MATH 251: Quiz 7

April 23, 2015

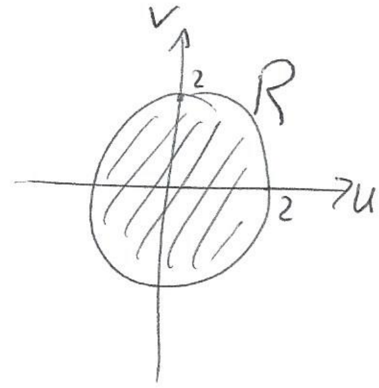
Name: Solutions Sec: ____________

1. Let $\mathcal{S} = G(u,v) = (u^2+v^2, -v, u)$ be a parametrically defined surface, where u and v satisfy $u^2+v^2 \le 4$.

(a) Compute $\vec{T}_u(u,v)$, $\vec{T}_v(u,v)$, and $\vec{n}(u,v)$ for this surface (as functions of u and v).

(b) Find the equation for the tangent plane to $\mathcal{S}$ at the point $\cancel{(2,0,1)}$ $(2,-1,0)$ $= G(1,1)$.

(c) Compute the surface area of $\mathcal{S}$.



(a)
$$\vec{T}_u = \frac{\partial G}{\partial u} = \langle 2u, 0, 1\rangle$$
$$\vec{T}_v = \frac{\partial G}{\partial v} = \langle 2v, -1, 0\rangle$$
$$\vec{n} = \vec{T}_u \times \vec{T}_v = \langle 1, 2v, -2u\rangle$$

(b) Normal vector: $\vec{n}(1,1) = \langle 1, 2, -2\rangle$

Base point: $(2, -1, 1)$

Eqn: $0 = \vec{n}\cdot\langle x-x_0, y-y_0, z-z_0\rangle = \langle 1,2,-2\rangle \cdot \langle x-2, y+1, z-1\rangle$
$$= x-2+2y+2-2(z-1)$$
$$\boxed{-2 = x+2y-2z}$$

(c)
$$\iint_S 1\,dS = \iint_R \|\vec{n}\|\,dA = \int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\cdot r\,dr\,d\theta$$

$u = r\cos\theta$
$v = r\sin\theta$

$w = 1+4r^2$
$dw = 8r\,dr$

$$\|\vec{n}\| = \sqrt{1+4u^2+4v^2} = \sqrt{1+4(u^2+v^2)}$$

$$= \frac{1}{8}\int_0^{2\pi}\int_1^{17}\sqrt{w}\,dw\,d\theta = \frac{1}{8}\cdot w^{3/2}\cdot\frac{2}{3}\Big|_1^{17}\int_0^{2\pi}d\theta$$
$$= \frac{2\pi}{8}\cdot\frac{2}{3}\left(17^{3/2}-1\right) = \boxed{\frac{\pi}{6}\left(17^{3/2}-1\right)}$$

2. Let $\vec{F} = \langle x + 3y^2, y + 4x, z \rangle$ be a vector field, and let $\mathcal{S}$ be the plane defined by the equation $z = 6 - 2x - y$ for $1 \le x \le 3$ and $1 \le y \le 3$. Compute the flux of $\vec{F}$ through $\mathcal{S}$, where $\mathcal{S}$ is defined with the upward (positive z-direction) normal, i.e., find

$2x+y+z=6$

$$\int_S \vec{F} \cdot d\vec{S}.$$

$G(u,v) = \langle u, v, 6-2u-v \rangle$

$\vec{n} = \langle 2, 1, 1 \rangle$

$\vec{T_u} = \langle 1, 0, -2 \rangle$

$\vec{T_v} = \langle 0, 1, -1 \rangle$

$F(G(u,v)) = \langle u+3v^2, v+4u, 6-2u-v \rangle$

$F(G(u,v)) \cdot \vec{n} = 2(u+3v^2) + 1(v+4u) + 1(6-2u-v)$

$= 6v^2 + 4u + 6$

Then

$$\iint_S \vec{F} \cdot d\vec{s} = \iint_R \vec{F} \cdot \vec{n} \, dA = \int_1^3 \int_1^3 6v^2 + 4u + 6 \, du \, dv$$

$$= \int_1^3 6uv^2 + 2u^2 + 6u \Big|_1^3 dv$$

$$= \int_1^3 12v^2 + 16 + 12 \, dv$$

$$= 4v^3 + 28v \Big|_1^3 = 26 \cdot 4 + 28 \cdot 2$$

$$= 104 + 56 = 160$$