Math 244 Learning Objectives

Core Objectives

1. I can draw or identify a direction field for first order differential equations and use that to discuss the behavior of solutions to a differential equation.
2. I can set up and solve applied modeling problems involving first order differential equations.
3. I can determine the type and order of a differential equation.
4. I can verify that a function does (or does not) solve a differential equation.
5. I can use integrating factors to solve first order linear differential equations, including those that can only be written using definite integrals.
6. I can solve separable differential equations and determine the interval on which such a solution is valid.
7. I can apply the existence and uniqueness theorems to determine the largest interval on which a first or second order linear DE is guaranteed to exist, and understand what it means for the theorems to not apply.
8. I can draw a phase line for an autonomous differential equation and use this to discuss the behavior of solutions and stability of critical points. I can sketch a representative set of solutions to these equations.
9. I can use Euler's method to numerically approximate the solution to a differential equation.
10. I can find the general solution to a second order linear, constant-coefficient, homogeneous differential equation as well as solve initial value problems of this type. This includes real and distinct roots, complex roots, and repeated roots.
11. I can solve second order, linear, non-homogeneous equations using the method of undetermined coefficients or variation of parameters.
12. I can analyze physical problems involving damped and undamped motion, including mechanical and electrical vibrations, generating and solving equations of this type.
13. I can properly utilize terminology around mechanical and electrical motion, including "transient", "steady-state solution", "underdamped", "overdamped", "critically damped".
14. I can solve a forced vibration problem, including an analysis of beats and resonance.
15. I can perform basic operations on matrices.
16. I can write a linear system of equation in matrix notation and use this to solve linear systems of equations or determine that the system has no solution.
17. I can row reduce a matrix in order to analyze matrices and linear systems of equations.
18. I can find eigenvalues and eigenvectors of small (2x2) matrices.
19. I can verify that a set of vector functions solves a given system of differential equations.
20. I can draw or identify a direction field in the phase plane for an autonomous system of differential equation and use this to discuss the behavior of solutions to the system of differential equations.
21. I can find the general solution and solve initial value problems for linear, constant coefficient homogeneous systems of differential equations (2x2). I can sketch phase planes or direction fields for these problems. This includes real and distinct eigenvalues, complex eigenvalues, and repeated eigenvalues.
22. I can find critical points for an autonomous, two-component system and use locally linear analysis to determine the stability and type of each of these critical points.

Supplementary Objectives

More Linear Algebra

1. I can find the determinant of a matrix and compute the inverse of a matrix.
2. I can find eigenvalues and eigenvectors of 3x3 matrices.
3. I can determine whether vectors or vector functions are linearly independent.

Other Types of Equations and Methods

1. I can solve a Bernoulli Equation using an appropriate substitution.
2. I can determine if an equation is exact or can be made exact using an integrating factor, and use that to solve these equations.
3. I can identify equations of Euler-type and find the solution these equations.
4. I can find critical points for an autonomous, two-component system and use the direction field to determine stability of critical points and approximate the basis of attraction for any asymptotically stable critical points.
5. I can determine periodic solutions to an autonomous system, find limit cycles, and determine their stability.
6. I can find the solution to a Hamiltonian System.

Theoretical Concepts

1. I can find the Wronskian of the solutions to a second order equation.
2. I can compute the Wronskian of a set of solutions to a system of differential equations.
3. I can find general solutions to higher order linear differential equations using the characteristic equation.
4. I can compute the Wronskian for a higher order system and verify that the solutions form a fundamental set.
5. I can solve non-homogeneous equations of higher order using undetermined coefficients.
6. I can plot trajectories and compute facts about chaotic solutions to the Lorenz system.

Details of Methods

1. I can find the real and imaginary parts of a complex function and use them to find linearly independent solutions to problems with complex roots.
2. I can use reduction of order to generate a second solution to a linear second order differential equation and use this to solve problems with repeated roots.
3. I can determine that two solutions to a second order ODE form a fundamental set, or a basis, of solutions.
4. I can transform a higher order differential equation into a first order system and vice-versa.
5. I can determine that a pair of vector functions forms a fundamental set of solutions to a 2x2 system of DEs.
6. I can analyze physical problems with competing species or predator-prey interaction using an autonomous system of differential equations.