Math 152 - Worksheet 10

Section 7.6 - Strategies for Integration

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Evaluate $\displaystyle\int \frac{dx}{x(x-1)^2}$
2. Evaluate $\displaystyle\int \frac{x}{(x^2-1)^{3/2}}\,dx$
3. Compute $\displaystyle\int \frac{x^3}{(x^2-1)^{3/2}}\,dx$
4. Compute $\displaystyle\int \frac{x^4+1}{x^2+1}\,dx$
5. Compute $\displaystyle\int \tan x \sec^{5/4} x\,dx$
6. Compute $\displaystyle\int e^x\sqrt{e^{2x}-1}\,dx$. Take your time on this one. It's going to need many different techniques.

Submission Problems

1. Compute $\displaystyle\int x \sec^{-1} x\,dx$.
2. Compute $\displaystyle\int \sqrt{x^2+6x}\,dx$

Hint #1

1. This integral should be done by Partial Fractions.
2. This is just a substitution problem.
3. This can be done with either trigonometric substitution or normal substitution.
4. There are a few ways to handle this integral here. The easiest way might be long division into using known integral formulas.
5. This is related to both trigonometric integrals and substitution.
6. Your best bet here is to start with a substitution, and see where that gets you.

Hint #2

1. The decomposition you want is $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$
2. Set $u = x^2 - 1$, so that $du = 2x\ dx$.
3. For trigonometric substitution, set $x = sec(\theta)$. For normal substitution, set $u = x^2 - 1$ and manipulate whatever is left in the numerator to be in terms of u.
4. Long division gives that $\frac{x^4+1}{x^2+1} = x^2 - 1 + \frac{2}{x^2+1}$
5. What choice could I make for u and du here to make this easy? Look for du.
6. Try $u = e^x$ and see what is left, which should leave you with needing to integrate $\sqrt{u^2-1}$, which is a trigonometric substitution.

Hint #3

1. The function you need to integrate is $\dfrac{1}{x} - \dfrac{1}{x-1} + \dfrac{1}{(x-1)^2}$

5. Take $du = \sec x \tan x$, which comes from $u = \sec x$.

6. After this, you'll get to integrating $\sec\theta \tan^2(\theta)$, which requires trigonometric integrals.

Hint #4

1. Once you're done, use a triangle to switch this back to being in terms of x.

5. This integral then becomes $\int u^{1/4}\, du$

6. This is the same as integrating $\sec^3(\theta) - \sec(\theta)$, which needs reduction, and then you need to convert back to x.

Answers

1. $\ln|x| - \ln|x-1| - \dfrac{1}{x-1} + C$
2. $\dfrac{1}{\sqrt{x^2-1}} + C$
3. $\sqrt{x^2-1} - \dfrac{1}{\sqrt{x^2-1}} + C$
4. $\dfrac{x^3}{3} - x + 2\tan^{-1}(x) + C$
5. $\dfrac{4}{5}\sec^{5/4}(x) + C$
6. $\dfrac{1}{2}(e^x\sqrt{e^{2x}-1} - \dfrac{1}{2}\ln|e^x + \sqrt{e^{2x}-1}| + C$