Math 152 - Worksheet 5

Section 6.4 - Solids of Revolution - Shell Method

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Find the volume of the solid of revolution obtained by revolving the graph of $y = 2x^2$ around the y-axis over the range $[1, 4]$.

2. Find the volume of the solid of revolution obtained by revolving the region enclosed between the graphs of $y = 3 - x$ and $y = x^2 - 3$ around the line $x = 5$.

3. Find the volume of the solid of revolution obtained by revolving the region in the first quadrant between the graph of $y = x^2 + 4$ and the line $y = 13$ around the x axis using the Shell Method.

4. Find the volume of the solid of revolution obtained by revolving the region in the first quadrant between the graph of $y = x^2 + 4$ and the line $y = 13$ around the y axis using the Shell Method.

5. Consider the solid of revolution formed by revolving the region between the curves $y = x^2$, $y = 4$ and $x = 1$ around the line $x = -2$. Work out the answer using both the washer method and the shell method.

Submission Problems

1. Find the volume of the solid of revolution obtained by revolving the region between the graphs of $x = 9 - y^2$ and $x = 5$ around the line $y = 8$.

2. Find the volume of the solid of revolution obtained by revolving the region between the graphs of $y = x^2 + 1$, $y = 1 - x$, and the lines $x = 1$ and $x = 5$ around the line $x = -2$.

Hint #1

1. Draw a picture of the region. What should the radius and height be for this region?
2. Sketch out a picture. Draw in the segment that is being rotated and try to set up the integral.
3. If we're doing this around the x axis via the Shell Method, this means we need an integral in y. Draw out the picture and the segment that is being rotated.
4. Since we want to revolve around the y axis with shells, this requires an integral in x. Draw out the picture and the segment being rotated.
5. Sketch out the region and look at what the segments are that will be rotated for each method.

Hint #2

1. From that, set up the integral using the formula for the shell method. (This should be an integral in x).
2. What should the height and radius be for doing this via the Shell Method?
3. What is the radius and height for this setup? What are the bounds on the integral?
4. What are the radius and height for this rotation? What are the bounds on the integral?
5. Washer Method: The inner radius should be the straight line, and outer radius the curve.

Hint #3

1. The integral to find the volume is $2\pi \int_1^4 x(2x^2)\ dx$
2. The radius should be $5-x$, where the height is the difference between the two functions.
3. The integral should be
$$2\pi \int_4^{13} y(\sqrt{4-y})\ dy$$
4. The integral should be
$$2\pi \int_0^3 x(13-(x^2+4))\ dx$$
5. Shell Method: The radius should be $x+2$, and the height should be the gap between the curve and the horizontal line.

Hint #4

2. The integral is $2\pi \int_{-3}^{2} (5-x)(3-x-(x^2-3))\ dx$

Answers

1. 255π
2. $\frac{1375}{6}\pi$
3. $\frac{1692}{5}\pi$
4. $\frac{81}{2}\pi$
5. $\frac{67}{6}\pi$