MATH 251: Quiz 8

April 30, 2015

Name: Solutions Sec: ____________

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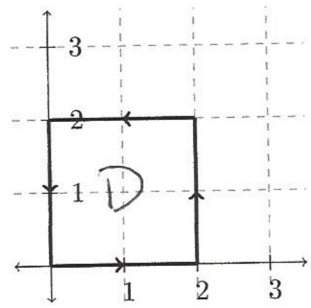
1. Use Green's Theorem to calculate the integral $\oint_C \vec{F} \cdot d\vec{s}$ for the vector field

$$\vec{F} = \langle 2xy + x^4, 3xy^2 - \sin(y) \rangle$$

and the curve



By Green's Thm. $\oint_C \vec{F} \cdot d\vec{s} = \iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, dA$ (2)

$\frac{\partial F_2}{\partial x} = 3y^2 \qquad \frac{\partial F_1}{\partial y} = 2x$ (1)

So

$$\oint_C \vec{F} \cdot d\vec{s} = \int_0^2 \int_0^2 3y^2 - 2x \, dx \, dy$$

(1)

$$= \int_0^2 3xy^2 - x^2 \Big|_0^2 \, dy$$

$$= \int_0^2 6y^2 - 4 \, dy = 2y^3 - 4y \Big|_0^2$$

$$= 16 - 8 = \boxed{8}$$ (1)

2. Use Stokes' Theorem to evaluate the integral

$$\iint_S \operatorname{curl}(\vec{F}) \cdot d\vec{S}$$

for the surface $\mathcal{S}$ with outward normal vector and vector field $\vec{F}$ below, where the boundary of $\mathcal{S}$ is the ellipse $4x^2 + y^2 = 16$ in the xy-plane. This boundary can be parametrized as $c(t) = \langle 2\cos(t), 4\sin(t), 0\rangle$. $0 \le t \le 2\pi$

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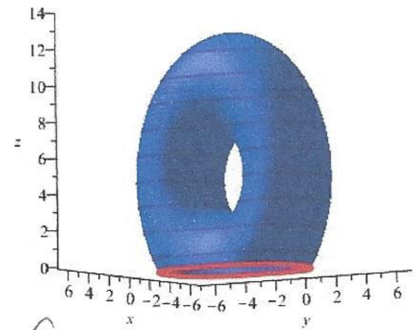
$$\vec{F} = \langle 3x + 4zx^2, x + y + z, x^2 + y^2 + z^2\rangle$$



By Stokes' thm: $\iint_S \operatorname{curl}(\vec{F}) \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}$ (1)

$c(t) = \langle 2\cos(t), 4\sin(t), 0\rangle$

$c'(t) = \langle -2\sin(t), 4\cos(t), 0\rangle$ (1)

$F(c(t)) = \langle 6\cos t + 0, 2\cos t + 4\sin t + 0, 4\cos^2 t + 16\sin^2 t + 0\rangle$

$F(c(t)) \cdot c'(t) = -12\cos(t)\sin t + 8\cos^2 t + 16\sin t\cos t + 0$

$= 4\cos t \sin t + 8\cos^2 t$ (1)

$= 4\cos t \sin t + 4 + 4\cos 2t$

So $\iint_S \operatorname{curl}(\vec{F}) \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s} = \int_0^{2\pi} \vec{F}(c(t)) \cdot c'(t)\, dt = \int_0^{2\pi} 4\cos t \sin t + 4 + 4\cos 2t \, dt$ (1)

$= 2\sin^2 t + 4t + 2\sin 2t \Big|_0^{2\pi} = 8\pi$ (1)