Math 152 - Worksheet 3

Section 6.2 - Setting Up Integrals

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Find the volume of a right circular cone of height 14 whose base is a circle of radius 6.
2. Find the volume of the solid whose base is the region bounded between $y = x^2$ and $y = 5$ and whose cross-sections perpendicular to the y-axis are squares.
3. Find the volume of the solid whose base is the circle $x^2 + y^2 = 1$, and whose cross-sections perpendicular to the x-axis are equilateral triangles.
4. Find the total mass of a 2 m rod whose linear density function is $\rho(x) = 10(x + 5)^{-1}$ for $0 \leq x \leq 2$
5. Find the average value of the function $f(x) = \frac{2x}{x^2+1}$ over the interval $[4, 7]$.
6. Let $f(x) = x^2$. Find a value c between 1 and 5 so that $f(c)$ equals the average value of f on $[1, 5]$. That is, find the value of c that is guaranteed to exist by the Mean Value Theorem for Integrals.

Submission Problems

1. Find the volume of the solid whose base is the region bounded by the curves $y = x^2 - 1$ and $y = 3 - x^2$, and whose cross sections perpendicular to the x-axis are semicircles with diameter on the xy-plane.
2. Find the average value of the function $f(x) = \sec^2 x$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$.

Hint #1

1. You're going to need the cross-sectional area here. How could you find that? The cross-sections should be circles.
2. Sketch out a picture. Which way are the cross-sections going, and how which variable should we integrate in?
3. What shape are the cross-sections? You're going to know the length of the base of this shape; try to find a formula for the area in terms of this length.
4. This should be computed by an integral. What does it look like?
5. How do you compute the average value of a function?
6. First, compute the average value of f on this interval.

Hint #2

1. Use similar triangles to figure out what the radius of each circle is as a function of height.

2. What is the length of the base of each cross section in terms of y? Given this, what is the area as a function of y?

3. If you have an equilateral triangle of base L, the height will be $\frac{\sqrt{3}}{2}L$. (Prove it)

4. The mass can be computed by $\int_0^2 \frac{10}{x+5}\, dx$.

5. You'll need to do a u-substitution in here. Make sure you set up the entire expression for average value before you do that.

6. Once that is done, you want to set $f(c)$ equal to that value and solve for c.

Hint #3

1. $r = 6 - \frac{3h}{7}$. If this is r, what is the area of each cross section?

2. What should the bounds be on the integral that you need to compute?

3. Since we have cross-sections perpendicular to the x axis, this should be an integral in x. How long is the base of the triangle at position x?

5. The expression should be
$$\frac{1}{3}\int_4^7 \frac{2x}{x^2+1}\,dx$$

6. You should end up with $c^2 = \frac{1}{4}\int_1^5 x^2\,dx = \frac{31}{3}$

Hint #4

1. $V = \int_0^{14} \pi \left(6 - \frac{3h}{7}\right)^2 \, dh$

2. The integral you will need to compute is $\int_0^5 4x \, dx$

3. The base is length $2\sqrt{1-x^2}$, which gives an area of the triangle as

$$\frac{1}{2}L\left(\frac{\sqrt{3}}{2}L\right) = \sqrt{3}(1-x^2)$$

.

5. After the substitution, this should be

$$\frac{1}{3}\int_{17}^{50} \frac{1}{u} \, du$$

Answers

1. 168π

2. 50

3. $\displaystyle\int_{-1}^{1} \sqrt{3}(1 - x^2)\ dx = \frac{4\sqrt{3}}{3}$

4. $10 \ln\left(\frac{7}{5}\right)$

5. $\frac{1}{3} \ln\left(\frac{50}{17}\right)$

6. $\sqrt{\frac{31}{3}} \approx 3.21455$