MATH 251: Quiz 5

April 2, 2015

Name: Solutions Sec: __________

1. Calculate

$$\iiint_D x + y \, dV$$

for the region in the first octant $x \geq 0$, $y \geq 0$, $z \geq 0$ between the planes $z = 2$ and $z = x + y + 1$. You should be looking at the region near the origin.

$x+y+1 \leq z \leq 2.$

For bondary $2 = x+y+1$

$x+y=1 \Rightarrow y = 1-x.$

$0 \leq x \leq 1$

So $$\int_0^1 \int_0^{1-x} \int_{x+y+1}^{2} (x+y) \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^{1-x} (x+y)z \Big|_{x+y+1}^{2} dy \, dx = \int_0^1 \int_0^{1-x} 2(x+y) - (x+y)(x+y+1)$$

$$= \int_0^1 \int_0^{1-x} 2(x+y) - (x+y)^2 - (x+y) \, dy \, dx = \int_0^1 \int_0^{1-x} (x+y) - (x+y)^2 \, dy \, dx$$

$$= \int_0^1 \frac{(x+y)^2}{2} - \frac{(x+y)^3}{3} \Big|_0^{1-x} dx = \int_0^1 \frac{1}{2} - \frac{1}{3} - \left(\frac{x^2}{2} - \frac{x^3}{3}\right)$$

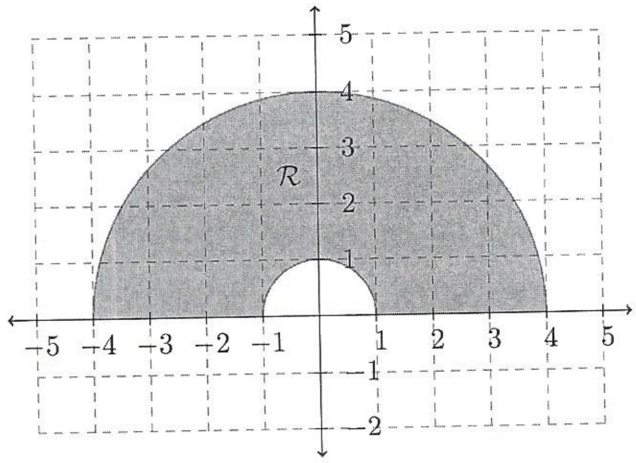
$$= \int_0^1 \frac{1}{6} + \int_0^1 \frac{x^3}{3} - \frac{x^2}{2} \, dx = \frac{1}{6} + \left(\frac{x^4}{12} - \frac{x^3}{6}\right)\Big|_0^1 = \frac{1}{6} + \frac{1}{12} - \frac{1}{6}$$

$$= 1/12$$

2. Compute

$$\iint_{\mathcal{R}} y \, dA$$

for the region $\mathcal{R}$ pictured below by changing to polar coordinates.



In polar coordinates

$1 \le r \le 4$

$0 \le \theta \le \pi$

So $\displaystyle\iint_R y \, dA = \int_0^{\pi}\int_1^4 r\sin\theta \; r\,dr\,d\theta$

$$= \int_0^{\pi}\int_1^4 r^2 \sin\theta \, dr\, d\theta$$

$$= \frac{1}{3}\int_0^{\pi} r^3 \sin\theta \Big|_1^4 \, d\theta$$

$$= \frac{1}{3}\int_0^{\pi} (64-1)\sin\theta \, d\theta$$

$$= \frac{63}{3}\left(-\cos\theta\right)\Big|_0^{\pi} = \frac{63 \cdot 2}{3} = \boxed{42}$$

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