Math 152 - Worksheet 9

Section 7.5 - Method of Partial Fractions

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Evaluate $\int \frac{2x-1}{x^2-5x+6}\,dx$
2. Evaluate $\int \frac{x^2-8x}{(x+1)(x+4)^3}\,dx$
3. Compute $\int \frac{x^2}{(x+1)(x^2+1)}dx$
4. Compute $\int \frac{3x^2-2}{x-4}\,dx$
5. Compute $\int \frac{10}{(x+1)(x^2+9)^2}\,dx$

Submission Problems

1. Compute $\int \frac{3x}{(x^2+4)(x-1)}\,dx$.
2. Compute $\int \frac{x^2-3x+1}{(x-1)^2(x+3)}\,dx$

Hint #1

1. You need to first factor the denominator, which is $(x-3)(x-2)$. What does this mean for the partial fraction decomposition of this rational function?

2. What is the partial fraction decomposition for this function?

3. The partial fraction decomposition here is of the form $\frac{A}{x+1} + Bx + Cx^2 + 1$

4. We can't go right into partial fractions here, because the degree in the numerator is higher than the degree in the denominator. How do we fix this?

5. Here we have a repeated quadratic factor, so what does that mean for the partial fraction decomposition of this function?

Hint #2

1. You need to find the coefficients A and B in $2x - 1 = A(x - 3) + B(x - 2)$
2. The decomposition is $\dfrac{A}{x+1} + \dfrac{B}{x+4} + \dfrac{C}{(x+4)^2} + \dfrac{D}{(x+4)^3}$
3. You should end up with $A = 1/2$, $B = 1/2$ and $C = -1/2$.
4. Long division gives that $\dfrac{3x^2 - 2}{x - 4} = 3x + 12 + \dfrac{46}{x - 4}$
5. The decomposition you should use is $\dfrac{A}{x+1} + \dfrac{Bx + C}{x^2 + 9} + \dfrac{Dx + E}{(x^2 + 9)^2}$

Hint #3

1. You should end up with $A = -3$ and $B = 5$
2. You can plug in -1 and -4 to get two of the coefficients, but you'll need to plug in two other numbers to get the last two (try -2 and -3)
3. The integral to be evaluated is then $\int \frac{1/2}{x+1} + \frac{x/2}{x^2+1} - \frac{1/2}{x^2+1}\, dx$
4. Then we evaluate the integral; the first part is a polynomial and the second is a logarithm.
5. Solving out for the coefficients should give $A = 1/10$, $B = -1/10$, $C = 1/10$, $D = -1$, $E = 1$.

Hint #4

1. The integral you then need to compute is $\displaystyle\int \frac{-3}{x-3} + \frac{5}{x-2}\,dx$
2. The integral you need to compute is $\displaystyle\int \frac{-1/3}{x+4} + \frac{16}{(x+4)^3} + \frac{1/3}{x+1}\,dx$
3. Your answer should have two logarithm terms and one inverse tangent term.
5. You'll need to use trigonometric substitution on the $\frac{1}{(x^2+9)^2}$ term, but the rest will be logarithms or inverse tangent.

Answers

1. $5\ln|x-3| - 3\ln|x-2| + C$
2. $-\frac{1}{3}\ln|x+4| + \frac{1}{3}\ln|x+1| - \frac{1}{8}\frac{1}{(x+4)^2} + C$
3. $\frac{1}{2}\ln|x+1| + \frac{1}{4}\ln|x^2+1| - \frac{1}{2}\tan^{-1}(x) + C$
4. $\frac{3}{2}x^2 + 12x + 46\ln|x-4| + C$
5. $\frac{1}{10}\ln|x+1| + -\frac{1}{20}\ln|x^2+9| + \frac{1}{30}\tan^{-1}\frac{x}{3} + \frac{1}{2}\frac{1}{x^2+9} + \frac{1}{54}\frac{3x}{x^2+9} + \frac{1}{54}\tan^{-1}\frac{x}{3} + C$