MATH 251: Quiz 2

February 12, 2015

Name: Solutions Sec: ______

Note: Question 2 is on the back.

1. Given the curve

$$\mathbf{r}_1(t) = \langle 8\cos(t), 6t, 8\sin(t)\rangle,$$

(a) Calculate the length of the curve from $t = 0$ to $t = \pi$.

(b) Find an arc-length parametrization for this curve.

$$r'(t) = \langle -8\sin t,\ 6,\ 8\cos t\rangle$$

$$\|r'(t)\| = \sqrt{(-8\sin t)^2 + 6^2 + (8\cos t)^2}$$

$$= \sqrt{64\sin^2 t + 36 + 64\cos^2 t}$$

$$= \sqrt{64+36} = \sqrt{100} = 10.$$

So $\overset{S(t)}{\ell(t)} = \int_0^t \|r'(t)\|\,dt = \int_0^t 10\,dt = 10t.$

a) $\underline{\ell(\pi) = 10\pi}$

b) $s = 10t \Rightarrow s = t/10.$

Plugging this in, the arc-length parametrization is

$$\boxed{r(s) = \langle 8\cos(t/10),\ 6t/10,\ 8\sin(t/10)\rangle.}$$

2. Find the equation of a plane that is parallel to $x + 3y + z = 0$ and goes through the point $(2, 3, 4)$.

$$\vec{n} = \langle 1, 3, 1 \rangle = \langle a, b, c \rangle$$

$$d = \vec{n} \cdot \langle x_0, y_0, z_0 \rangle = a x_0 + b y_0 + c z_0$$

$$= 1 \cdot 2 + 3 \cdot 3 + \overset{1 \cdot 4}{\cancel{4 \cdot 1}}$$

$$= 2 + 9 + 4 = 15.$$

So the plane is

$$\boxed{x + 3y + z = 15}$$