Active Learning in Calculus: Clickers and Worksheets

RUTGERS

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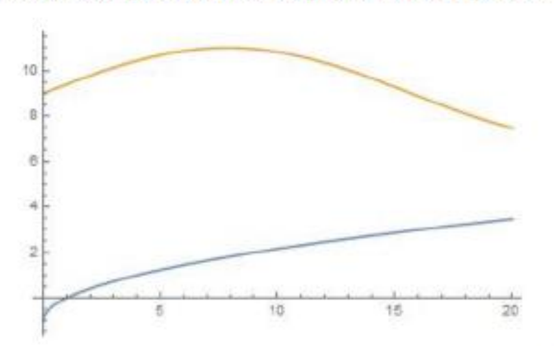
Course Format

- Lectures of 75-105 students
- Each lecture is paired with 3 recitations/workshops
- Workshops involve group problem solving
- Lectures are mostly focused on content delivery

2. Students in a Calculus class are given two functions, graphed in the image below.

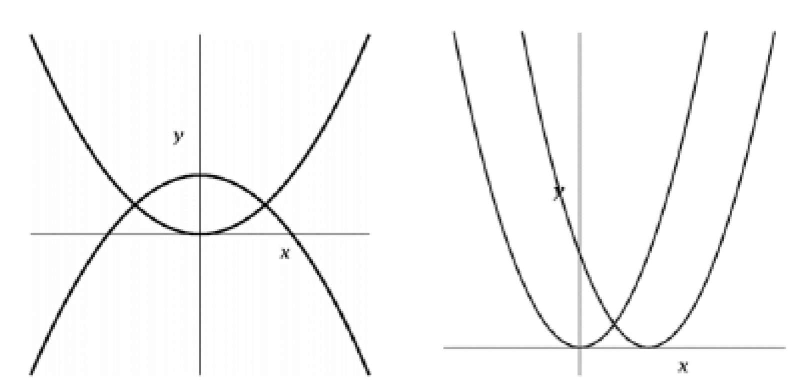


The students were asked to find the area between the curves on $[5, 15]$. Their answers are below:

- Student A: 25
- Student B: 100
- Student C: 75
- Student D: -75

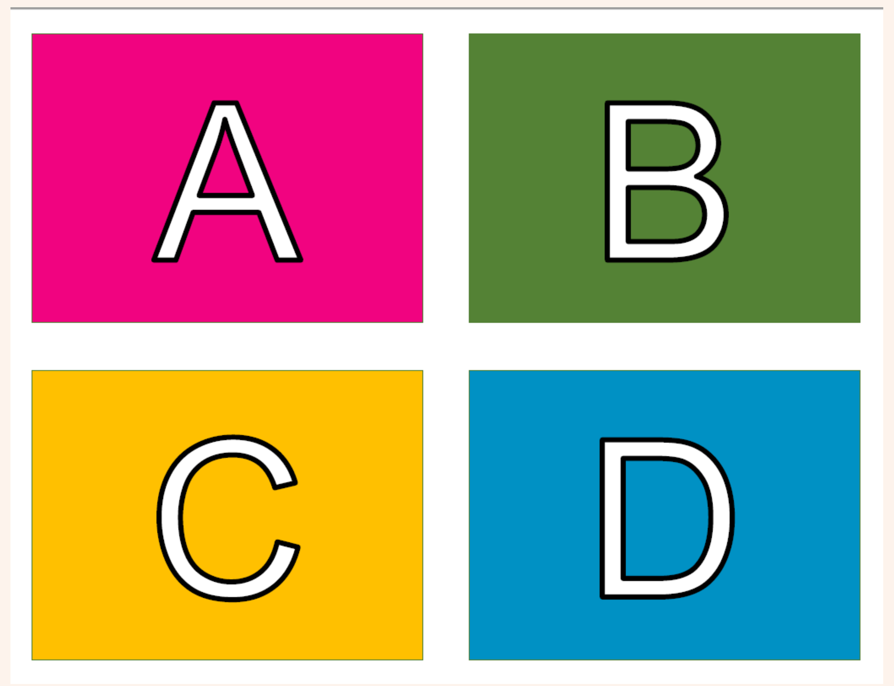
Only one student got the problem correct. Which student got it correct? What mistake did each of the other students likely make? Justify your claims.

1. This problem investigates the intersection of two graphs.
   (a) Find the value of the positive constant C so that the parabolas $y = x^2$ and $y = C - x^2$ (shown in the picture below on the left) intersect at right angles (that is, the graphs are *orthogonal*).
   (b) Find the value of the positive constant D so that the parabolas $y = x^2$ and $y = (D - x)^2$ (shown in the picture below on the right) intersect at right angles (that is, the graphs are *orthogonal*).



Clicker Questions

- Multiple choice questions to break up lecture
- Check for understanding
- Gets students thinking and solving problems
- Part of a participation grade



Which of the following is an equilibrium solution for the system

$$\frac{dx}{dt} = (x + y^2)(y - 3)$$
$$\frac{dy}{dt} = (y - 1)(x + 4)$$

A. (0,0)
B. (1,1)
C. (-9, 3)
D. (-4,2)

$\frac{dx}{dt} = 0 \to y^2 = -x \quad y = 3$

$\frac{dy}{dt} = 0 \quad y = 1 \quad x = -4$

$x = -4$

$y^2 = -x \quad y^2 = 4$

$y = 1 \quad y = \pm 2$

Worksheets

- Problems for students to work with neighbors in class
- Break up the lecture and encourage discussion
- Discuss answers afterwards
- Use pre-class videos to make time

Math 151: Worksheet 9

March 5, 2020

Part 1: Read each problem and figure out what quantities are changing. Then, draw a picture that models the situation and write an equation that relates all of these changing quantities to each other. (Hint: Triangles)

Part 2: Now solve the related rates problems given.

1. Car A passed through an intersection at 1pm at a speed of 48 mph heading north. Car B passes through the same intersection exactly 1 minute later at a speed of 45 mph heading east, and both cars continue at the same speed and in the same direction for the next 10 minutes. At exactly 1:05pm, how fast is the distance between the cars changing?
2. A 6 foot tall man is walking away from a 15 foot tall lamppost at a rate of 5 ft/sec. When he is 8 feet from the lamppost, how long is his shadow? How fast is the length of his shadow changing at this time?
3. A 10 foot ladder is leaning up against a wall, and the base begins to slide away from the wall at a rate of 1.5 ft/sec. How fast is the top of the ladder moving down the wall when the base is 6 feet away from the wall?

In Class Activity

Compute each of the following integrals in terms of R, which is one of the limits of integration.

1. $\int_4^R e^{-2x}\,dx$

   $-\frac{1}{2}(e^{-2R} - e^{-8})$
2. $\int_3^R x^2\,dx$

   $\frac{R^3}{3} - 9$
3. $\int_3^R \frac{1}{\sqrt{x}}\,dx$
4. $\int_1^R x^{-3}\,dx$
5. $\int_R^4 \frac{1}{\sqrt{x}}\,dx$
6. $\int_R^5 x^{-3}\,dx$
7. $\int_2^R \frac{1}{\sqrt{4-x}}\,dx$
8. $\int_R^0 e^{-x}\,dx$

Online Format

- Zoom Polls and Canvas Quizzes
- Worksheets and Breakout Rooms were less common
- Practice Problems at the end of class
- Time was more of an issue online

Practice Problem

Convert $(3, \sqrt{3})$ to polar coordinates.

Convert $(2, \frac{3\pi}{4})$ to rectangular coordinates.

Find a rectangular coordinate equation that is equivalent to the polar equation $r = \frac{1}{2 - \cos\theta}$.

$r = \sqrt{3^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12}$

Practice Problem

Find the general solution of and sketch a phase portrait for the system of differential equations

$$\vec{x}\,' = \begin{bmatrix} 2 & 1 \\ -4 & -3 \end{bmatrix} \vec{x}.$$

Find eigenvalues