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MATH 251: Quiz 3

February 26, 2015

Name: Solutions Sec: ____________

Note: Questions 2 and 3 are on the back.

1. Find an equation for the tangent plane to the graph of

$$z = f(x, y) = y^3 + 2ye^x$$

/4 at the point $(0, 3)$. [Hint: This plane must contain the vectors $\langle 1, 0, f_x\rangle$ and $\langle 0, 1, f_y\rangle$. Or you can use any other method to find this plane.]

$f(0,3) = 3^3 + 2(3)(1) = 27+6 = 33$

$f_x = 2ye^x \qquad f_x(0,3) = 6$ (1)

$f_y = 3y^2 + 2e^x \qquad f_y(0,3) = 27+2 = 29$ (1)

So $\langle 1,0,6\rangle$ and $\langle 0,1,29\rangle$ lie in the plane

Therefore $\vec{n} = \langle 1,0,6\rangle \times \langle 0,1,29\rangle = \langle -6,-29,1\rangle$

is a normal vector. Thus, we have

$0 = 6(x-0) + 29(y-3) - (z-33)$ (2)

as an equation for the tangent plane.

or $z = L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

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$6x + 29y - z = 54$

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2. Compute the gradient of the function

$$g(x,y,z) = 2z^2e^{xy} + x^2y^2.$$

$g_x = 2yz^2e^{xy} + 2xy^2$ (1)

$g_y = 2xz^2e^{xy} + 2x^2y$ (1)

$g_z = 4ze^{xy}$ (1)

So $\nabla g = \langle 2yz^2e^{xy} + 2xy^3;\ 2xz^2e^{xy} + 2x^2y,\ 4ze^{xy} \rangle$

3. Given the function

$$h(x,y) = x^3y^2 + 2xy$$

and the parametrizations

$$x(s,t) = 3s + 2t \qquad y(s,t) = 4s - t$$

$s = 3/11$ $t = 1/11$ ie. $x,y = (1,1)$

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use the Chain Rule to compute the derivatives $\frac{\partial h}{\partial s}$ and $\frac{\partial h}{\partial t}$.

$\frac{\partial h}{\partial x} = 3x^2y^2 + 2y$ $\qquad \frac{\partial h}{\partial y} = 2x^3y + 2x$ 1 ( )

$\frac{\partial x}{\partial s} = 3 \qquad \frac{\partial x}{\partial t} = 2 \qquad \frac{\partial y}{\partial s} = 4 \qquad \frac{\partial y}{\partial t} = -1$

$\frac{\partial h}{\partial s} = \frac{\partial h}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial h}{\partial y}\frac{\partial y}{\partial s} = 3(3x^2y^2 + 2y) + 4(2x^3y + 2x)$ (1)

$\left.\frac{\partial h}{\partial s}\right|_{(3/11,\,1/11)} = (\overset{15}{3(3+2)} + \overset{16}{4(2+2)}) = \boxed{31}$

$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial h}{\partial y}\frac{\partial y}{\partial t} = 2(3x^2y^2 + 2y) - 1(2x^3y + 2x)$ (1)

$\left.\frac{\partial h}{\partial t}\right|_{(3/11,\,1/11)} = 2(3+2) - 1(2+2) = \boxed{16}$