Math 152 - Worksheet 26

Section 20.1 - Complex Numbers

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Given complex numbers $z_1 = 3-4i$, and $z_2 = 2+i$, compute $2z_1 - z_2$, $z_1 \cdot z_2$ and $z_2 - z_1$
2. For $z_1 = 3-i$ and $z_2 = 2+3i$, calculate $\bar{z}_1$, $\bar{z}_2$, $\dfrac{1}{z_1}$ and $\dfrac{z_2}{z_1}$
3. Convert the complex number $z = -2\sqrt{3} + 2i$ into exponential form, and find the exponential form of $\bar{z}$.
4. Find the partial fraction decomposition of $\dfrac{6x^4 - 13x^3 + 37x^2 - 37x - 1}{(x-2)(x^2+1)(x^2+9)}$.
5. Find the four complex numbers where the function $f(x) = \dfrac{1}{(x^2+1)(x^2+2x+5)}$ does not exist, and use this to determine an upper bound on the radius of convergence of the power series expansion of $f(x)$ centered at $x = -3$.

Submission Problems

1. Use complex numbers to help compute $\displaystyle\int \frac{x^2 - 5x + 9}{(x+1)(x^2+4)}\, dx$
2. Use complex numbers to find an upper bound on the radius of convergence of the power series expansion of $\frac{\sin x}{x^2+9}$ centered at $x = 1$. Do not compute the power series expansion.

Hint #1

1. For addition and subtraction, we need to add and subtract the real and imaginary parts separately.
2. To find the complex conjugate, we need to switch the sign of the imaginary part.
3. We need to find r and θ for the point $(-2\sqrt{3}, 2)$ in Cartesian coordinates.
4. The decomposition here looks like $\frac{A}{x-2} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{x^2+9}$
5. The denominator of this function is $(x^2+1)(x^2+2x+5)$, so we can find where this part is zero.

Hint #2

1. When multiplying by a real number, we can just distribute that number to each of the real and imaginary parts.

2. For any complex number $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$.

3. We get that $r^2 = (-2\sqrt{3})^2 + 2^2 = 16$ and $\theta = \tan^{-1}\left(\frac{2}{-2\sqrt{3}}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

4. To set up value substitution, we end up with $6x^4 - 13x^3 + 37x^2 - 37x - 1 = A(x^2 + 1)(x^2 + 9) + (Bx + C)(x - 2)(x^2 + 9) + (Dx + E)(x - 2)(x^2 + 1)$.

5. The roots we get here are $z_1 = i$, $z_2 = -i$, and $z_3 = -1 + 2i$, $z_4 = -1 - 2i$.

Hint #3

1. For multiplying complex numbers, we distribute everything out, treating i like a variable, and then using the fact that $i^2 = -1$ to simplify the expression.

2. Once you get $\frac{1}{z}$, the quotient of two complex numbers can be found by multiplying them.

3. So $r = 4$ and $\theta = -\dfrac{\pi}{6}$, but this θ is in the wrong quadrant. So we actually want $\dfrac{5\pi}{6}$ for θ.

4. To set this up, we need to plug in $x = 2$, $x = i$, and $x = 3i$.

5. To find the radius of convergence, or an upper bound on it, we need to figure out how far each of these numbers are from -3, because the radius can not extend past any of these points.

Hint #4

2. For z_1, we get that $\dfrac{1}{z_1} = \dfrac{3+i}{10} = \dfrac{3}{10} + \dfrac{1}{10}i$

3. To get $\bar{z}$, we need to reflect over the x axis. This results in taking the negative of the θ angle. We can also just start with $\bar{z} = -2\sqrt{3} - 2i$ and go from there.

4. Each of the complex numbers gives rise to a system of two equations to solve for two coefficients, with these equations coming from the real and imaginary parts of the complex numbers.

5. The distances are $\sqrt{10}$, $\sqrt{10}$, $\sqrt{8}$ and $\sqrt{8}$.

Answers

1. $2z_1 - z_2 = 4 - 9i$, $z_1 \cdot z_2 = 10 - 5i$ and $z_2 - z_1 = -1 + 5i$.

2. $\bar{z}_1 = 3 + i$, $\bar{z}_2 = 2 - 3i$, $\dfrac{1}{z_1} = \dfrac{3}{10} + \dfrac{1}{10}i$ and $\dfrac{z_2}{z_1} = \dfrac{3}{10} + \dfrac{11}{10}i$

3. $z = 4e^{i\frac{5\pi}{6}}$, $\bar{z} = 4e^{-i\frac{5\pi}{6}} = 4e^{i\frac{7\pi}{6}}$

4. $\dfrac{1}{(x-2)} + \dfrac{2x+1}{x^2+1} + \dfrac{3x-4}{x^2+9}$

5. $f(x)$ does not exist at $z_1 = i$, $z_2 = -i$, and $z_3 = -1 + 2i$, $z_4 = -1 - 2i$. An upper bound on the radius of convergence is $\sqrt{8}$.