Math 152 - Worksheet 8

Section 7.3 - Trigonometric Substitution

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Evaluate $\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$
2. Evaluate $\int \frac{1}{x^2\sqrt{25-x^2}}\,dx$
3. Compute $\int x^3\sqrt{36-x^2}dx$
4. Compute $\int \frac{1}{y^3\sqrt{y^2-16}}\,dy$
5. Compute $\int \frac{1}{(121-x^2)^2}\,dx$
6. Compute $\int \sqrt{x^2+6x}\,dx$

Submission Problems

1. Compute $\int \frac{1}{(4-x^2)^{3/2}}\,dx.$
2. Compute $\int_0^1 \frac{dt}{(t^2+9)^2}$

Hint #1

1. That part under the square root should tell us what trigonometric substitution to use here. What is that?

2. That part under the square root should tell us what trigonometric substitution to use here. What is that?

3. That part under the square root should tell us what trigonometric substitution to use here. What is that?

4. That part under the square root should tell us what trigonometric substitution to use here. What is that?

5. This may not look like trigonometric substitution at first, but it turns out that this is a good way to solve this integral.

6. This also does not look like trigonometric substitution right off the bat, but there's no other way to solve this problem. You'll need to complete the square first.

Hint #2

1. Use the substitution $x = 3\sec\theta$ because $\sec^2(\theta) - 1 = \tan^2(\theta)$
2. Use the substitution $x = 5\sin\theta$ to make this work out.
3. Use the substitution $x = 6\sin\theta$ to make this work out.
4. Use the substitution $y = 4\sec\theta$ to solve this integral, because the square root on the bottom becomes $4\tan\theta$.
5. Try the substitution $x = 11\sin(\theta)$ to solve out this integral.
6. The part under the square root can be rewritten as $(x+3)^2-9$, so that the appropriate trigonometric substitution is $x + 3 = 3\sec\theta$.

Hint #3

1. The square root in the denominator becomes $3\tan\theta$, and since $dx = 3\sec\theta\tan\theta$ some things will cancel.

2. Make sure to also change dx to $d\theta$ and use that to cancel things out.

3. Once you simplify this out, it falls into the trigonometric integrals category. Thankfully, this one is an easy one.

4. The expression that needs to be simplified is $\dfrac{4\sec\theta\tan\theta}{4^3\sec^3\theta 4\tan\theta}$.

5. The resulting integral you want to solve should be $\dfrac{1}{1331}\displaystyle\int \sec^3\theta\ d\theta$

6. This substitution will result in needing to solve a trigonometric integral, which will need reduction formulas.

Hint #4

1. Once you're done, use a triangle to switch this back to being in terms of x.
2. Once you're done, use a triangle to switch this back to being in terms of x.
3. You need to evaluate $6^5 \displaystyle\int \sin^3(\theta)\cos^2(\theta)\, d\theta$ and then convert back to x.
4. The resulting integral will be $\frac{1}{64}\int \cos^2\theta\, d\theta$, and when you have $\sin(2\theta)$, you'll want to rewrite it as $2\sin(\theta)\cos(\theta)$ so that you can convert it back into x.
5. This will need the reduction formulas and the fact that the antiderivative of $\sec(\theta)$ is $\ln|\sec\theta + \tan\theta|$ to finish up the problem.
6. You end up needing to solve $9 \displaystyle\int \sec^3(\theta) - \sec(\theta)\, d\theta$ and then convert back to x.

Answers

1. $\dfrac{\sqrt{x^2-9}}{9x}+C$

2. $-\dfrac{\sqrt{25-x^2}}{25x}+C$

3. $\dfrac{1}{5}(36-x^2)^{5/2}-12(36-x^2)^{3/2}+C$

4. $\dfrac{1}{128}\sec^{-1}(\dfrac{y}{4})+\dfrac{\sqrt{y^2-16}}{16y^2}+C$

5. $\dfrac{1}{2662}\left(\dfrac{11x}{121-x^2}+\ln\left|\dfrac{11+x}{\sqrt{121-x^2}}\right|\right)+C$

6. $\dfrac{(x+3)(\sqrt{x^2+6x})}{2}-\dfrac{9}{2}\ln\left(\dfrac{x+3+\sqrt{x^2+6x}}{3}\right|+C$