{L}\left\{ \int_0^t \tau \sin \tau \, d\tau \right\} = \mathcal{L}\{1\} \cdot \mathcal{L}\{t \sin t\}$$

$$\int_0^t \tau \sin \tau \, d\tau = f * g = \int_0^t f(t-\tau) g(\tau) \, d\tau$$

So choose: $f(t) = 1 \quad g(t) = t \sin t$

421 Workshop 1

December 13, 2017

1 Chapter 4: The Laplace Transform

1.1 Introduction

Over the course of chapter 4, we learned many different techniques for computing Laplace Transforms and Inverse Laplace Transforms. For reference, some/most of them are listed here.

1. Computing from the definition of Laplace Transform (4.1)

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\ dt$$

2. Using linearity from a table of known transforms (4.1)

3. Computing Inverse Transforms using Partial Fractions (4.2)

4. Formulas for computing the transform of a derivative, or the derivative of a transform (4.2, 4.5)

$$\mathcal{L}[f'(t)] = sF(s) - f(0) \qquad\qquad \mathcal{L}[tf(t)] = -\frac{d}{ds}F(s)$$

5. Translation Theorems (4.3)

$$\begin{aligned}\mathcal{L}[f(t-a)\mathcal{U}(t-a)] &= e^{-as}F(s)\\ \mathcal{L}[e^{at}f(t)] &= F(s-a)\\ \mathcal{L}[f(t)\mathcal{U}(t-a)] &= e^{-as}\mathcal{L}[f(t+a)]\end{aligned}$$

6. Convolutions (4.4)

$$\mathcal{L}[f * g(t)] = F(s)G(s) \qquad \text{where} \quad f * g(t) = \int_0^\infty f(\tau)g(t-\tau)\ d\tau$$

7. Periodic functions (4.4)

$$\mathcal{L}[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\ dt$$

8. Delta functions (4.5)

$$\mathcal{L}[\delta(t-t_0)] = e^{-t_0 s}$$

Use these properties to compute the Laplace Transforms and Inverse Laplace transforms on the following pages. You'll have to decide which method to use.

1.2 Problems

Compute the Laplace Transform for each of the following functions $f(t)$

1. $$f(t) = \begin{cases} t & 0 < t < 5 \\ 3 & t > 5 \end{cases}$$

2. $$f(t) = \begin{cases} 0 & 0 < t < \pi \\ \cos(t) & t > \pi \end{cases}$$

3. $$f(t) = \begin{cases} 2 & 0 < t < 1 \\ t & 1 < t < 3 \\ e^{5t} & t > 3 \end{cases}$$

4. $f(t) = t^4 + 3t^2 + \sin(2t)$
5. $f(t) = (\sin(t) + t^2)\mathcal{U}(t-\pi)$
6. $f(t) = t^2 \cos(t)$
7. $f(t) = e^{2t}(t^2 + 3t)$
8. $f(t) = (1 + e^{3t} - 2e^{-t})\cos(5t)$
9. $f(t) = t^2(e^t + e^{2t})^2$
10. $f(t) = (t^2 - 4t + 1)\mathcal{U}(t-2)$
11. $f(t) = \int_0^t \tau \sin(t-\tau)\ d\tau$
12. $f(t) = t\int_0^t \tau^2 e^{t-\tau}\ d\tau$
13. $f(t) = t^2 \int_0^t \cos(\tau)\ d\tau$
14. $f(t) = te^{3t}\sin(2t)$
15. $f(t) = e^{-2t}t^3\mathcal{U}(t-4)$

Compute the inverse Laplace Transform for each of the following functions $F(s)$.

1. $F(s) = \frac{2s+5}{s^2+9}$
2. $F(s) = \frac{4s-3}{s^2+2s+17}$
3. $F(s) = \frac{s^2+4s-1}{s^3-5s^2+6s}$
4. $F(s) = e^{-s}\frac{3s}{(s+1)^2+4}$
5. $F(s) = \frac{2}{5s+3}$
6. $F(s) = \frac{32}{s^5} + \frac{6}{s^2}$
7. $F(s) = \frac{s+3}{(s^2+1)(s^2-4)}$
8. $F(s) = \frac{4}{(s+3)^2}$
9. $F(s) = \frac{(s+1)^2}{(s+2)^4}$
10. $F(s) = \frac{e^{-2s}}{s^3}$
11. $F(s) = (1-e^{-s})\frac{1}{s^2+1}$
12. $F(s) = \frac{(2s+1)e^{-\pi s}}{(s^2+25)(s^2+1)}$
13. $F(s) = \frac{1}{s(s^2+1)}$
14. $F(s) = \frac{2s+1}{s^2(s^2+4)}$

1.3 ODEs

We also saw how the Laplace Transform can be used to solve ODEs and integral equations. Solve the following differential equations and systems using Laplace Transforms.

1. $y' - 3y = 4\cos(t) \qquad y(0) = 2$
2. $y'' - 2y' + y = 3e^t \qquad y(0) = 1, y'(0) = 3$
3. $y' - 3y = 1 + te^{-t} \qquad y(0) = 0$
4. $y'' - y' = e^t \cos(2t) \quad y(0) = 0, y'(0) = 0$
5. $y' - 2y = f(t) \qquad y(0) = 1$ where
$$f(t) = \begin{cases} 0 & 0 < t < 2 \\ 3 & t > 2 \end{cases}$$
6. $y'' - 4y' - 5y = f(t) \qquad y(0) = 0, y'(0) = 1$ where
$$f(t) = \begin{cases} t & 0 < t < 4 \\ 1 & t > 4 \end{cases}$$
7. $y'' + y = f(t) \qquad y(0) = 2, y'(0) = 1$ where
$$f(t) = \begin{cases} 0 & 0 < t < \pi \\ \sin(t) & t > \pi \end{cases}$$
8. $y'' - 4y = f(t) \qquad y(0) = 3, y'(0) = 0$ where
$$f(t) = \begin{cases} 0 & 0 < t < 1 \\ 2 & 1 < t < 3 \\ 1 & t > 3 \end{cases}$$
9. $f(t) = 3t + \int_0^t f(t-\tau)\, d\tau$
10. $f'(t) - 1 = \sin(t) + \int_0^t f(\tau)\, d\tau \qquad f(0) = 0$
11. $t + 2f(t) = \int_0^t (e^{t-\tau} - e^{\tau - t}) f(\tau)\, d\tau$
12. $y'' - 6y' + 9y = 1 + \delta(t-2) \qquad y(0) = 1, y'(0) = 0$
13. $y'' + 4y = \delta(t - 2\pi) + \delta(t - 4\pi) \qquad y(0) = 0, y'(0) = 0$
14. $x' = 3x - 2y \quad y' = x + 4y \qquad x(0) = 2, y(0) = 1$
15. $2x' + y' - 2x = 1 \quad x' + y' - 3x - 3y = 2 \qquad x(0) = 0, y(0) = 0$
16. $x'' + x - y = 0 \quad y'' + y - x = 0 \quad x(0) = 0, x'(0) = -2, y(0) = 0, y'(0) = -1$

2 Chapter 8: Matrices

2.1 Introduction

2.1.1 Row Reduction

In the first part of chapter 8, we learned about row reduction of matrices, and in later sections, we saw what it allowed us to do in terms of simplifying calculations using matrices.

- Solve a linear system:

  To solve the given system

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

  we row reduce the augmented matrix

$$\begin{bmatrix} a_1 & b_1 & c_1 & | & d_1 \\ a_2 & b_2 & c_2 & | & d_2 \\ a_3 & b_3 & c_3 & | & d_3 \end{bmatrix}$$

- Compute the rank of a matrix: The **rank** of a matrix is the number of nonzero rows in its row-reduced form. Equivalently, it is the number of columns in its row-reduced form which have pivots.

- Find the determinant of a matrix by row reducing to triangular form: Row-reduce the matrix, keeping track of your steps to use the properties of determinants under row operations (see below). The determinant of an upper- or lower-triangular matrix is the product of the elements on the diagonal.

- Find the inverse of a matrix: For a 2x2 matrix, you may memorize the formula:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{1}$$

  For an $n \times n$ matrix A, we can set up the augmented matrix with A on the left and I_n (the $n \times n$ identity matrix) on the right. Row reduce this augmented matrix. If you end up with I_n on the left, then whatever is on the right at that point is A^{-1}. If you end up with a row of zeros, the matrix is **singular**, i.e. not invertible.

2.1.2 Properties of Determinant

- Invertibility: A matrix A is invertible *if and only if* $\det(A) \neq 0$. In other words, both of the following two statements are true:
  - If you already know that A is invertible, then you automatically also know that $\det(A) \neq 0$.
  - If you compute that $\det(A) \neq 0$, then that immediately tells you that A is invertible.

  When A is invertible, $\det(A^{-1}) = \frac{1}{\det(A)}$.
- Determinant of a product: $\det(AB) = \det(A)\det(B)$
- Determinant of the transpose: $\det(A) = \det(A^T)$
- Change in determinant under row operations:
  - Suppose that matrix B matches matrix A, except that two of the rows are swapped. Then

$$\det(B) = -\det(A) \tag{2}$$

  - Suppose that matrix C matches matrix A, except that one of the rows is multiplied by some number c. Then,

$$\det(C) = c\det(A)$$

  - Suppose that matrix D is obtained from matrix A by taking one row and adding a multiple of another row to it. Then

$$\det(D) = \det(A)$$

2.1.3 Eigenvalues and Eigenvectors

- Find eigenvalues and eigenvectors: For a given $n \times n$ matrix A, we say that the nonzero vector K is an **eigenvector** for A with **eigenvalue** λ if

$$AK = \lambda K \tag{3}$$

  To find the eigenvalues of A, we start by creating a polynomial $p(\lambda)$, which is often called the **characteristic polynomial** of A:

$$p(\lambda) = \det(A - \lambda I_n)$$

  Then, the eigenvalues of A are the zeros of p. In other words, the eigenvalues of A are the values of λ that solve the **characteristic equation** of A:

$$0 = p(\lambda) = \det(A - \lambda I_n)$$

Then, to find the eigenvectors that correspond to each λ, we reduce the augmented matrix with $A - \lambda I_n$ on the left and a column of 0's on the right, namely $(A - \lambda I_n|0)$.

- Repeated roots: For some matrices, the characteristic polynomial may have repeated roots, ie roots with multiplicity 2 or more. For example, $p(\lambda) = \lambda^2(1-\lambda)$ has $\lambda = 0$ as a root with multiplicity 2. In these cases, it is possible that you will be able to find two or more linearly independent eigenvectors corresponding to the same eigenvalue, up to the multiplicity of that root.

- A note on the zero vector: Suppose I had the vector Z, all of whose entries are 0, ie

$$Z = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

Then, for any matrix A, $AZ = Z$, because the zeros cancel everything out. Furthermore, for any number λ, we also have that $\lambda Z = Z$, again because the zeros cancel everything out. It is for this reason that we don't consider Z an eigenvector: an eigenvector K can have some entries be 0, but it must have at least one entry which is nonzero. The zero vector satisfies $AZ = \lambda Z$ for *every* A and *every* λ, so the fact that it satisfies the equation tells us nothing about A. Furthermore, we want each eigenvector to correspond to *one* eigenvalue. If we allow Z to be an eigenvector, it would have *every* possible λ as an eigenvalue. There are also many other reasons that we don't include Z as a possibility in our definition of an eigenvector, which I won't detail here.

2.2 Computational Problems

For the problems below, either solve the given system or show that no solution exists.

1. $3x_1 - 2x_2 = 4$

   $x_1 - x_2 = -2$

2. $10x_1 + 15x_2 = 1$

   $3x_1 + 2x_2 = -1$

3. $x_1 + 2x_2 - x_3 = 0$

   $2x_1 + x_2 + 2x_3 = 9$

   $x_1 - x_2 + x_3 = 3$

4. $x_1 - x_2 - 2x_3 = 0$
   $2x_1 + 4x_2 + 5x_3 = 0$
   $6x_1 - 3x_3 = 0$

Find the rank of each of the below matrices.

5. $\begin{pmatrix} 2 & -2 \\ 0 & 0 \end{pmatrix}$

6. $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 4 \\ -1 & 0 & 3 \end{pmatrix}$

7. $\begin{pmatrix} 3 & -2 & 2 & 0 \\ 6 & 2 & 4 & 5 \end{pmatrix}$

8. $\begin{pmatrix} 1 & -2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 0 & 1 & 0 & 0 \\ 2 & 5 & 6 & 8 \end{pmatrix}$

Find the inverse of each of the below matrices, or show that one does not exist.

9. $\begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix}$

10. $\begin{pmatrix} 1 & 0 & -1 \\ 0 & -2 & 1 \\ 2 & -1 & 3 \end{pmatrix}$

11. $\begin{pmatrix} 8 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

Row reduce each of the following matrices to a triangular form. Then, use that to find the determinant of the original matrix.

12. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 5 \\ -3 & 0 & 2 \end{pmatrix}$

13. $\begin{pmatrix} 2 & 4 & 5 \\ 4 & 2 & 0 \\ 8 & 7 & -2 \end{pmatrix}$

14. $\begin{pmatrix} -2 & 2 & 6 \\ 5 & 0 & 1 \\ 1 & -2 & 2 \end{pmatrix}$

15. $\begin{pmatrix} 6 & 2 & 7 \\ -4 & -3 & 2 \\ 2 & 4 & 8 \end{pmatrix}$

Find all the eigenvalues and eigenvalues of the matrices below.

16. $\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$

17. $\begin{pmatrix} 1 & 1 \\ \frac{1}{4} & 1 \end{pmatrix}$

18. $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

19. $\begin{pmatrix} 7 & 0 \\ 0 & 13 \end{pmatrix}$

20. $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 0 & 1 \end{pmatrix}$

21. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

22. $\begin{pmatrix} 2 & -1 & 0 \\ 5 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix}$

2.3 Conceptual Problems

1. Suppose $\det(A) = 1$. Then what is $\det(2A)$ if

   (a) A is a 2×2 matrix?

   (b) A is a 3×3 matrix?

   (c) A is a 4×4 matrix?

   (d) A is a 5×5 matrix?

   (e) A is a $n \times n$ matrix?

2. A permutation matrix is a matrix A each of whose columns and rows contain exactly one entry of 1, and all other entries 0. Some examples of permutation matrices are

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

For a given permutation matrix A, what are all the possible values of $\det(A)$? Hint: use the properties of determinant under row reduction. You can try the examples above to get some ideas.

3. Without doing any computation, either find the inverse of the below matrix or argue why one does not exist:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & \pi & .2 \\ 0 & 0 & 0 \end{pmatrix}$$

4. Without doing any computation, argue why the below system of equations does not have a unique solution:

$$\begin{aligned} 3x_1 + x_2 - x_3 &= 0 \\ x_1 - x_2 - x_3 &= 0 \end{aligned}$$

5. What is $\det(A)$ in each of the below cases?
   (a) The equation $AX = 0$ has a nontrivial solution.
   (b) $\lambda = 0$ is an eigenvalue of A.
   (c) A is singular.
   (d) A is a 3×3 matrix whose eigenvalues are 1, 2, and 3.

6. In this problem, we investigate orthogonal matrices:
   (a) Find the inverse of the matrix

$$\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$$

   (b) We say that a matrix A is orthogonal if $A^{-1} = A^T$. Verify that the matrix above is an orthogonal matrix.
   (c) Verify that the matrix below is an orthogonal matrix:

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{pmatrix}$$

   (d) If A is an orthogonal matrix, what are the possibilities for $\det(A)$? Prove your answer. Hint: use properties of determinants and the fact that $AA^{-1} = I$.

7. Suppose I tell you that λ is an eigenvalue for the matrix A. What are the possibilities for the number of eigenvectors of A which correspond to λ? Select all that apply.

(a) 0
(b) 1
(c) 2
(d) infinitely many

8. Suppose A is an $n \times n$ matrix. Which of the following senarios are possible? Select all that apply.

   (a) A has less than n different eigenvalues.
   (b) A has exactly n different eigenvalues.
   (c) A has more than n different eigenvalues.

9. Suppose A is singular. Is it possible to find a matrix B so that AB is invertible? Why or why not?

D.12 Spring 2018 - Math 574 and 575

This section contains the following documents from my teaching assistant assignment for Math 574 and 575:

1. My recitation syllabus from Math 574
2. A solution set to an optional homework set for Math 574 at the end of the semester that was posted for the students to use for review
3. My recitation syllabus from Math 575
4. A diagnostic linear algebra quiz that was given to students early in the semester
5. A sample homework set solution that was given to the students and the homework grader

MATH 574 - Spring 2018 - Workshop Syllabus

Contact Information

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Office Hours

Mondays from 2-4pm in Hill 606
or by appointment.

Course

This class is MATH 574 - Numerical Analysis II, lectured by Michael Vogelius. The course will cover numerical methods for solving linear systems of equations, matrix eigenvalues, minimization, and finite difference methods. See the instructor syllabus for more complete and accurate information.

Class Meetings

Lecture: W 1:40PM - 4:40PM, Hill 423
Workshop: Th 1:40PM - 3:00PM, ARC 203

Goals

The goals for these workshop sessions are to give you extra help and guidance in solving the types of problems covered in this course. While a lot of things will look familiar from 573, there are quite a few new ideas in this class that can be tricky to grasp at first. My role, and the role of these sessions, is to show you these ideas in more detail by working out problems so that you can figure out these concepts and understand how to use it in the future. I will also be able to provide support with any computer or programming issues you may have with the assignments.

Outline of Session

The general plan for each session is to begin by going over the material covered in lecture during the previous week. I will then take time to go over some problems from the previous week's homework that were particularly difficult. There will also be time at the end (as well as during the body of the sections) for questions from you, which can either be about material from lecture or problems on the upcoming homework set. In the lead-up to exams, these weekly sessions will turn into a review session for the exam.

Math 574 - Optional Homework Solutions

Matt Charnley

April 25, 2018

8. See the attached code. This computes the largest eigenvalue and corresponding eigenvector by the power method. This agrees with the information computed algebraically.

10. The same proof goes through as in the book. The only difference is that none of the terms x_i with eigenvalue λ_1 vanish. However, this is fine, because if x_1 and x_2 are both eigenvectors with eigenvalue λ_1, so is any linear combination of x_1 and x_2. Therefore, the most we can say is that the sequence of iterates converge to a linear combination of $x_1, x_2, ..., x_r$, but this is still an eigenvector with eigenvalue λ_1, which is the eigenvalue with largest magnitude.

11. Let $\{v_1, v_2, ..., v_n\}$ be an orthonormal basis of eigenvectors corresponding to eigenvalues $\lambda_1, ..., \lambda_n$. Then, for $x = \sum \alpha_i v_i$, we have

$$\mathcal{R}(x) = \frac{(Ax, x)}{(x, x)} = \frac{(\sum \alpha_i A v_i, \sum \alpha_j v_j)}{(\sum \alpha_i v_i, \sum \alpha_j v_j)} = \frac{\sum \lambda_i \alpha_i^2}{\sum \alpha_i^2}$$

Since $\lambda_1 \geq \lambda_i \geq \lambda_n$ for all i, we can see that

$$\lambda_1 = \frac{\lambda_1 \sum \alpha_i^2}{\sum \alpha_i^2} \geq \mathcal{R}(x) \geq \frac{\lambda_n \sum \alpha_i^2}{\sum \alpha_i^2} = \lambda_n$$

which clearly gives that $\max \mathcal{R}(x) \leq \lambda_1$ and $\min \mathcal{R}(x) \geq \lambda_n$. On the other hand, the same calculation as above gives that

$$\mathcal{R}(v_1) = \lambda_1 \qquad \mathcal{R}(v_n) = \lambda_n$$

Therefore, those inequalities are equalities, and we have

$$\max \mathcal{R}(x) = \lambda_1 \qquad \min \mathcal{R}(x) = \lambda_n$$

14. See the attached code. The functions go through the Householder matrices to find the QR factorization.

22. See the attached code. Looking at the end results, we can conclude that the eigenvalues for each matrix are:

(a) 3.7321, 2.0000, 0.2679

(b) 3.6180, 2.6180, 1.3820, 0.3820

(c) 2.9021, 2.1756, 1.0000, -0.9021, -0.1756

Optional Homework - MATH 574

Matt Charnley

Contents

Problem 8 - Power Method

```
% Part a)

clear all;

A = [[6,4,4,1];[4,6,1,4];[4,1,6,4];[1,4,4,6]];

[l1, x1] = PowerMethod(A);

disp(A);
disp(['Largest Eigenvalue: ' num2str(l1)]);
disp(['Eigenvector: [' num2str(x1') ']' ]);
disp(' ');
disp(' ');

% Part b)

clear all;

A = [[2,1,3,4];[1,-3,1,5];[3,1,6,-2];[4,5,-2,-1]];

[l1, x1] = PowerMethod(A);

disp(A);
disp(['Largest Eigenvalue: ' num2str(l1)]);
disp(['Eigenvector: [' num2str(x1') ']' ]);

disp(' ');
disp(' ');

% Part c)

clear all;

A = [[1,3,-2]; [-1,-2,3];[1,1,2]];

[l1, x1] = PowerMethod(A);

disp(A);
disp(['Largest Eigenvalue: ' num2str(l1)]);
disp(['Eigenvector: [' num2str(x1') ']' ]);

disp(' ');
disp(' ');
```

```
     6     4     4     1
     4     6     1     4
     4     1     6     4
     1     4     4     6

Largest Eigenvalue: 15
Eigenvector: [1           1           1           1]


     2     1     3     4
     1    -3     1     5
     3     1     6    -2
     4     5    -2    -1

Largest Eigenvalue: -8.0286
Eigenvector: [-0.38998    -0.97553      0.2955           1]


     1     3    -2
    -1    -2     3
     1     1     2

Largest Eigenvalue: 2.6289
Eigenvector: [-0.024463     0.65338           1]
```

Problem 14 - QR Factorization

```
A = [[1,1,1];[2,-1,-1];[2,-4,5]];

[Q,R] = QR_HH(A);
A
Q
R
Q*R

A = [[1,3,-2];[-1,-2,3];[1,1,2]];

[Q,R] = QR_HH(A);
A
Q
R
Q*R
```

```
A =

     1     1     1
     2    -1    -1
     2    -4     5


Q =

   -0.3333   -0.6667   -0.6667
   -0.6667   -0.3333    0.6667
   -0.6667    0.6667   -0.3333
```

```
R =

   -3.0000    3.0000   -3.0000
   -0.0000   -3.0000    3.0000
   -0.0000   -0.0000   -3.0000


ans =

    1.0000    1.0000    1.0000
    2.0000   -1.0000   -1.0000
    2.0000   -4.0000    5.0000


A =

     1     3    -2
    -1    -2     3
     1     1     2


Q =

   -0.5774   -0.7071    0.4082
    0.5774    0.0000    0.8165
   -0.5774    0.7071    0.4082


R =

   -1.7321   -3.4641    1.7321
         0   -1.4142    2.8284
         0         0    2.4495


ans =

    1.0000    3.0000   -2.0000
   -1.0000   -2.0000    3.0000
    1.0000    1.0000    2.0000
```

Problem 22

```
vals = [2,4,8,16,32, 64,128];

% Matrix (a) No Shift

A = [[3,1,0];[1,2,1];[0,1,1]];
disp('Matrix (a), No Shift');
disp(' ');

Ai = A;
for i=2:max(vals)
    [Q,R] = QR_HH(Ai);
    Ai = R*Q;
    if ismember(i,vals)
        disp(['After ' num2str(i) ' iterations:']);
```

```
        disp(Ai);
       disp(' ');
    end
end

% Matrix (a) with shift
disp(' ');
disp('Matrix (a), With Shift');
disp(' ');
Ai = A;
n = length(Ai);
for i=2:max(vals)
    a = Ai(n-1,n-1);
    b = Ai(n-1,n);
    c = Ai(n,n);
    l = a+c+sign(a)*sqrt((a+c)^2 + 4*(b^2 - a*c))/2;
    [Q,R] = QR_HH(Ai - l*eye(n));
    Ai = R*Q + l*eye(n);
    if ismember(i,vals)
        disp(['After ' num2str(i) ' iterations:']);
        disp(Ai);
        disp('');
    end
end
```

```
Matrix (a), No Shift

After 2 iterations:
    3.5000    0.5916    0.0000
    0.5916    2.2143   -0.1807
    0.0000   -0.1807    0.2857


After 4 iterations:
    3.7103    0.1927    0.0000
    0.1927    2.0217   -0.0031
    0.0000   -0.0031    0.2680


After 8 iterations:
    3.7319    0.0161    0.0000
    0.0161    2.0001   -0.0000
    0.0000   -0.0000    0.2679


After 16 iterations:
    3.7321    0.0001    0.0000
    0.0001    2.0000   -0.0000
   -0.0000   -0.0000    0.2679


After 32 iterations:
    3.7321    0.0000    0.0000
    0.0000    2.0000   -0.0000
   -0.0000   -0.0000    0.2679


After 64 iterations:
    3.7321   -0.0000    0.0000
```

```
    0.0000    2.0000   -0.0000
   -0.0000   -0.0000    0.2679


After 128 iterations:
    3.7321   -0.0000    0.0000
    0.0000    2.0000   -0.0000
   -0.0000   -0.0000    0.2679



Matrix (a), With Shift

After 2 iterations:
    1.5617    0.9023    0.0000
    0.9023    1.3650   -1.1455
    0.0000   -1.1455    3.0733

After 4 iterations:
    0.8769    0.8349   -0.0000
    0.8349    1.4451   -0.3322
   -0.0000   -0.3322    3.6780

After 8 iterations:
    0.3315    0.3257    0.0000
    0.3257    1.9373   -0.0386
   -0.0000   -0.0386    3.7312

After 16 iterations:
    0.2683    0.0261    0.0000
    0.0261    1.9996   -0.0009
    0.0000   -0.0009    3.7321

After 32 iterations:
    0.2679    0.0002    0.0000
    0.0002    2.0000   -0.0000
    0.0000   -0.0000    3.7321

After 64 iterations:
    0.2679    0.0000    0.0000
    0.0000    2.0000   -0.0000
    0.0000   -0.0000    3.7321

After 128 iterations:
    0.2679   -0.0000    0.0000
    0.0000    2.0000    0.0000
    0.0000   -0.0000    3.7321
```

```
% Matrix (b) No Shift

disp('Matrix (b), No Shift');
disp(' ');
A = [[2,1,0,0];[1,2,1,0];[0,1,2,1];[0,0,1,2]];

Ai = A;
for i=2:max(vals)
    [Q,R] = QR_HH(Ai);
    Ai = R*Q;
```

```
    if ismember(i,vals)
        disp(['After ' num2str(i) ' iterations:']);
        disp(Ai);
        disp(' ');
    end
end

% Matrix (a) with shift
disp(' ');
disp('Matrix (b), With Shift');
disp(' ');
Ai = A;
n = length(Ai);
for i=2:max(vals)
    a = Ai(n-1,n-1);
    b = Ai(n-1,n);
    c = Ai(n,n);
    l = a+c+sign(a+eps)*sqrt((a+c)^2 + 4*(b^2 - a*c))/2;
    [Q,R] = QR_HH(Ai - l*eye(n));
    Ai = R*Q + l*eye(n);
    if ismember(i,vals)
        disp(['After ' num2str(i) ' iterations:']);
        disp(Ai);
        disp(' ');
    end
end
```

```
Matrix (b), No Shift

After 2 iterations:
    2.8000    0.7483   -0.0000   -0.0000
    0.7483    2.3429    0.8748    0.0000
   -0.0000    0.8748    2.1905   -0.6236
   -0.0000   -0.0000   -0.6236    0.6667


After 4 iterations:
    3.3333    0.4689   -0.0000    0.0000
    0.4689    2.7585    0.4270    0.0000
   -0.0000    0.4270    1.5249   -0.0361
   -0.0000    0.0000   -0.0361    0.3832


After 8 iterations:
    3.5906    0.1635   -0.0000    0.0000
    0.1635    2.6447    0.0320    0.0000
   -0.0000    0.0320    1.3828   -0.0002
   -0.0000    0.0000   -0.0002    0.3820


After 16 iterations:
    3.6179    0.0126   -0.0000    0.0000
    0.0126    2.6182    0.0002    0.0000
   -0.0000    0.0002    1.3820   -0.0000
   -0.0000   -0.0000   -0.0000    0.3820


After 32 iterations:
    3.6180    0.0001   -0.0000    0.0000
```

```
    0.0001    2.6180    0.0000    0.0000
   -0.0000    0.0000    1.3820   -0.0000
   -0.0000   -0.0000   -0.0000    0.3820


After 64 iterations:
    3.6180    0.0000   -0.0000    0.0000
    0.0000    2.6180    0.0000    0.0000
    0.0000    0.0000    1.3820   -0.0000
    0.0000   -0.0000   -0.0000    0.3820


After 128 iterations:
    3.6180    0.0000   -0.0000    0.0000
    0.0000    2.6180    0.0000    0.0000
    0.0000    0.0000    1.3820   -0.0000
    0.0000   -0.0000   -0.0000    0.3820



Matrix (b), With Shift

After 2 iterations:
    1.4000    0.8602    0.0000   -0.0000
    0.8602    1.8703    0.9698         0
    0.0000    0.9698    1.9724   -0.9187
   -0.0000   -0.0000   -0.9187    2.7573


After 4 iterations:
    0.8939    0.6278    0.0000   -0.0000
    0.6278    1.5280    0.8038    0.0000
    0.0000    0.8038    2.1801   -0.4933
   -0.0000   -0.0000   -0.4933    3.3979


After 8 iterations:
    0.5352    0.3679    0.0000   -0.0000
    0.3679    1.3150    0.3284    0.0000
    0.0000    0.3284    2.5515   -0.1419
   -0.0000   -0.0000   -0.1419    3.5983


After 16 iterations:
    0.3931    0.1049   -0.0000   -0.0000
    0.1049    1.3721    0.0393    0.0000
   -0.0000    0.0393    2.6170   -0.0150
   -0.0000   -0.0000   -0.0150    3.6178


After 32 iterations:
    0.3820    0.0068   -0.0000   -0.0000
    0.0068    1.3819    0.0006    0.0000
    0.0000    0.0006    2.6180   -0.0002
   -0.0000   -0.0000   -0.0002    3.6180


After 64 iterations:
    0.3820    0.0000   -0.0000   -0.0000
    0.0000    1.3820    0.0000    0.0000
   -0.0000    0.0000    2.6180   -0.0000
```

```
    -0.0000   -0.0000   -0.0000    3.6180


After 128 iterations:
    0.3820    0.0000   -0.0000   -0.0000
    0.0000    1.3820    0.0000    0.0000
   -0.0000    0.0000    2.6180   -0.0000
    0.0000   -0.0000   -0.0000    3.6180
```

```
% Matrix (c) No Shift

A = [[0,1,0,0,0];[1,1,1,0,0];[0,1,1,1,0];[0,0,1,1,1];[0,0,0,1,2]];

disp('Matrix (c), No Shift');
disp(' ');

Ai = A;
for i=2:max(vals)
    [Q,R] = QR_HH(Ai);
    Ai = R*Q;
    if ismember(i,vals)
        disp(['After ' num2str(i) ' iterations:']);
        disp(Ai);
        disp(' ');
    end
end

% Matrix (c) with shift
disp(' ');
disp('Matrix (c), With Shift');
disp(' ');
Ai = A;
n = length(Ai);
for i=2:max(vals)
    a = Ai(n-1,n-1);
    b = Ai(n-1,n);
    c = Ai(n,n);
    l = a+c+sign(a)*sqrt((a+c)^2 + 4*(b^2 - a*c))/2;
    [Q,R] = QR_HH(Ai - l*eye(n));
    Ai = R*Q + l*eye(n);
    if ismember(i,vals)
        disp(['After ' num2str(i) ' iterations:']);
        disp(Ai);
        disp(' ');
    end
end
```

```
Matrix (c), No Shift

After 2 iterations:
    1.0000   -1.4142         0         0         0
   -1.4142    0.5000   -0.8660   -0.0000    0.0000
   -0.0000   -0.8660    1.5000    0.8165    0.0000
   -0.0000   -0.0000    0.8165    2.0000    0.5774
    0.0000    0.0000    0.0000    0.5774    0.0000
```

```
After 4 iterations:
    2.4000   -0.5538    0.0000   -0.0000   -0.0000
   -0.5538    0.7739   -1.6547    0.0000    0.0000
    0.0000   -1.6547    0.9596    0.5072   -0.0000
   -0.0000   -0.0000    0.5072    1.0420    0.0105
   -0.0000   -0.0000    0.0000    0.0105   -0.1755


After 8 iterations:
    2.8078   -0.2443   -0.0000   -0.0000   -0.0000
   -0.2443    2.2675   -0.0822    0.0000    0.0000
   -0.0000   -0.0822   -0.7605    0.4959   -0.0000
    0.0000    0.0000    0.4959    0.8608    0.0000
    0.0000    0.0000   -0.0000    0.0000   -0.1756


After 16 iterations:
    2.9010   -0.0279   -0.0000   -0.0000   -0.0000
   -0.0279    2.1766   -0.0001    0.0000   -0.0000
    0.0000   -0.0001   -0.3480    0.8643   -0.0000
   -0.0000   -0.0000    0.8643    0.4459    0.0000
   -0.0000   -0.0000   -0.0000    0.0000   -0.1756


After 32 iterations:
    2.9021   -0.0003   -0.0000   -0.0000   -0.0000
   -0.0003    2.1756   -0.0000    0.0000   -0.0000
   -0.0000   -0.0000    0.8429    0.5236   -0.0000
   -0.0000   -0.0000    0.5236   -0.7450   -0.0000
   -0.0000   -0.0000   -0.0000    0.0000   -0.1756


After 64 iterations:
    2.9021   -0.0000   -0.0000    0.0000   -0.0000
   -0.0000    2.1756    0.0000    0.0000   -0.0000
    0.0000   -0.0000    0.9998    0.0211   -0.0000
    0.0000   -0.0000    0.0211   -0.9019   -0.0000
    0.0000   -0.0000    0.0000    0.0000   -0.1756


After 128 iterations:
    2.9021   -0.0000   -0.0000    0.0000   -0.0000
   -0.0000    2.1756    0.0000    0.0000   -0.0000
    0.0000   -0.0000    1.0000    0.0000   -0.0000
    0.0000   -0.0000    0.0000   -0.9021   -0.0000
    0.0000   -0.0000    0.0000    0.0000   -0.1756



Matrix (c), With Shift

After 2 iterations:
   -0.4029    0.7003    0.0000    0.0000   -0.0000
    0.7003    0.7406    0.9413    0.0000   -0.0000
         0    0.9413    0.9480    0.9887    0.0000
         0    0.0000    0.9887    1.1215   -0.5925
         0   -0.0000   -0.0000   -0.5925    2.5928


After 4 iterations:
```

```
   -0.6638    0.4328   -0.0000    0.0000    0.0000
    0.4328    0.2651    0.7533    0.0000   -0.0000
    0.0000    0.7533    0.7815    0.7219    0.0000
    0.0000   -0.0000    0.7219    1.7830   -0.2397
   -0.0000    0.0000   -0.0000   -0.2397    2.8341


After 8 iterations:
   -0.8162    0.2434   -0.0000   -0.0000    0.0000
    0.2434   -0.1310    0.3816    0.0000   -0.0000
   -0.0000    0.3816    0.9063    0.2109    0.0000
   -0.0000    0.0000    0.2109    2.1466   -0.0756
    0.0000   -0.0000    0.0000   -0.0756    2.8943


After 16 iterations:
   -0.8891    0.0965   -0.0000   -0.0000    0.0000
    0.0965   -0.1855    0.0608    0.0000   -0.0000
    0.0000    0.0608    0.9971    0.0174    0.0000
    0.0000    0.0000    0.0174    2.1755   -0.0100
   -0.0000   -0.0000    0.0000   -0.0100    2.9020


After 32 iterations:
   -0.9018    0.0140   -0.0000   -0.0000    0.0000
    0.0140   -0.1758    0.0014    0.0000   -0.0000
    0.0000    0.0014    1.0000    0.0001    0.0000
    0.0000    0.0000    0.0001    2.1756   -0.0002
   -0.0000   -0.0000    0.0000   -0.0002    2.9021


After 64 iterations:
   -0.9021    0.0003   -0.0000   -0.0000    0.0000
    0.0003   -0.1756    0.0000    0.0000   -0.0000
    0.0000    0.0000    1.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    2.1756   -0.0000
   -0.0000   -0.0000    0.0000   -0.0000    2.9021


After 128 iterations:
   -0.9021    0.0000   -0.0000   -0.0000    0.0000
    0.0000   -0.1756    0.0000    0.0000   -0.0000
   -0.0000    0.0000    1.0000   -0.0000    0.0000
   -0.0000    0.0000    0.0000    2.1756   -0.0000
    0.0000   -0.0000    0.0000   -0.0000    2.9021
```

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```
function [ lam1, x1, Rvec, lit ] = PowerMethod(A)
%PowerMethod - Uses the Power Method to compute the largest eigenvalue of
%the matrix A

n = length(A);
x0 = rand(n,1);
Rvec= [1,1,1];
step = 1;
tol = 10^(-7);
k = floor(n/2);
z = x0;
lm2 = 0;
lm1 = 0;
lit = [];
while step < 5 || abs(Rvec(step) - Rvec(step-1)) > tol
    w = A*z;
    while z(k) < tol
        k = k+1;
        if k == n+1
            k=1;
        end
    end
    lm = w(k)/z(k);
    lit(end+1) = lm;
    if step > 2
        if abs(lm1 - lm2) < tol^2;
            Rvec(step+1) = Rvec(step);
        else
            Rvec(step+1) = (lm - lm1)/(lm1 - lm2);
        end
    end
    lm2 = lm1;
    lm1 = lm;
    z = w/max(abs(w));
    step  = step + 1;
end

lam1 = lm;
x1 = z;


end
```

```
Error using PowerMethod (line 5)
Not enough input arguments.
```

Published with MATLAB® R2015a

```
function [ Q,R ] = QR_HH( A )
%QR_HH - does the QR factorization of a matrix A using Householder matrices
%   Detailed explanation goes here

n = length(A);
Q = eye(n);
R = A;

for i=1:n-1
    b = R(:,i);
    w = HH_w(b, i);
    P = eye(n) - 2*w*w';
    R = P*R;
    Q = Q*P.';
end

end
```

```
Error using QR_HH (line 5)
Not enough input arguments.
```

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```
function [ w ] = HH_w( b, r )
%HH_w - returns the vector w so that for U = I - 2ww^*, we have Ub with the
%last r+1 entries zero.
%   Detailed explanation goes here
n = length(b);
w = zeros(n,1);
if r <= n
   d = b(r:n);
   v = zeros(length(d),1);
   alpha = -norm(d, 2)*sign(d(1)+eps);
   v(1) = sqrt(1/2 * (1 - d(1)/alpha));
   p = -alpha*v(1);
   v(2:end) = d(2:end)/(2*p);
   w(r:n) = v;
end

end
```

```
Error using HH_w (line 5)
Not enough input arguments.
```

Published with MATLAB® R2015a

MATH 575 - Spring 2018 - Workshop Syllabus

Contact Information

Name: Matt Charnley
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Office: Hill 606

Office Hours

Mondays from 2-4pm in Hill 606
or by appointment.

Course

This class is MATH 575 - Numerical Solution of Partial Differential Equations, lectured by Paul Feehan. The course will cover both finite element and finite difference methods for elliptic, parabolic, and hyperbolic equations. See the instructor's syllabus for more complete and accurate information.

Class Meetings

Lecture: TF 12:00PM - 1:20PM, CORE 101
Workshop: Th 10:20AM - 11:40AM, Hill 005

Goals

The goals for these workshop sessions are to give you extra help and guidance in solving the types of problems covered in this course. While a lot of things will look familiar from 573, there are quite a few new ideas in this class that can be tricky to grasp at first. My role, and the role of these sessions, is to show you these ideas in more detail by working out problems so that you can figure out these concepts and understand how to use it in the future. I will also be able to provide support with any computer or programming issues you may have with the assignments.

Outline of Session

The general plan for each session is to begin by going over the material covered in lecture during the previous week. I will then take time to go over some problems from the previous week's homework that were particularly difficult. There will also be time at the end (as well as during the body of the sections) for questions from you, which can either be about material from lecture or problems on the upcoming homework set. In the lead-up to exams, these weekly sessions will turn into a review session for the exam.

MATH 575 - Linear Algebra Diagnostic Quiz

June 6, 2018

Name: ____

1. Let A be an $n \times n$ matrix over $\mathbb{R}$. What does it mean for A to be
   (a) symmetric?

   (b) orthogonal?

2. Let A be an $n \times n$ matrix over $\mathbb{C}$. What does it mean for A to be
   (a) Hermitian?

   (b) unitary?

3. There are several properties of a matrix A that are equivalent to A being invertible. State the three of these related to
   (i) determinants,

   (ii) eigenvalues, and

   (iii) solutions to $Ax = 0$.

4. Let A, B, and P be matrices of appropriate sizes. For each computation below, relate it to related computations for the individual matrices, or say that there's no way to simplify it.
   (a) $\mathrm{tr}(AB)$

   (b) $\mathrm{tr}(P^{-1}AP)$

   (c) $\det(AB)$

   (d) $\det(P^{-1}AP)$

5. What does it mean for a real matrix A to be positive definite? You should have two definitions here, one involving eigenvalues, and one involving arbitrary vectors in $\mathbb{R}^n$.

6. Let A be an $n \times n$ matrix with eigenvalues $\lambda_1, ..., \lambda_n$, possibly repeated. What are $\mathrm{tr}(A)$ and $\det(A)$?

7. Let A be an $n \times n$ real matrix and assume that there is a matrix B so that

$$\langle Ax, y \rangle = \langle x, By \rangle$$

for all vectors x and y, where this is a real inner product. What relation must A and B satisfy?

8. Let A be an $n \times n$ complex matrix and assume that there is a matrix B so that

$$\langle Ax, y \rangle = \langle x, By \rangle$$

for all vectors x and y, where this is a complex inner product. What relation must A and B satisfy?

9. What conditions will guarantee that a matrix A has real eigenvalues?

10. What conditions will guarantee that a matrix A is diagonalizable?

11. Show that if A is a real matrix and λ is an eigenvalue of A, $\bar{\lambda}$ is also an eigenvalue of A.

12. If a matrix is not diagonalizable, what is the best replacement for diagonalizing A? What does this matrix look like?

Math 575 - Homework 9 Solutions

Matt Charnley

April 19, 2018

1. Using the fact that the vertices are $\mathbf{a}_1 = (1,0)$, $\mathbf{a}_2 = (1,1)$, and $\mathbf{a}_3 = (0,1)$, we can see that the function $1-x_1$ is zero on both $\mathbf{a}_1$ and $\mathbf{a}_2$ and is 1 at $\mathbf{a}_3$. Therefore $\lambda_3 = 1-x_1$. By a similar argument, $\lambda_1 = 1-x_2$, and then by the fact that all of the all of the barycentric coordinates need to add to 1, we have $\lambda_2 = x_1 + x_2 - 1$.

2.

(a) Since $\mathbf{m}_{ij}$ is the point where $\lambda_i = \lambda_j = \frac{1}{2}$, we get that if ϕ_k is of the form $c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3$, then it must satisfy

$$1 = \phi_k(\mathbf{m}_{ij}) = \frac{1}{2}(c_i + c_j) \qquad 0 = \phi_k(\mathbf{m}_{ik}) = \frac{1}{2}(c_i + c_k) \qquad 0 = \phi_k(\mathbf{m}_{jk}) = \frac{1}{2}(c_k + c_j)$$

Solving this system gives $c_i = c_j = 1$ and $c_k = -1$. Thus, we have that

$$\phi_k = \lambda_i + \lambda_j - \lambda_k = 1 - 2\lambda_k$$

because the sum of all of the lambdas is 1. Therefore

$$\phi_1 = 1 - 2\lambda_1 \qquad \phi_2 = 1 - 2\lambda_2 \qquad \phi_3 = 1 - 2\lambda_3$$

(b) Therefore, if we want to find a polynomial on T so that $P(\mathbf{m}_{12}) = 4$, $P(\mathbf{m}_{23}) = 5$, $P(\mathbf{m}_{31}) = 6$, then we can just do this by

$$P = 4\phi_3 + 5\phi_1 + 6\phi_2$$

3.

(a) Let P be a quadratic polynomial on a triangle T with value zero at all the vertices, and average value zero on all of the edges. Then, since the quadrature formula is exact, we know that

$$0 = \int_{e_{ij}} P(t)\ dt = C[P(\mathbf{a}_i) + 4P(\mathbf{m}_{ij}) + P(\mathbf{a}_j)] = 4CP(\mathbf{m}_{ij})$$

because the value at the vertices are zero. Thus, we have that $P(\mathbf{m}_{ij}) = 0$ as well for each edge. Therefore, from the representation formula

$$P(x) = \sum_{i=1}^{3} \lambda_i(2\lambda_i - 1)P(a_i) + \sum_{i<j} 4\lambda_i\lambda_j P(m_{ij})$$

since the value at each vertex and midpoint is zero, we have that $P \equiv 0$.

(b) If g is a quadratic polynomial, then for both of the integrals

$$\int_a^b g(t)\ dt \qquad \text{and} \qquad \int_a^b g(t)(t - [a+b]/2)\ dt$$

the quadrature formula exactly holds. Thus, we have that

$$0 = \frac{b-a}{6}[g(a) + 4g([a+b]/2) + g(b)]$$
$$0 = \frac{b-a}{6}[g(a)(a-[a+b]/2) + 4g([a+b]/2)[a+b]/2 - [a+b]/2) + g(b)(b-[a+b]/2)]$$

Simplifying these equations, we get

$$0 = g(a) + 4g([a+b]/2) + g(b) \qquad 0 = g(a)\frac{a-b}{2} + g(b)\frac{b-a}{2}$$

The second of these implies that $g(a) = g(b)$, and plugging this into the first equation gives that $g([a+b]/2) = -\frac{1}{2}g(a)$, as desired.

(c) Based on the motivation from the previous parts, we want to define a polynomial so that

$$g(a_i) = 2 \qquad g(m_{ij}) = -1$$

for all vertices a_i and midpoints m_{ij}. Thus, the polynomial

$$P(x) = \sum_{i=1}^{3} 2\lambda_i(2\lambda_i - 1) - 4\sum_{i<j} \lambda_i \lambda j$$

will be non-zero, but will satisfy

$$\int_e P\ ds = 0 \qquad \int_e Ps\ ds = 0$$

because these can be reduced to integrals of the form

$$\int_a^b g(t)\ dt \qquad \text{and} \int g(t)t\ dt$$

which are zero by the way we chose our endpoints and midpoints.

MATH 575 - Quiz 10

April 19, 2018

Name: Key.

1. Write out a pseudocode procedure for the Solve-Estimate-Mark-Refine method for adaptive mesh refinement. Assume that the method will stop then the total error is less than *tol*.

Start with an initial mesh T_0.

$k=0$.

SOLVE: Find the solution u_k on the mesh T_k.

ESTIMATE: Compute the total error and error on each triangle.

If the total error is $\leq$ tol, then stop.

MARK: Pick a minimal set of triangles $M_k \subset T_k$ so that the total error on M_k is at least $\Theta \cdot$ the error on the entire mesh, for a chosen $\Theta \in (0,1)$.

REFINE: Refine the triangles in M_k to get the next mesh T_{k+1}.

Increment k.

2. Prove that the set

$$K := \{v \in H_0^1(\Omega)\ v \geq \psi \text{ a.e. in } \Omega\}$$

is a convex subset of $H_0^1(\Omega)$.

For any $u, v \in K$ and any $t \in (0,1)$, we know that

$$tu + (1-t)v \in H_0^1(\Omega) \quad \text{and}$$

$$tu + (1-t)v \geq t\psi + (1-t)\psi = \psi \quad \text{a.e. in } \Omega$$

Therefore $tu + (1-t)v \in K$, and so K is convex.

3. Consider the set K from the previous page,

$$K := \{v \in H_0^1(\Omega)\ v \geq \psi \text{ a.e. }\}$$

and define $u \in K$ by the solution to the variational inequality

$$\int_\Omega (\nabla u) \cdot \nabla(v-u)\,dx \geq \int_\Omega f(v-u)\,dx \qquad \forall v \in K$$

Prove that:

(a) $-\Delta u \geq f$ a.e. in Ω. (Hint: Integrate by parts and choose v appropriately)

Integration gives $\int_\Omega -\Delta u\,(v-u) \geq \int_\Omega f\,(v-u)$

For any $w \in H_0^1(\Omega)$, with $w \geq 0$, we can pick $v = u + w$ so that this reads $\int_\Omega (-\Delta u) w \geq \int_\Omega f w \quad \forall w \in H_0^1(\Omega)\ w \geq 0$

Thus $-\Delta u \geq f$. ✓.

(b) $u \geq \psi$ a.e. in Ω.

This is in the definition of K.

(c) $(-\Delta u - f)(u - \psi) = 0$ a.e. in Ω (Hint: Assume $u(x) > \psi(x)$. I can then pick a function $v \in K$ so that $v \leq u$, and strictly less near x. What does this do to the inequality?)

If $u(x) > \psi(x)$, $\exists$ a function $\eta(x) \geq 0$, compact support, $\eta(x) > 0$. so that $u(x) - \eta(x) > \psi$. Since η is positive and smooth with compact support, we also know that $u + \eta \in K$, $u - \eta \in K$.

Thus, applying the variational inequality to both $v = u + \eta$ and $v = u - \eta$, we get

$$\int -\Delta u(\eta) \geq \int f\eta \quad \text{and} \quad \int -\Delta u(-\eta) \geq \int f(-\eta).$$

which implies that $\int_\Omega -\Delta u\,\eta = \int_\Omega f\eta$

Therefore $-\Delta u = f$ on the support of η, i.e. where $u > \psi$.

Appendix E

Full Student Feedback

Included are the full SIRS student feedback surveys for each of the classes for which I was either the Teaching Assistant or the instructor. They have each been summarized in their corresponding sections, but the full results are shown here for reference. The classes are presented in the following order:

1. Summer 2015 - Math 251
2. Summer 2016 - Math 477
3. Summer 2017 - Math 244
4. Summer 2018 - Math 252
5. Fall 2014 - Math 135
6. Spring 2015 - Math 251
7. Fall 2015 - Math 251
8. Fall 2017 - Math 421
9. Spring 2018 - Math 575

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Summer 2015, 01:640:251:C1 — MULTIVARIABLE CALCULUS (index #01954) Enrollment= 23, Responses= 8	Student Responses						Weighted Means			
Part A: University-wide Questions:	Strong Disagree 1				Strong Agree 5	No response	Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner	0	0	0	1	7	0	**4.88**	4.66	4.51	**4.52**
2. The instructor responded effectively to student comments and questions	0	0	0	0	7	1	**5.00**	4.56	4.54	**4.49**
3. The instructor generated interest in the course material	0	0	0	1	7	0	**4.88**	4.40	4.27	**4.40**
4. The instructor had a positive attitude toward assisting all students in understanding course material	0	0	0	1	7	0	**4.88**	4.63	4.65	**4.56**
5. The instructor assigned grades fairly	0	0	0	2	6	0	**4.75**	4.51	4.48	**4.45**
6. The instructional methods encouraged student learning	0	0	0	1	7	0	**4.88**	4.46	4.29	**4.37**
7. I learned a great deal in this course	0	0	1	1	6	0	**4.63**	4.40	4.38	**4.38**
8. I had a strong prior interest in the subject matter and wanted to take this course	0	1	1	3	3	0	**4.00**	3.86	3.68	**3.55**
	Poor				Excellent					
9. I rate the teaching effectiveness of the instructor as	0	0	0	2	6	0	**4.75**	4.37	4.37	**4.41**
10. I rate the overall quality of the course as	0	0	1	2	5	0	**4.50**	4.23	4.25	**4.27**

What do you like best about this course?:

“Matt was an awesome instructor and very approachable! Always had answers to our questions!”

“It's very useful for describing motions. And it's closely relevant to physics, which makes it easier to understand intuitively. ”

“I liked his way of teaching the best. It was easy to understand, especially as we progressed to the more difficult material. ”

“The professor ”

“I love to see teachers who are passionate about what they are teaching it makes me wish I could enjoy the subject as much as they do. I liked that he would do his best to present the material in the best way possible given the time constraints.”

If you were teaching this course, what would you do differently?:

“Nothing.”

“I would not do anything differently if I were teaching the course.”

“There is very few things I would do differently. Mr. Charnley was excellent in explaining all the concepts, presenting examples and applications, and answering any questions I had.”

“not a thing”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“He is always showing us how even the hardest of topics can be tackled in the simplest of ways.”

“He tries to make everybody at the class understand the materials.”

“The instructor helped me understand the fundamental material so that when we started to cover the more difficult material, I did not have a hard time understanding it.”

“Mr. Charnley had a great, positive attitude throughout the summer. For 8 AM, it was awesome to have someone with such a great attitude.”

“I was just trying to pass so it was kind of hard to be encouraged intellectually when my mind was set on finishing this class without wasting the thousands of dollars that was spent on taking these summer classes.”

Other comments or suggestions::

“Matt did a really great job teaching this summer math session. He was always encouraging and made sure everyone had a fair chance at asking their questions as well as doing a great job answering them. By far the best Calculus course I've taken here at Rutgers. ”

“Mr. Charnley was an excellent instructor. He was, by far, the best instructor I've had here at Rutgers. I hope that he is teaching Differential Equations next semester, because he can explain any math concept very well.”

“This course was not easy for me considering the last math class I took was damn near 2yrs ago. I forgot a lot of things that my classmates breezed over but I realize that I can't complain since all that I needed to succeed was at my disposal. If I had taken this class during the semester with Matt I'm sure I would have done well. If I didn't need this for P-chem I wouldn't have taken it. Side Note:I hate filling these things out bc they always show up when I'm supposed to be preparing for the final exam. *sighs*”

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Summer 2016, 01:640:477:B1 — MATHEMATICAL THEORY OF PROBABILITY (index #02048) Enrollment= 25, Responses= 14	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	1	0	1	12	0	**4.71**	4.74	4.88	**4.63**
2. The instructor responded effectively to student comments and questions.	0	0	0	3	11	0	**4.79**	4.74	4.84	**4.50**
3. The instructor generated interest in the course material.	0	0	3	1	10	0	**4.50**	4.62	4.75	**4.42**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	0	14	0	**5.00**	4.93	4.93	**4.61**
5. The instructor assigned grades fairly.	0	0	1	4	9	0	**4.57**	4.74	4.74	**4.50**
6. The instructional methods encouraged student learning.	0	0	0	4	9	1	**4.69**	4.77	4.73	**4.39**
7. I learned a great deal in this course.	0	0	0	4	10	0	**4.71**	4.74	4.79	**4.48**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	1	2	6	5	0	**4.07**	4.30	4.27	**3.67**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	1	3	10	0	**4.64**	4.59	4.77	**4.41**
10. I rate the overall quality of the course as:	0	0	1	5	8	0	**4.50**	4.48	4.68	**4.31**

What do you like best about this course?:

"in class practice problems that reinforced what the class learned that day"

"Even though this was an 8 a.m. summer class and many of the students were tired and distracted, the instructor seemed to put a lot of effort and enthusiasm into the material he was teaching. He offered a lot of office hours which were very helpful. "

"Although there was little time to learn the material, I think I acquired an accurate understanding of the subject matter."

"The teacher is really patient and passionate."

"There are enough review materials for preparing the exams. Easy to understand for the professor's explanation."

"The practice problems will helped introduce the material and set a good foundation! "

If you were teaching this course, what would you do differently?:

"go over the harder homework problems assigned. give more examples that were not in the textbook "

"When presenting new material, I would give more simple examples to explain concepts. I felt like often, we would learn a new theorem or proposition, and be given one extremely simple example followed by one hard, overly complicated example. I understand that there is more of a time constraint when going through material during a summer course, but I think it would be more effective to give less complicated examples and more simple/moderate examples to demonstrate concepts. Additionally, when it comes to grading, I think quizzes should be a smaller percentage or equal percentage to homework/practice problems. I find it unfair that we only have 4 quizzes worth 20% while there were 18 homeworks only worth 10%. I did poorly on only 1 quiz and it pulled my entire grade down, while I spent hours competing all of my homework and do not have much to show for it. For future classes, I think either these grading policies should be adjusted, or one quiz grade should be dropped to compensate."

"I found that the last three chapters seemed rushed. A longer focus on this material may be more beneficial. "

"I would assign less homework; maybe one assignment every other day. Even though you should be studying (i.e. doing problems) every day in a summer course, some days it was hard to complete homework on time if you have other obligations in addition to the course."

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

"The use of practice problems during every lecture definitely helped to learn the material"

"I've learned countless of new ways to think about probability and even find myself thinking about what I've learned during this course when I play card games with my family."

Other comments or suggestions::

"This class was tough and had dry material but the professor definitely made it more bearable"

"The layout of the course was very effective with daily homework and weekly quizzes."

"Overall, Matt Charnley is a really good instructor! He is really passionate about teaching and wants his students to do well. "

"Maybe can be less homework and hope the exams are straight forward as what professor said."

"The only problem I had with this course was the amount of formulas that we had to memorize. That might just be because I'm not great with memorization, but having a small formula sheet in my previous statistics courses really helped me personally. "

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew Summer 2017, 01:640:244:C1 — DIFFERENTIAL EQUATIONS FOR ENGINEERING AND PHYSICS (index #00276) Enrollment= 25, Responses= 13	Student Responses						Weighted Means			
Part A: University-wide Questions:	Strong Disagree 1				Strong Agree 5	No response	Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	0	1	12	0	**4.92**	4.61	4.54	**4.68**
2. The instructor responded effectively to student comments and questions.	0	0	1	2	10	0	**4.69**	4.39	4.48	**4.56**
3. The instructor generated interest in the course material.	0	0	0	1	12	0	**4.92**	4.30	4.37	**4.54**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	0	13	0	**5.00**	4.63	4.60	**4.64**
5. The instructor assigned grades fairly.	0	1	1	2	9	0	**4.46**	4.30	4.43	**4.56**
6. The instructional methods encouraged student learning.	0	0	0	2	11	0	**4.85**	4.37	4.34	**4.50**
7. I learned a great deal in this course.	0	0	0	3	10	0	**4.77**	4.49	4.43	**4.55**
8. I had a strong prior interest in the subject matter and wanted to take this course.	1	2	3	3	4	0	**3.54**	3.57	3.75	**3.61**
	Poor				Excellent					
9. I rate the teaching effectiveness of the instructor as:	0	0	0	4	9	0	**4.69**	4.34	4.33	**4.53**
10. I rate the overall quality of the course as:	0	0	0	5	8	0	**4.62**	4.26	4.25	**4.41**

What do you like best about this course?:

"The flipped class setting alllowed for better and more learning."

"The reverse classroom style"

"The method of doing video lectures at home then problems in class"

"The flipped classroom made learning easier. "

"He posted his lectures as YouTube videos to be watched at home so we spent our time in the classroom practicing problems and working together which was a great way to learn the material. "

"The format of learning and in class work."

"I like the structure of the class. It is very different than the other calc classes I have taken at Rutgers. The way this class is set up encourages learning in a great new way. All math classes at Rutgers should be taught in this way. Active collaborative learning is the best!"

"I liked that the instructor posted videos of the course material online. It allowed me to learn on my own prior to the next class meeting. "

"What I liked best about this course is the problems given to us. I felt like working on exercises everyday in class was an effective way to learn the material."

"The online pre lecture videos were very beneficial to learning because most of class time was spent working on problems in groups "

"I like the "flipped classroom" style."

"The teaching style of this class was great for actually learning the material and understanding it."

If you were teaching this course, what would you do differently?:

"Hace fewer write ups."

"I would have five more partial credit on exams like all my past professors have done. If there was one silly mistake but the logic was correct I would still get 0 points of out 8, even though I knew what I was doing so it was frustrating at some times."

"Nothing, done very well"

"Do an example before class to make sure students understand the videos"

"Nothing"

"Nothing"

"I believe that although there are online videos available, a brief lecture in class is still necessary. "

"I would assign various problems that are easy, medium, and hard, than just one hard question for homework."

"Nothing"

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

"This method of teaching worked very well and helped me understand the material much better."

"Very helpful in class, encouraged questions, and was every enthusiastic about this course in class. "

"I am better at articulating my answerings and recognizing graphing patterns when given a function or set of functions "

"He has reinvigorated my passion for the subject."

"The instructor had taught us how to apply math problems in a real world setting related to our majors. "

"working on the material everyday has progressed and encouraged my intellectual growth."

"Group work and projects relating to applications "

Other comments or suggestions::

"Very well done"

"The use of canvas and how the class was taught in respect to the use of videos and other materials helped me learn like no other class before"

"Best class I took at rutgers "

"The videos were very helpful."

"Great teacher!!"

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew mpc163 Summer 2018, 01:640:252:B6 — Elem Diff Equations (index #00268) Enrollment= 27, Responses= 21	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	1	6	14	0	**4.62**	4.61	4.65	**4.62**
2. The instructor responded effectively to student comments and questions.	0	0	0	6	15	0	**4.71**	4.67	4.55	**4.54**
3. The instructor generated interest in the course material.	0	0	2	6	13	0	**4.52**	4.58	4.37	**4.40**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	6	15	0	**4.71**	4.67	4.58	**4.61**
5. The instructor assigned grades fairly.	0	0	1	8	11	1	**4.50**	4.50	4.46	**4.39**
6. The instructional methods encouraged student learning.	0	1	0	6	14	0	**4.57**	4.61	4.40	**4.40**
7. I learned a great deal in this course.	0	0	3	4	14	0	**4.52**	4.55	4.46	**4.45**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	1	5	7	8	0	**4.05**	4.09	3.72	**3.48**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	1	3	17	0	**4.76**	4.82	4.55	**4.45**
10. I rate the overall quality of the course as:	0	0	1	6	14	0	**4.62**	4.70	4.43	**4.26**

What do you like best about this course?:

“The combination of lecture with in-class practice of problems made it so there were few occasions in which I left class with questions remaining. ”

“I really enjoyed the active learning activities that we did. Each of the practice problems and mini quizzes helped reinforce what we learned and helped me have a better sense of what problems I understood and which I needed to review again. ”

“As a math major I love Calculus and the professors own love and enthusiasm for the topic made the course one of the best I have taken at Rutgers. ”

“The instructor explains the concepts very clearly with effective examples. He also does a great job answering student questions and clarifying any topics that may be confusing.”

“Matt is very enthusiastic.”

“the instructor put so much efforts to make sure we understand the stuff and the best thing is that he wants to make sure we absorb all of the in-class knowledge at the end of each class, which is really responsible. ”

“I like how the professor would engage the students and would always be receptive to questions. I also liked how I never was afraid to ask questions or felt stupid for asking. ”

“Everything is good.”

“How interactive everything was. Everything that was being taught, was easily understandable because we were constantly practicing them in class, at home, and through small assessments. ”

“I liked the active learning aspect of the course, I felt like being able to do the problems in class let me figure out what I really needed to work on and what I knew.”

“we have a lot of practices that helped me a lot to understand the materials.”

“Everything”

“The professor was excellent. He genuinely wanted students to learn. I appreciated his efforts. I also found that his class was engaging because of the problems we would receive at the end of each lesson. They were somewhat difficult problems that involved really understanding the material we learn in class so it got me to pay attention more. I really enjoyed the overall format of the class because it made me less lazy. It challenged me but at the same time allowed me to REALLY learn the material. I think the professor was very fair and it was actually refreshing watching him teach us. Professors with enthusiasm and effort like his allow me to feel more open to learning. Honestly, I think he is better than A LOT of the current full time mathematics professors at Rutgers; some of those professors are extremely lousy and do not deserve their positions. If I was on the administrative team... I would hire him in a heartbeat. ”

“This course helped me gain the foundations and understanding to take on more challenging upper level math courses. It broke down many concepts I couldn't understand and linked them back up in a more defined manner. ”

If you were teaching this course, what would you do differently?:

“Maybe shift the class towards being slightly more lecture based but I think the course worked well as is. Also, the Matlab component did not really aid in the understanding (outside of visualization), which I think is a result of the inconsistent software usage in Rutgers courses.”

“It was frustrating to wait until the last 5 minutes of class to get practice problems that sometimes took over 10 minutes to complete. The class period is already long enough. I didn't like staying after to complete a problem that could have been given to us just a few minutes earlier. I also think the Matlab assignments should have been due on Fridays. Having the Matlab assignments due the same day as our quizzes made it hard to find time to study. Some of the Matlab assignments took several hours to figure out so I would put off studying to work on them. If they were due on Fridays instead of Wednesdays we would have had more time to focus on one thing at a time. ”

“I honestly don't think I would do anything differently. I really enjoyed all aspects. ”

“Matlab assignments weren’t helpful since the assignments were intended to be done in order but we were skipping around. Seemed like there was important information explained in other assignments that we didn't do ”

“I would teach more examples.”

“I do not think the matlab thing really helps us because we never really learn about the basic principles and how to code in matlab and the HW just present the code and ask us to modify it, which is really harder than writing it by ourselves because we need to figure out what are the coder's thoughts. Besides, I believe that there is another way to organize the readiness assessments. It would be better for posting them online before everyday's class and then asking us to submit it at the beginning of the class.Because this book, honestly speaking, does not organize its thoughts very well and when we read about this book we feel so confused. Additionally, although we read about this book, we still do not know what the instructor wants us to know so we do not do good in RA. If the instructor can put the RA in advance, we can at least know what he want us to know and for the other confusing part, we can make it clear in the class because this instructor is clear enough.”

“I probably wouldn't give readiness assessments.”

“Maybe I wouldn't assign matlab, and have more workshop style assessments. ”

“While I really liked the active learning, I would do more example problems before giving us time to work on our own. I think if we were shown how to do one basic and one advanced type question before having to do them on our own it would've worked out a little better”

“none”

“nothing”

“I was not really pleased with the readiness assignments he gave us because they were unnecessary. I do not think we need to be tested on material based on the book that he still needed to teach us. Some students like myself have minor dyslexia and reading from the book does not necessarily help me grasp concepts very well. The professor even explained that some parts in the book were confusing yet still tested us on it. Maybe an alternative is to make readiness assignments based on concept videos to watch as homework as well? I also think some of the Matlabs were unfair. We should be tested on applying our knowledge on the CONCEPTS we've learned so far to the Matlabs; I think using intensive calculation-based numbers defeats the purpose. Furthermore, sometimes the professor gave us a problem when there were 5 minutes left in class... I do not think this was fair. ”

“Since there was a very restricted time limit on so many aspects, I wouldn't have made the course any different than the professor. He did a great job teaching and helping us practice the material. ”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“While Differential Equations is not my particular area of interest, it has proven to be useful in considering other kinds of problems, such as alternate means of comparing the relation between two factors in an experiment.”

“So many ways. I want to be a high school math teacher so every class I take I'm always learning classroom management and teaching techniques from the teacher even if it isn't an education class. This class deepened my love of calculus and my want to be a teacher. The professor is inspired, engaging and implemented many different learning techniques. ”

“Reading before class begins.”

“The quizzes and practices in this class. I have never done so much quizzes and in-class practices in this university.”

“active studying helps me a lot. ”

“He has shown me the importance of active learning in classrooms, and obviously helped me improve my knowledge of differential equations. ”

“This class encouraged my growth because the Instructor not only knew what he was doing but he knew how to teach it to the class well. Mr. Charnley is hands down one of the best instructors I've had at Rutgers because his teaching style was very easy to follow and made difficult concepts easier to grasp. ”

“good”

“This class constantly made me actively learning. Since we would have quizzes every day, I retained information better and when it came time for the midterms, I was not cramming or stressing prior to the midterms because all these quizzes and work problems really prepared me well. This is the first time in a while that I felt confident studying for a mathematics exams. On the first day of class, I remember writing that i wanted to gain my confidence for learning math back and getting over math test anxiety — I definitely did. I'm very thankful for Mr. Charney for helping me heal and get over this mental obstacle. As a human to human influence, this has really re-encouraged me to be more hopeful in this path. (:”

“I feel more driven to understand the concept of computational mathematics rather than just plug and chug. Matthew helped break down formulas and concepts which I struggled in. He helped relate things we learned in class to real life scenarios and put his heart in making sure we understood the concept. Great professor!”

Other comments or suggestions::

“The course was well taught and ultimately more enjoyable than I would have suspected. ”

“I really enjoyed this class and would recommend this summer section to anyone. It was fast passed but our professor broke it down to seem a lot less scary. I never felt afraid to ask questions or for help. This has been my favorite math class at Rutgers thus far. The professor made me not hate going to class on beautiful Friday nights, that takes a very good teacher. ”

“If you do one more example after the explanations of the context will be better ”

“For the quiz section of the grade add more quizzes or make it worth less, I think it's a little off balanced that the mini quizzes, practice problems, in class activities, and readiness assessments, and the Matlab's both account for the same amount of the grade but both areas had many more opportunities to make up for a bad assignment. ”

“great class. ”

“no”

“Suggestion: Spend more times on showing students examples rather than giving them problems to try in class before the hand-in practice problem.”

“No other comments or suggestions. ”

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew mpc163 Fall 2018, 01:640:104:02 — INTRO TO PROBABILITY (index #06597) Enrollment= 26, Responses= 13	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	0	1	12	0	**4.92**	4.87	4.37	**4.37**
2. The instructor responded effectively to student comments and questions.	0	0	0	2	11	0	**4.85**	4.87	4.21	**4.24**
3. The instructor generated interest in the course material.	1	0	1	0	10	1	**4.50**	4.77	4.08	**4.09**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	1	1	11	0	**4.77**	4.87	4.33	**4.35**
5. The instructor assigned grades fairly.	0	0	1	2	10	0	**4.69**	4.61	4.17	**4.19**
6. The instructional methods encouraged student learning.	1	0	0	2	10	0	**4.54**	4.71	3.98	**4.01**
7. I learned a great deal in this course.	0	1	1	1	10	0	**4.54**	4.68	3.96	**4.03**

Charnley Matthew mpc163 Fall 2018, 01:640:104:02 — INTRO TO PROBABILITY (index #06597) Enrollment= 26, Responses= 13	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
8. I had a strong prior interest in the subject matter and wanted to take this course.	1	0	3	0	9	0	**4.23**	4.19	3.41	**3.56**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	1	3	9	0	**4.62**	4.71	4.02	**4.04**
10. I rate the overall quality of the course as:	0	1	1	2	9	0	**4.46**	4.58	3.85	**3.91**
Part B: Questions Added by Department or Instructor										
	Strongly disagree	**Disagree**	**Uncertain**	**Agree**	**Strongly agree**					
15. The course objectives were well explained.	0	1	0	2	10	0	**4.62**	-	-	**-**
16. The course assignments were related to the course objectives.	0	1	0	1	11	0	**4.69**	-	-	**-**
17. I understood what was expected of me in this course.	0	0	1	2	10	0	**4.69**	-	-	**-**
18. The instructor encouraged me to do my best work.	0	1	0	1	11	0	**4.69**	-	-	**-**
19. I learned perspectives, principles, and practices from this course that I expect to apply to new situations.	1	0	1	1	10	0	**4.46**	-	-	**-**

Charnley Matthew mpc163 Fall 2018, 01:640:104:02 — INTRO TO PROBABILITY (index #06597) Enrollment= 26, Responses= 13 **Part A: University-wide Questions:**	**Student Responses** Strong Disagree 1				Strong Agree 5	No response	**Weighted Means** Section	Course	Level	Dept
20. The course challenged me to think critically and communicate clearly about the subject.	1	0	0	2	10	0	**4.54**	-	-	-

What do you like best about this course?:

"Nothing."

"Perfect"

"The effective teaching method"

"The material was interesting "

"I love my prof Charnley as well as the course context."

"The fairness of Professor Charnley, and his dedication to making sure we learn all the material."

"The concept of probability is very interesting and it was just interesting to learn a little more about it. "

If you were teaching this course, what would you do differently?:

"Give homework as a participation grade as many points can be lost if you make even the smallest mistake. Same with practice problems at the end of class. Also much of the homework was not what we did in class. Difficulty went from 0-100."

"Nothing"

"I would give less assignments. Also if you missed one day of class, your grade would decrease significantly."

"I will apply the information I learned in the daily life."

"I think that I would assign chapters to read in the textbook as additional material."

"If anything outside resources would be valuable. If I didn't fully understand something I would have had liked somewhere else to go and be able to spend time trying to understand that topic. "

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

"The only upside I can think of is I have never put more effort both in and out of the classroom in order to pass."

"Just math knowledge "

"I am more interested in probability "

"Yes, he is very patient and full of enegetic which makes the class very vivid."

"I learned that I can persevere and still learn something new against all odds."

"I was not particularly interested in this course due to my not finding it useful, but by the end I can see how probability relates to the workforce and how it might be applied in my field of study. So this course expanded my understanding of the importance of probability. "

Other comments or suggestions::

"You go too fast in class, the only time I somewhat understood concepts was in office hours. Also stop making exams cumulative, it's literally the worst."

":))"

"Great teacher and interesting class"

"I love him, Charnley, the best teacher I have ever seen!"

"This class was truly amazing! I would highly recommend it."

"The course was well done. Professor was very motivated to have students understand the material. Good class overall. "

Rutgers University Student Instructional Rating
(Online Survey)

Charnley M Fall 2014, 01:640:135:70 — Calculus I (index #14802) Enrollment= 30, Responses= 17 **Part A: University-wide Questions:**	Student Responses						Weighted Means			
	Strong Disagree 1				Strong Agree 5	No response	Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner	0	0	0	3	14	0	**4.82**	4.14	4.26	**4.29**
2. The instructor responded effectively to student comments and questions	0	0	0	1	16	0	**4.94**	3.97	4.14	**4.17**
3. The instructor generated interest in the course material	0	0	2	2	13	0	**4.65**	3.79	3.99	**4.01**
4. The instructor had a positive attitude toward assisting all students in understanding course material	0	0	0	1	16	0	**4.94**	4.17	4.30	**4.32**
5. The instructor assigned grades fairly	0	0	0	2	15	0	**4.88**	4.11	4.19	**4.21**
6. The instructional methods encouraged student learning	0	0	1	1	15	0	**4.82**	3.74	3.93	**3.96**
7. I learned a great deal in this course	0	0	0	4	13	0	**4.76**	3.68	3.89	**3.96**
8. I had a strong prior interest in the subject matter and wanted to take this course	0	1	3	2	11	0	**4.35**	3.34	3.45	**3.53**
	Poor				Excellent					
9. I rate the teaching effectiveness of the instructor as	0	0	0	4	13	0	**4.76**	3.74	3.93	**3.98**
10. I rate the overall quality of the course as	0	0	0	5	12	0	**4.71**	3.68	3.82	**3.87**

What do you like best about this course?:

“Teacher is very friendly; he closes the distance by memorizing every single students' name after one or two classes albeit having a total of 7,80+ students - impressive. When asking for help in office hours, he provides much assistance and simplifies explanations in a good way.”

“The small size of the recitation allowed for many questions to be answered personally. ”

“It helped review subjects in calculus class that might have otherwise been left confusing to understand”

“Matt did a great job of answering our questions, and helping us through the hard concepts. He was very relatable. ”

“Mr. Charnley knows a lot about calc and it really helped me understand the material better when he went over it in recitation.”

“The clarity in explaining the information.”

“I liked my instructor's teaching style and the way he thoroughly taught us what we needed to know before the quiz. I also appreciated his willingness to help us by having office hours and helping me over email.”

If you were teaching this course, what would you do differently?:

“Possibly check what the homework problems were to better understand the confusion that students have.”

“Nothing, I thought he did a good job.”

“Nothing”

“Keep it the same!”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“It helped as a sort of review on subjects and give the same lesson from a different teacher.”

“Attitude towards class is very positive and instructor was always in a good mood”

“He has helped me understand calculus better.”

“None”

“He helped my intellectual growth and progress by teaching me different methods to do problems, and encouraging us to ask questions. ”

Other comments or suggestions::

“None”

“Thank you for everything! ”

Rutgers University Student Instructional Rating
(Online Survey)

Charnley M Fall 2014, 01:640:135:71 — Calculus I (index #14804) Enrollment= 29, Responses= 15	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner	0	0	1	5	9	0	**4.53**	4.14	4.26	**4.29**
2. The instructor responded effectively to student comments and questions	0	0	1	3	11	0	**4.67**	3.97	4.14	**4.17**
3. The instructor generated interest in the course material	0	0	3	2	9	1	**4.43**	3.79	3.99	**4.01**
4. The instructor had a positive attitude toward assisting all students in understanding course material	0	0	1	2	12	0	**4.73**	4.17	4.30	**4.32**
5. The instructor assigned grades fairly	0	0	2	2	11	0	**4.60**	4.11	4.19	**4.21**
6. The instructional methods encouraged student learning	0	0	1	4	10	0	**4.60**	3.74	3.93	**3.96**
7. I learned a great deal in this course	0	1	1	4	9	0	**4.40**	3.68	3.89	**3.96**
8. I had a strong prior interest in the subject matter and wanted to take this course	3	1	2	3	6	0	**3.53**	3.34	3.45	**3.53**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as	0	0	1	5	9	0	**4.53**	3.74	3.93	**3.98**
10. I rate the overall quality of the course as	0	0	2	4	9	0	**4.47**	3.68	3.82	**3.87**

What do you like best about this course?:

“he was very effective in helping everyone who didn't understand the information in lecture understand it through his class”

“ TA is really helpful!”

“He goes over things quickly”

“Definitely not my grades but I do like finding things out and linking concepts. The TA is helpful and supportive of that, even when its not during his office hours I have emailed him and received a quick response with help but not a totally detailed solution-- he helps you solve the problem.”

“The effective teaching. ”

“The reviewing of the material learned in class”

“practice is helpful”

“It was a great supplement to the regular lectures. Being able to go over various questions and topics that maybe were not covered to the desired depth in lecture was greatly helpful.”

“I improved on simple algebra… ”

“That it was very easy”

“helped me understand further understand materials I learned in lecture”

If you were teaching this course, what would you do differently?:

“nothing”

“Maybe use better examples or relate more to the professor”

“I would immediately quit because I suck at calculus. But seriously I would probably not do anything differently. The TA is great.”

“Nothing. ”

“nothing”

“more practice and review”

“Nothing comes to mind.”

“I really don't know considering the fact that I can hardly do math, so probably nothing.”

“Probably make webwork more user friebdky”

“nothing”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“made me understand the content even more”

“I mean it taught me math. So. There's that.”

“He just helped me understand the material. It isn't a easy subject for me to understand, but he made it easy for me to understand. ”

“He helped clear up any of the material that was confusing to me in lecture” “practice and review”

“He worked to make sure that every student's questions were answered to their satisfaction. He helped me achieve a deeper understanding of calculus, a topic I expected to struggle with.”

“Every time I failed a quiz, it would encourage to try to do better. ”

“He taught in a very easy to understand manner”

“I know now how to study for math classes”

Other comments or suggestions::

“The TA is really nice and chill. Give him a raise or something lol”

“I would definitely recommend this TA to other students. ”

“more classes, it's too short ”

“Calculus is hard, I liked the TA, it was the subject that was difficult for me. ”

“Hes a very good TA, very helpful”

“Great job Charnley”

Rutgers University Student Instructional Rating
(Online Survey)

Charnley M Fall 2014, 01:640:135:72 — Calculus I (index #14805) Enrollment= 31, Responses= 11 **Part A: University-wide Questions:**	Student Responses						Weighted Means			
	Strong Disagree 1				Strong Agree 5	No response	Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner	0	0	0	3	7	1	**4.70**	4.14	4.26	**4.29**
2. The instructor responded effectively to student comments and questions	0	0	1	2	7	1	**4.60**	3.97	4.14	**4.17**
3. The instructor generated interest in the course material	0	0	1	1	7	2	**4.67**	3.79	3.99	**4.01**
4. The instructor had a positive attitude toward assisting all students in understanding course material	0	0	0	1	9	1	**4.90**	4.17	4.30	**4.32**
5. The instructor assigned grades fairly	0	0	0	2	8	1	**4.80**	4.11	4.19	**4.21**
6. The instructional methods encouraged student learning	0	0	2	1	7	1	**4.50**	3.74	3.93	**3.96**
7. I learned a great deal in this course	0	1	0	3	6	1	**4.40**	3.68	3.89	**3.96**
8. I had a strong prior interest in the subject matter and wanted to take this course	0	3	2	2	3	1	**3.50**	3.34	3.45	**3.53**
	Poor				Excellent					
9. I rate the teaching effectiveness of the instructor as	0	0	1	3	6	1	**4.50**	3.74	3.93	**3.98**
10. I rate the overall quality of the course as	0	0	1	4	5	1	**4.40**	3.68	3.82	**3.87**

What do you like best about this course?:

“I hate calculus, but the TA did his best in his teachings. I appreciated that.”

“The instructor was very prepared and taught in a way which assisted in the understanding of the concepts taught in lecture.”

“His teaching style”

“He went over the lectures in summary and answered specific questions”

“He was extremely helpful when anybody had questions.”

If you were teaching this course, what would you do differently?:

“Nothing.”

“I honestly wouldn't do anything differently; for an instructor teaching for the first time, he did an excellent job and his teaching methods are fine as they are.”

“Provide more explanation”

“Nothing”

“There is nothing more to do differently.”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“None.”

“I learned new ways to solve problems and it reinforced my understanding of the course material that was taught in lecture.”

“Helped me understand the concepts faster”

“Michael made a calculus easier to understand. He was very approachable and I didn't ever feel hesitant asking questions in class. I think you would make a great professor one day because he's very enthusiastic about this class and clearly cares a lot about his students. The one thing I would encourage him to do is to speak a bit more clearly when explaining the steps to a problem. Other than that I thought Michael was an excellent recitation instructor”

“Instructor answered any questions asked”

“He really helped me whenever i had a problem, thus he helped me improve on difficult questions. ”

Other comments or suggestions::

“None.”

“None.”

“He was very fun and the class was enjoyable. ”

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Spring 2015, 01:640:251:22 — Multivariable Calc (index #19841) Enrollment= 26, Responses= 12 **Part A: University-wide Questions:**	**Student Responses** Strong Disagree 1				Strong Agree 5	No response	**Weighted Means** Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner	1	0	2	0	9	0	**4.33**	4.47	4.46	**4.37**
2. The instructor responded effectively to student comments and questions	1	0	1	1	9	0	**4.42**	4.42	4.36	**4.29**
3. The instructor generated interest in the course material	1	0	2	1	8	0	**4.25**	4.24	4.16	**4.13**
4. The instructor had a positive attitude toward assisting all students in understanding course material	1	0	1	1	9	0	**4.42**	4.54	4.47	**4.41**
5. The instructor assigned grades fairly	1	0	0	3	8	0	**4.42**	4.32	4.35	**4.24**
6. The instructional methods encouraged student learning	1	0	1	3	7	0	**4.25**	4.16	4.13	**4.08**
7. I learned a great deal in this course	1	0	1	2	8	0	**4.33**	4.26	4.24	**4.10**
8. I had a strong prior interest in the subject matter and wanted to take this course	1	0	3	1	7	0	**4.08**	4.03	3.94	**3.62**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as	2	0	0	1	9	0	**4.25**	4.27	4.20	**4.11**
10. I rate the overall quality of the course as	1	1	1	1	8	0	**4.17**	4.14	4.14	**3.99**

What do you like best about this course?:

“Matt has a very upbeat personality and really seems like he enjoy teaching.”

“The TA.”

“Professor Charnley was very helpful”

“I liked the way he taught us it was very easy and i didnt had any difficulty understanding him ”

If you were teaching this course, what would you do differently?:

“Alternate quizzes and maple labs every week. It was like that for a while but then we would have a quiz and lab due the same day.”

“Nothing.”

“Nothing”

“I would do reviews before every exams ”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“Eh.”

“It has allowd me to study harder ”

Other comments or suggestions::

“Went to office hours ever change I got, learned everything there and nothing in lecture”

“stop being condescending ”

“Overall this is a very nice class ”

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Spring 2015, 01:640:251:23 — Multivariable Calc (index #19842) Enrollment= 28, Responses= 8	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner	0	0	0	0	8	0	**5.00**	4.47	4.46	**4.37**
2. The instructor responded effectively to student comments and questions	0	0	0	0	8	0	**5.00**	4.42	4.36	**4.29**
3. The instructor generated interest in the course material	0	0	1	2	5	0	**4.50**	4.24	4.16	**4.13**
4. The instructor had a positive attitude toward assisting all students in understanding course material	0	0	0	0	8	0	**5.00**	4.54	4.47	**4.41**
5. The instructor assigned grades fairly	0	1	0	1	6	0	**4.50**	4.32	4.35	**4.24**
6. The instructional methods encouraged student learning	0	0	0	2	6	0	**4.75**	4.16	4.13	**4.08**
7. I learned a great deal in this course	0	0	1	2	5	0	**4.50**	4.26	4.24	**4.10**
8. I had a strong prior interest in the subject matter and wanted to take this course	0	0	1	1	6	0	**4.63**	4.03	3.94	**3.62**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as	0	0	0	1	7	0	**4.88**	4.27	4.20	**4.11**
10. I rate the overall quality of the course as	0	0	1	1	6	0	**4.63**	4.14	4.14	**3.99**

What do you like best about this course?:

"Charnleye xplanation"

"I liked that it was not an overbearing workload, but it still taught me alot."

"Everything"

"the recitations summarized what was gone over in lecture and presented the material in a different, more to the point way. "

"I'll love math to my death, and this TA did everything to make sure the students understood"

If you were teaching this course, what would you do differently?:

"Na"

"The recitations were a perfect supplement to the lectures, which were sometimes very confusing. Matt usually had a much clearer and organized way of teaching and explaining things."

"Nothing, the instructor does a great job at making recitation what it's supposed to be."

"Nothing"

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

"explanined many diffilcut concepts intuitiveyl in reciation"

"Taught me how to integrate e^x^2 which was really cool"

"Further encouraged my mathematics major"

Other comments or suggestions::

"Na"

"Great teaching style and super helpful."

"Great guy and great instructor!"

"very helpful, a caring TA and enthusiastic about the material."

"Nothing"

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Spring 2015, 01:640:251:24 — Multivariable Calc (index #19843) Enrollment= 24, Responses= 9 **Part A: University-wide Questions:**	**Student Responses** **Strong Disagree 1**				**Strong Agree 5**	**No response**	**Weighted Means** **Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner	0	0	0	0	9	0	**5.00**	4.47	4.46	**4.37**
2. The instructor responded effectively to student comments and questions	0	0	0	0	9	0	**5.00**	4.42	4.36	**4.29**
3. The instructor generated interest in the course material	0	0	1	1	7	0	**4.67**	4.24	4.16	**4.13**
4. The instructor had a positive attitude toward assisting all students in understanding course material	0	0	0	1	8	0	**4.89**	4.54	4.47	**4.41**
5. The instructor assigned grades fairly	0	0	0	1	8	0	**4.89**	4.32	4.35	**4.24**
6. The instructional methods encouraged student learning	0	0	1	0	8	0	**4.78**	4.16	4.13	**4.08**
7. I learned a great deal in this course	0	0	0	1	8	0	**4.89**	4.26	4.24	**4.10**
8. I had a strong prior interest in the subject matter and wanted to take this course	0	0	0	2	7	0	**4.78**	4.03	3.94	**3.62**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as	0	0	0	2	7	0	**4.78**	4.27	4.20	**4.11**
10. I rate the overall quality of the course as	0	0	0	0	9	0	**5.00**	4.14	4.14	**3.99**

What do you like best about this course?:

“Information was presented in an organized manner, and explanations were clear”

“the instructor made the material easier to understand”

If you were teaching this course, what would you do differently?:

“Mr. Charnley is the best TA I've had so far. He always has positive attitude towards his students and is always willing to help. One comment though: Mr. Charnley used the same examples during recitations as professor did which was not that helpful. Other then that, great teacher! Thank you”

“i would remind students the day before class to have questions ready to ask during class”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“he has helped me learn to take a complicated concept and break it down into simpler parts”

“TA Charnley taught us the meaning behind our math, explaining to us the proofs and applications of the theories we learned.”

Other comments or suggestions::

“Best TA I've ever had. Very helpful.”

“Thank you so much for a great semester! You made the material so much easier to understand and you made calc 3 as fun as it could possibly be.”

“you're awesome”

“Excellent job on part of the TA.”

“Awesome. Takes time to schedule office hours with students ”

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Fall 2015, 01:640:251:H1 — Multivariable Calc (index #03919) Enrollment= 10, Responses= 8 **Part A: University-wide Questions:**	**Student Responses**						**Weighted Means**			
	Strong Disagree 1				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	1	0	7	0	**4.75**	4.35	4.25	**4.30**
2. The instructor responded effectively to student comments and questions.	0	0	1	0	7	0	**4.75**	4.29	4.19	**4.21**
3. The instructor generated interest in the course material.	0	0	1	0	7	0	**4.75**	4.13	4.00	**4.06**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	1	0	7	0	**4.75**	4.47	4.33	**4.34**
5. The instructor assigned grades fairly.	0	0	1	1	6	0	**4.63**	4.29	4.21	**4.21**
6. The instructional methods encouraged student learning.	0	0	1	1	6	0	**4.63**	4.03	3.95	**3.98**
7. I learned a great deal in this course.	0	0	1	0	7	0	**4.75**	4.17	4.08	**3.98**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	0	1	2	5	0	**4.50**	3.95	3.87	**3.56**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	0	0	8	0	**5.00**	4.08	3.98	**4.00**
10. I rate the overall quality of the course as:	0	0	0	2	6	0	**4.75**	3.99	3.92	**3.90**

What do you like best about this course?:

> "While I learned virtually nothing from the lectures, I benefited greatly from the recitations."
>
> "I liked that we reviewed homework problems in detail and discussed basic concepts of each section in detail. I was able to understand most topics covered."
>
> "It cleared up anything I didn't understand in lecture. Was very clear and easy to follow."
>
> "The recitation. It was by far the most useful tool available for understanding the material due to the instructor."
>
> "This course has shown me the origins of the mathematical formulas and concepts from the Multivariable Calculus H1 lecture in a very simple manner."
>
> "I liked how Matt was always positive in class and clearly explained the material."

If you were teaching this course, what would you do differently?:

> "I would not do anything differently."
>
> "nothing."
>
> "I wouldn't change a thing."
>
> "I would show some more applications of the mathematical concepts that allow for them."
>
> "None"

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

> "The instructor helped me to fully understand the material that I did not comprehend in lecture."
>
> "Encouraged good work ethic because homework is gone over in class, so it is beneficial to have it mostly completed."
>
> "This course and instructor have shown me that higher level courses in mathematics can in fact tell one more about the basic roots of math itself."
>
> "He made the material much easier to learn, which made me more motivated to listen and stay attentive in class."

Other comments or suggestions::

> "Recitation was very helpful and the instructor's teaching style was clear and easy to understand. "
>
> "Very effective!"
>
> "Overall excellent TA"
>
> "None"

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Fall 2015, 01:640:251:H2 — Multivariable Calc (index #07513) Enrollment= 17, Responses= 8 **Part A: University-wide Questions:**	**Student Responses** **Strong Disagree 1**				**Strong Agree 5**	**No response**	**Weighted Means** **Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	0	0	8	0	**5.00**	4.35	4.25	**4.30**
2. The instructor responded effectively to student comments and questions.	0	0	0	0	8	0	**5.00**	4.29	4.19	**4.21**
3. The instructor generated interest in the course material.	0	0	1	2	5	0	**4.50**	4.13	4.00	**4.06**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	0	8	0	**5.00**	4.47	4.33	**4.34**
5. The instructor assigned grades fairly.	0	0	0	1	7	0	**4.88**	4.29	4.21	**4.21**
6. The instructional methods encouraged student learning.	0	0	2	0	6	0	**4.50**	4.03	3.95	**3.98**
7. I learned a great deal in this course.	0	0	0	2	6	0	**4.75**	4.17	4.08	**3.98**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	0	0	1	7	0	**4.88**	3.95	3.87	**3.56**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	0	1	7	0	**4.88**	4.08	3.98	**4.00**
10. I rate the overall quality of the course as:	0	0	1	0	7	0	**4.75**	3.99	3.92	**3.90**

What do you like best about this course?:

“This recitation was a lot more clear on the material than the lecture.”

“He explained everything so much better than the professor. Matt always made himself available to help us learn more outside of the classroom. He knows exactly what he is talking about. In one recitation I learn more than in two lectures from the professor. ”

“I liked two things: that everything was covered again so that it was better learned and that multiple approaches were shown for a few difficult things.”

“Matt was one of the best math TAs I've ever had. He was detailed and extremely clear with all the lessons that he taught. ”

“Matt was fantastic, the best TA i have had at Rutgers. ”

“Matt helped us understand the course material very well. He often made up for the instructor's shortcomings.”

“Mr. Charnley was very clear with everything he taught and was good at simplifying concepts we may not have understood at first.”

If you were teaching this course, what would you do differently?:

“N/A”

“Nothing. ”

“Maybe a few details on the more abstract things. Better coordination with the professor, if possible.”

“nothing at all”

“Nothing”

“I think it did a perfect job as a TA.”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“Instructor taught with clarity and even created an online review session before an exam.”

“He made calc entertaining and easy to learn. Instead of telling us the material was extremely tough he would make it very doable and provided positive encouragement. ”

“This course offered a nice tie-up of what the professor showed and made the material much more accessible.”

“Matt has really sparked my interest in mathematics and made multivariable calculus an easy to understand and fun course.”

“Gave me a different insight to Calculus not seen in the book”

“In many ways. :O”

“I wasn't really pushed towards growth outside the class, but I don't really see a problem with that in terms of this class.”

Other comments or suggestions::

“He should be a professor. ”

“Matt was a fantastic TA. :)”

“Mr. Charnley did a fantastic job. He was very helpful and knowledgeable, and very personable.”

Rutgers University Student Instructional Rating
(Online Survey)

Charnley Matthew Fall 2015, 01:640:251:H3 — Multivariable Calc (index #09825) Enrollment= 9, Responses= 6 **Part A: University-wide Questions:**	Student Responses Strong Disagree 1				Strong Agree 5	No response	Weighted Means Section	Course	Level	Dept
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	0	1	5	0	**4.83**	4.35	4.25	**4.30**
2. The instructor responded effectively to student comments and questions.	0	0	0	0	6	0	**5.00**	4.29	4.19	**4.21**
3. The instructor generated interest in the course material.	0	0	1	1	4	0	**4.50**	4.13	4.00	**4.06**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	1	5	0	**4.83**	4.47	4.33	**4.34**
5. The instructor assigned grades fairly.	0	0	0	0	6	0	**5.00**	4.29	4.21	**4.21**
6. The instructional methods encouraged student learning.	0	0	0	2	4	0	**4.67**	4.03	3.95	**3.98**
7. I learned a great deal in this course.	0	0	1	1	4	0	**4.50**	4.17	4.08	**3.98**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	0	2	2	2	0	**4.00**	3.95	3.87	**3.56**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	0	0	6	0	**5.00**	4.08	3.98	**4.00**
10. I rate the overall quality of the course as:	0	0	0	1	5	0	**4.83**	3.99	3.92	**3.90**

What do you like best about this course?:

“He clearly explained lecture material as well as how to solve difficult homework problems. It was very helpful. ”

“The teacher used colored chalk to draw easily understandable pictures, took all of our homework questions, and covered all of the material that was covered in lecture clearly and in a different way than the professor, solidifying and enhancing our understanding.”

“I liked resuscitation the best because it's awesome!”

“I liked that it corresponded well with what I was learning in Physics class.”

“I liked how everything was always explained very clearly with graphs usually and there were never any tricks on the quizzes.”

If you were teaching this course, what would you do differently?:

“n/a”

“Nothing.”

“More Michigan jokes.”

“More communication between the professor and the TA”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“I can understand how certain formulas and theorems were derived, so I understand the material much better. ”

“The instructor did a great job in answering questions from the homework and explained the subject matter in a very clear and understandable way that allowed me to better understand the subject.”

“Class was not only informative and interesting but fun as well, making me excited for calculus class each week.”

“The instructor has encouraged us to go on and use multivariable calculus to the best of our abilities! Thanks!”

“Mr. Charnley is really knowledgeable and does his best to help us fully understand the topics presented in class. He is so prepared that he even has the difficult homework problems memorized. He has also brought in colored chalk that really helped us picture the curves and surfaces we were supposed to integrate.”

Other comments or suggestions::

“Use more colored chalk, it helps a lot.”

“:)”

“Thank you very much for teaching me this semester”

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew mpc163 Fall 2017, 01:640:421:04 — ADV CALC FOR ENGRNG (index #05351) Enrollment= 87, Responses= 32	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	1	6	17	8	**4.67**	4.30	4.27	**4.34**
2. The instructor responded effectively to student comments and questions.	0	0	1	6	20	5	**4.70**	4.27	4.26	**4.22**
3. The instructor generated interest in the course material.	0	0	4	4	19	5	**4.56**	4.08	4.10	**4.05**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	1	7	21	3	**4.69**	4.44	4.40	**4.33**
5. The instructor assigned grades fairly.	0	1	6	3	14	8	**4.25**	4.14	4.19	**4.17**

6. The instructional methods encouraged student learning.	0	0	2	7	18	5	**4.59**	4.16	4.07	**3.99**
7. I learned a great deal in this course.	1	0	6	5	17	3	**4.28**	4.12	4.11	**4.02**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	2	6	6	17	1	**4.23**	3.91	4.05	**3.61**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	4	6	17	5	**4.48**	4.08	4.07	**4.02**
10. I rate the overall quality of the course as:	0	1	6	10	12	3	**4.14**	3.99	3.99	**3.90**
Part B: Questions Added by Department or Instructor										
	Strongly disagree	**Disagree**	**Uncertain**	**Agree**	**Strongly agree**					
15. The TA was a valuable resource for this course.	0	0	2	3	22	5	**4.74**	-	-	**-**
16. The online office hours were a valuable resource for this course.	0	0	5	0	18	9	**4.57**	-	-	**-**
17. The recordings of the online office hours were helpful for this course.	0	0	4	3	16	9	**4.52**	-	-	**-**

What do you like best about this course?:

“this is interesting.”

“I enjoyed the earlier material and the application of using Laplace Transform in solving differential equations. ”

“Matt is the best TA I have ever had and is one of the smartest people I have ever talked to. No matter how poorly you word a question, Matt somehow knows what you mean and has an answer that goes above and beyond. Matt effortlessly guides students from no understand and complete confusion to clarity and competence. Matt has been by far the best thing about the class and I wish him the all the best in the future.”

“I did not attend any office hours or reach out to Matthew Charnley (TA). However, from all the emails he has sent and trying to keep everyone updated on how they can contact him for more help shows that he was dedicated to trying to aid the students. ”

“I liked how the material was presented in a straightforward way.”

“I liked all of the review sessions.”

“N/A - See Terrence Butler”

“Topics toward the end of the class were fairly interesting”

“It deals with various information that can be applied in real life.”

“N/A”

“The TA is very knowledgeable about the subject and tries his best to help the students the best he can.”

“The time of the course ”

If you were teaching this course, what would you do differently?:

“I think this proffesor’s teaching is pretty effective i don’t want to do something different. ”

“I would make the latter half of the material presented in a manner which was more easily understood. There seemed to be a lot of different cases for solving Boundary Value Problems which kind somewhat blended together in terms of applying different techniques. ”

“offer recitation to some sections at least ”

“I think he did a good job from what I could tell. As stated above, I did not get to experience his teaching style first hand. ”

“I would not change anything about the course.”

“Recitations would have been helpful.”

“It would have been nice to see Matt in class more because he was always so helpful (also so that he can actually keep up with what we are doing in the class so Butler doesn't surprise him)”

“How homework was graded”

“Have everything in the class build upon each other.”

“N/A”

“There was a lack of communication between the instructor and the TA which lead to uncertainties regarding the material on the exams.”

“I would give extra credit for group study ”

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

“he give full attention in iffice hours and help to deal with challenges we face in this course.”

“No matter how hard Rutgers makes me want to hate math, Matt keeps that light alive because he is terrific.”

“Matthew Charnley has not encouraged my intellectual growth or progress. This is not due to his style of teaching but more because of the lack of meetings I have had with him. ”

“He is always so enthusiastic about everything and he always has a smile on his face. I enjoy being in the same room as him since he truly does bring a positive energy wherever he goes. ”

“Always helpful and showed me the "behind the scenes" of how we could get to certain conclusions, made me more interested in math”

“None”

“The enthusiasm of the instructor made the class more interesting.”

“N/A”

“Tough grader”

Other comments or suggestions::

“overall excellent professor.”

“Matthew Charnley seemed enthusiastic and flexible to the students. He was very active and I am sure that carried over to his teaching style. ”

“Great guy.”

“Keep doing the workshop and review sessions, they are very helpful”

“One of the best TA's that I've had so far at Rutgers”

“Professor shouldn’t make mistakes in class to much. Students are supposed to ”

What was your level of interaction with the TA?:

“Average”

“not that bad”

“I did not go to any of the TA's office hours, nor did I participate in the online office hours, although I did look at some of the notes which he posted and attempted to watch some of the online office hours after they had been conducted for further insight into topics which I saw were covered in that office hour.”

“Fairly often”

“frequent ”

“I never went to office hours for this course.”

“I rarely interacted with the TA. My class/work schedule conflicted with the TA's availability.”

“Office hours fairly often”

“Occasional office hours and review sessions”

“None”

“high”

“Moderate”

“Occasionally asked him a question related to a concept or homework question outside of class”

“Very minimal but good when I had to.”

“Almost none, except for review sessions before exams”

“Went almost every week”

“10/10”

Do you think there should be a TA for this course? Why or why not?:

“Yes, extra help is always great. Also, students can get a different explanation or examples in order to better understand the problem.”

“i don’t think TA is needed for this course.”

“I think there definitely needs to be a TA for the course because it does get very complicated and I think there needs to be someone to go to besides the professor to get assistance form. Professor Butler is also quite old and therefore may not be as reliable during his office hours as some of the TAs would be”

“YES ”

“Yes I think the TA was useful to many students.”

“I personally did not require a TA for this course. However, a TA in any math course should be required. Students have different learning preferences. Some may not find lectures effective but much rather 1 on 1 or small group learning environments which is why a TA proves useful in this course. ”

“Yes because the professor was awful”

“Yes, very helpful for homework questions and to get a second way to explain how to approach the problems”

“Yes because there should definitely be someone else to teach the material besides the professor since the professor makes so many mistakes in every example he does. ”

“yes, to help you understand the material you don't get from the lecture. ”

“Yes. Review sessions were helpful! It was nice having additional resources.”

“Absolutely, Matt was an invaluable resource and was always helpful at explaining things that could not be cleared up from the class or book”

“Yes because the professor may be busy dealing with other students and work.”

“Yes, there should be a TA. The exam review sessions are very helpful.”

“Yes, because the TA's sometimes explain concepts in a way that the professor can't, and the way the TA explains it just clicks.”

“Yes, it is very helpful and is a necessity. ”

“They should because the professor can’t teach ”

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew mpc163 Fall 2017, 01:640:421:05 — ADV CALC FOR ENGRNG (index #09872) Enrollment= 82, Responses= 22	**Student Responses**						**Weighted Means**			
Part A: University-wide Questions:	**Strong Disagree 1**				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	3	4	9	6	**4.38**	4.30	4.27	**4.34**
2. The instructor responded effectively to student comments and questions.	0	0	3	3	12	4	**4.50**	4.27	4.26	**4.22**
3. The instructor generated interest in the course material.	0	0	4	8	6	4	**4.11**	4.08	4.10	**4.05**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	3	3	13	3	**4.53**	4.44	4.40	**4.33**
5. The instructor assigned grades fairly.	0	0	3	7	8	4	**4.28**	4.14	4.19	**4.17**

6. The instructional methods encouraged student learning.	0	0	4	6	8	4	**4.22**	4.16	4.07	**3.99**
7. I learned a great deal in this course.	0	1	5	5	8	3	**4.05**	4.12	4.11	**4.02**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	2	7	3	6	4	**3.72**	3.91	4.05	**3.61**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	3	7	8	4	**4.28**	4.08	4.07	**4.02**
10. I rate the overall quality of the course as:	0	1	6	6	6	3	**3.89**	3.99	3.99	**3.90**
Part B: Questions Added by Department or Instructor										
	Strongly disagree	**Disagree**	**Uncertain**	**Agree**	**Strongly agree**					
15. The TA was a valuable resource for this course.	0	1	3	3	11	4	**4.33**	-	-	**-**
16. The online office hours were a valuable resource for this course.	1	1	7	3	4	6	**3.50**	-	-	**-**
17. The recordings of the online office hours were helpful for this course.	1	0	8	3	4	6	**3.56**	-	-	**-**

What do you like best about this course?:

"Good sense of humor with a very positive attitude toward students."

"As a lover of calculus, I found the topics very interesting "

"Matt gives the most practical essence of the course"

"The organization of material"

"He held review sessions before our exams, which allowed us to go over key problems that we had questions on. This greatly helped prepare us for the exam."

If you were teaching this course, what would you do differently?:

"Nothing."

"Be more accessible to the students "

"Do it more often, I wish this course had a recitation"

"More help regarding PDEs "

"I would not do anything differently."

In what ways, if any, has this course or the instructor encouraged your intellectual growth and progress?:

"None."

"It has not "

"None"

"The TA was very knowledgeable about the material and helped answer our questions. "

Other comments or suggestions::

"I didn't see the ta at all this semester "

"NA"

What was your level of interaction with the TA?:

"None."

"Didn't know him "

"I didn't really need to go see the TA for anything but when we came into class a few times, he seemed to want to make himself as available as possible. He offered online resources and a lot of extra help which I thought was a very good idea for this class."

"went to the ta before the test"

"Went to the review sessions"

"Attended review sessions for exams I and II and the final exam workshops"

"I attended the exam review sessions and they were pretty helpful. "

"Frequent during exams"

"I didn't go to office hours, but I went to all the review sessions and found them very helpful. "

“I went to his review sessions for the exams.”

“very low; I have never seen him in action because I do not go to his office hours.”

“not a lot”

Do you think this course should have a TA? Why or why not?:

“Yes, it helps students digest what they learned.”

“Yes. Easier access to one on one help.”

“It should have a recitation where the ta teaches the material ”

“Yes. The material is difficult sometimes and if a student needed help, having a TA would definitely help”

“I think the TA should be more involved”

“Yes, this course should have a TA. ”

“Yes, this course should have recitations.”

“Yes because sometimes the professor can't cover everything in a clear way and the TA is great to answer any questions that the students might have. ”

“Yes. The TA taught me more material than the professor. ”

“Yes, there should be a TA for the class. Having an extra person that can explain concepts differently from the professor is very helpful and useful. The TA provides a different perspective and approach to problems that are helpful for topics we do not fully understand.”

“Yes, considering that a lot of people in this course asks for help all the time.”

“Doesn't really matter for me tbh. I never went to any office hour etc. I'm doing fine. ”

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew Spring 2018, 16:642:574:01 — Numerical Analysis II (index #05056) Enrollment= 21, Responses= 10 **Part A: University-wide Questions:**	**Student Responses**						**Weighted Means**			
	Strong Disagree 1				**Strong Agree 5**	**No response**	**Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	1	0	9	0	**4.80**	4.90	4.76	**4.77**
2. The instructor responded effectively to student comments and questions.	0	0	0	0	10	0	**5.00**	5.00	4.82	**4.83**
3. The instructor generated interest in the course material.	0	0	1	0	9	0	**4.80**	4.89	4.70	**4.71**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	1	9	0	**4.90**	4.95	4.76	**4.77**
5. The instructor assigned grades fairly.	0	0	0	0	10	0	**5.00**	5.00	4.78	**4.78**
6. The instructional methods encouraged student learning.	0	0	0	0	10	0	**5.00**	5.00	4.73	**4.73**
7. I learned a great deal in this course.	0	0	1	0	9	0	**4.80**	4.90	4.61	**4.62**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	0	0	1	9	0	**4.90**	4.90	4.66	**4.67**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	0	1	9	0	**4.90**	4.90	4.73	**4.73**
10. I rate the overall quality of the course as:	0	0	1	0	9	0	**4.80**	4.85	4.69	**4.69**

What do you like best about this course?:

> “Matt is very nice and his solution is very explicit and easy to understand. He really put a lot of effort to the course”

If you were teaching this course, what would you do differently?:

> “I may be trying to communicate with the professor more and trying to know what professor should expect us to learn and what the exam will look like etc.”

Other comments or suggestions::

> “Matt helps me a lot in the problem sessions.Hope you can tell more about Matlab coding.”

Rutgers University Student Instructional Rating
(Online Survey - Sakai)

Charnley Matthew Spring 2018, 16:642:575:01 — Numerical Solutions of Partial Differential Equati (index #06743) Enrollment= 19, Responses= 4 **Part A: University-wide Questions:**	**Student Responses** **Strong Disagree 1**				**Strong Agree 5**	**No response**	**Weighted Means** **Section**	**Course**	**Level**	**Dept**
1. The instructor was prepared for class and presented the material in an organized manner.	0	0	1	2	1	0	**4.00**	4.14	4.76	**4.77**
2. The instructor responded effectively to student comments and questions.	0	0	0	2	2	0	**4.50**	4.29	4.82	**4.83**
3. The instructor generated interest in the course material.	0	0	0	2	2	0	**4.50**	4.29	4.70	**4.71**
4. The instructor had a positive attitude toward assisting all students in understanding course material.	0	0	0	2	2	0	**4.50**	4.14	4.76	**4.77**
5. The instructor assigned grades fairly.	0	0	0	3	1	0	**4.25**	4.14	4.78	**4.78**
6. The instructional methods encouraged student learning.	0	0	0	3	1	0	**4.25**	3.86	4.73	**4.73**
7. I learned a great deal in this course.	0	0	1	2	1	0	**4.00**	3.86	4.61	**4.62**
8. I had a strong prior interest in the subject matter and wanted to take this course.	0	0	1	2	1	0	**4.00**	4.00	4.66	**4.67**
	Poor				**Excellent**					
9. I rate the teaching effectiveness of the instructor as:	0	0	0	2	2	0	**4.50**	4.29	4.73	**4.73**
10. I rate the overall quality of the course as:	0	0	0	3	1	0	**4.25**	4.00	4.69	**4.69**

Other comments or suggestions::

> "For the first half of the semester there seemed to be a bit of a disconnect between teacher and teacher's assistant. Once Matt took over the class it improved, as he knew what was going on in both."
