Abstract:

I will give a talk summarizing some results of Duyckaerts, Kenig, and Merle concerning type II blow-up solutions to the Energy-Critical Focusing Wave Equation. This is the hottest material in Wave Equation research. The Energy-Critical Focusing Wave Equation in N dimensions, $N = 3, 4, 5$, is given by

$$\begin{cases} \partial_t^2 u - \Delta u - u|u|^{\frac{4}{N-2}} = 0 \\ u|_{t=0} = u_0 \in \dot{H}^1(\mathbb{R}^N) \\ \partial_t u|_{t=0} = u_1 \in L^2(\mathbb{R}^N) \end{cases}$$

The result that I will speak about roughly says that if a type II blow-up solution to this equation has energy that doesn't get too big, then the solution is asymptotically the sum of a regular solution and a Lorentz boosted stationary solution of this equation. I will make some of these notions precise (e.g. what it means to be a type II blow-up and what the stationary solution I am talking about is). Beyond this, I will outline the flow of the proof, explaining the main ideas of its arguments and doing my best to stay clear of technicalities.