Math 152 - Worksheet 27

Section 20.2 - Complex Numbers Part 2

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. For the complex number $z = 5\cos\left(\frac{\pi}{13}\right) + 5i\sin\left(\frac{\pi}{13}\right)$, find the complex number z^4 and z^{-2}.
2. Find the three complex solutions to the equation $z^3 = 1 - i$.
3. Use De Moivres Formulas to find an expression for $\cos(3\theta)$ in terms of $\cos(\theta)$ and $\sin(\theta)$
4. Find the exponential forms of the complex numbers $z_1 z_2$ and $\frac{z_1}{z_2}$ for the numbers $z_1 = \cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)$ and $z_2 = \sqrt{5}\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$
5. Use the fact that the polynomial $x^3 - 4x^2 + x + 26$ has a root at $x = -2$ to compute all three complex roots of this polynomial.

Submission Problems

1. Find all of the 5th roots of the complex number $-4 + 4i$.
2. Find all roots of the equation $z^4 - 2z^3 - 2z^2 + 8 = 0$, with the fact that $-1 + i$ is a root of this polynomial.

Hint #1

1. This complex number is basically given in polar form. What does this look like in exponential form?

2. The first step in this process is converting $1 - i$ into exponential form.

3. De Moivres Formulas tell us that $e^{in\theta} = (\cos(\theta) + i\sin(\theta))^n$. What does this mean for $n = 3$?

4. To find products and quotients, it is easier to convert these numbers into exponential form. What do we get for these two numbers?

5. Since we know this has a root at $x = -2$, we know that $x + 2$ is a factor of this polynomial. So we can use long division to get the other factor.

Hint #2

1. The number we have here is $z = 5e^{i\frac{\pi}{13}}$.
2. For this, we get that $1 - i = \sqrt{2}e^{i\frac{7\pi}{4}}$.
3. For $n = 3$, this says that $\cos(3\theta) + i\sin(3\theta) = (\cos(\theta) + i\sin(3\theta))^3$.
4. We get that $z_1 = e^{-i\frac{\pi}{4}}$ and $z_2 = \sqrt{5}e^{\frac{\pi}{2}}$.
5. We see that $x^3 - 4x^4 + x + 26 = (x+2)(x^2 - 6x + 13)$.

Hint #3

1. This means that the powers can be computed by $z^n = 5^n e^{i\frac{n\pi}{13}}$.

2. Therefore, the three solutions here are $z = 2^{1/6} e^{i\frac{7\pi}{12}}$ and then two more solutions where the angle is shifted by 2π before dividing by 3.

3. Expand out the right-hand side, and then look at the real part of the equation. What does this tell you?

5. We can find the roots of that quadratic polynomial using the quadratic formula.

Hint #4

3. The right-hand side expands to $\cos^3\theta + 3i\cos^2\theta\sin\theta + 3i^2\cos(\theta)\sin^2(\theta) + i^3\sin^3(\theta)$. Using the fact that $i^2 = -1$, this can be simplified to $\cos^3(\theta) - 3\cos(\theta)\sin^2(\theta) + i(3\cos^2(\theta)\sin(\theta) - \sin^3(\theta))$.

Answers

1. $z^4 = 625\cos\left(\frac{4\pi}{13}\right) + 625i\sin\left(\frac{4\pi}{13}\right)$ and $z^{-2} = \frac{1}{25}\cos\left(\frac{2\pi}{13}\right) - \frac{1}{25}i\sin\left(\frac{2\pi}{13}\right)$
2. The three solutions are $2^{1/6}e^{i\frac{7\pi}{12}}$, $2^{1/6}e^{i\frac{15\pi}{12}}$ and $2^{1/6}e^{i\frac{23\pi}{12}}$.
3. $\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin^2(\theta)$
4. $z_1 z_2 = \sqrt{5}e^{i\frac{\pi}{4}}$ and $\frac{z_1}{z_2} = \frac{1}{\sqrt{5}}e^{-i\frac{3\pi}{4}}$
5. The three zeros are $z = -2$, $z = 3 + 2i$ and $z = 3 - 2i$.