MATH 251: Quiz 4

March 12, 2015

Name: Solutions Sec: ____________

1. Evaluate the integral

$$\int_0^4 \int_1^3 3x^2 + 2y + 1 \; dx \; dy$$

$$= \int_0^4 x^3 + 2xy + x \Big|_1^3 \, dy$$

$$= \int_0^4 27 + 6y + 3 - 1 - 2y - 1 \, dy$$

$$= \int_0^4 28 + 4y \, dy$$

$$= 28y + 2y^2 \Big|_0^4 = 28 \cdot 4 + 2 \cdot 16 = 112 + 32 = \boxed{144}$$

2. Let $\mathcal{D} = \{x^2 + y^2 \leq 4, \; x \geq 0\}$. Evaluate the integral

$$\iint_{\mathcal{D}} 1 \, dA$$

[Hint: Sketch the region and think about what this integral computes.]

This is the area of a semicircle with radius 2

$$A = \frac{\pi r^2}{2} = \frac{\pi \cdot 4}{2} = \boxed{2\pi}$$

3. Evaluate the following integral by changing the order of integration. You will also likely need integration by parts.

$$\int_0^1 \int_0^2 \frac{x^3}{1+x^2y}\, dx\, dy.$$

$$= \int_0^2 \int_0^1 \frac{x^3}{1+x^2y}\, dy\, dx$$

$u = 1+x^2y$
$du = x^2 dy$

$$= \int_0^2 \int_{\cancel{1}}^{\cancel{1+x}} \frac{x}{u}\, du\, dx = \int_0^2 x \ln u \Big|\, dx$$

$$= \int_0^2 x \ln(1+x^2y)\Big|_{y=0}^{y=1} dx$$

$v = 1+x^2$
$dv = 2x\,dx$

$$= \int_0^2 x \ln(1+x^2) - 0 \, dx$$

$$= \frac{1}{2}\int_{\cancel{0}}^{\cancel{2}} \ln v\, dv = \frac{1}{2}\left[\int_{\cancel{0}}^{\cancel{0}} 1 \cdot \ln v\, dv\right]$$ Integrate by parts

$$= \frac{1}{2}\left[v \cdot \ln v\Big|_{\cancel{0}}^{\cancel{2}} - \int_{\cancel{0}}^{\cancel{2}} 1\, dv\right]$$

$$= \frac{1}{2}\left[v \ln v - v\right]\Big|$$

$$= \frac{1}{2}\left[(1+x^2)\ln(1+x^2) - (1+x^2)\right]\Big|_0^2$$

$$= \frac{1}{2}\left[5\ln 5 - 5 - 1\ln 1 + 1\right] = \boxed{\frac{1}{2}\left[5\ln 5 - 4\right]}$$