# MATH 251: Quiz 6

April 9, 2015

Name: Solution Sec: ____________

**1.** Let $\vec{F} = \langle yz, xz, xy \rangle$. Compute the integral

$$\int_C \vec{F} \cdot ds$$

for the curve $C(t) = \langle 3t^2 + \cos(\pi t^3), 2^t, t^4 + t^3 - t + 1 \rangle$ for $t = 0$ to $1$. Hint: Is $\vec{F}$ conservative?

$\vec{F}$ is conservative with $\vec{F} = \nabla(xyz)$. Thus

$$\int_C \vec{F} \cdot ds = \phi(C(1)) - \phi(C(0))$$

$$= \phi(3-1, 2, 2) - \phi(1,1,1)$$

$$= \phi(2,2,2) - \phi(1,1,1) = 8 - 1 = \boxed{7}$$

**2.** Determine whether or not the following vector fields are conservative. If they are conservative, find a potential function.

(a) $\vec{F} = \langle y^2, x^2, \sin(z) \rangle$.

(b) $\vec{G} = \langle 3x^2 + \sin(z), 2yz, y^2 + x\cos(z) \rangle$.

(a) No. $\partial F_1/\partial y = 2y \neq 2x = \partial F_2/\partial x$

(b) Yes $\frac{\partial \phi}{\partial x} = 3x^2 + \sin z \rightarrow \phi \sim x^3 + x\sin z$

$\frac{\partial \phi}{\partial y} = 2yz \rightarrow \phi \sim y^2 z$

$\frac{\partial \phi}{\partial z} = y^2 + x\cos z \rightsquigarrow \phi = y^2 z + x\sin z$



Thus $\boxed{\phi = x^3 + y^2 z + x\sin z}$

**3.** Let $f(x,y,z) = 9z + 2x$ and let $C$ be the curve $c(t) = \langle t, t^2, t^3 \rangle$ for $t = 0$ to $1$. Compute

$$\int_C f(x,y,z)\, ds.$$

$f(c(t)) = 9t^3 + 2t.$

$c'(t) = \langle 1, 2t, 3t^2 \rangle.$

$\|c'(t)\| = \sqrt{1 + 4t^2 + 9t^4}$

$\int_C f(x,y,z)\, ds = \int_0^1 f(c(t))\, \|c'(t)\|\, dt$

$u = 1 + 4t^2 + 9t^4$

$= \int_0^1 9t^3 + 2t \sqrt{1 + 4t^2 + 9t^4}\, dt \quad du = 36t^3 + 8t\, dt$

$= \frac{1}{4} \int \sqrt{u}\, du$

$= \frac{2}{3} \cdot \frac{1}{4} u^{3/2} = \frac{1}{6} \left(1 + 4t^2 + 9t^4\right)^{3/2} \Big|_0^1$

$= \frac{1}{6}\left( [1 + 4 + 9]^{3/2} - 1^{3/2} \right) = \boxed{\frac{1}{6}\left(14^{3/2} - 1\right).}$