Math 152 - Worksheet 22

Section 11.1 - Parametric Equations

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Write the parametric equation $x = \frac{1}{1+t}$ and $y = te^t$ in the form $y = f(x)$ by eliminating the parameter. Where is the curve at $t = 0$? What about at $t = 6$?

2. A particle follows the trajectory $x(t) = 6t - 5$, $y(t) = 10 + 3t - t^2$, with t in seconds and x and y in meters. What is the maximum height of the particle? When does it hit the ground and how far from the origin does it land?

3. Find a parametrization for the curve $3y = 7x^2 - 2x$.

4. Find a parametrization $c(t)$ for the curve $x^2 + y^2 = 4$, satisfying $c(0) = (-1, \sqrt{3})$.

5. Find the equation of the tangent line to the parametric curve $x = \sec\theta$, $y = \tan\theta$ at the point $(2, \sqrt{3})$.

6. Find the area under the graph of $c(t) = (\ln t, 2 - t)$ between $t = 1$ and $t = 2$

Submission Problems

1. Find a parametrization for the ellipse $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$.

2. Find all points where the tangent lines to the curve $c(t) = (\frac{t^3}{3} + t^2 - 2t - 1, 2t^3 - 4t^2 + 4t + 3)$ have slope 2.

Hint #1

1. To do this, we need to solve out for t in one equation and plug this into the other one. One of these equations is much easier to solve out for t than the other one.

2. What does it mean for the particle to be at its maximum height? It means that the function $y(t)$ is at a maximum. How do we find this?

3. We need to introduce a parameter here to make this happen. Since it is almost already written as $y = f(x)$, we have an easy choice for parameter.

4. The curve here is a circle. How can we write a parametrization for a circle?

5. The first thing we need to figure out is what value of θ corresponds to the point $(2, \sqrt{3})$, so we need to figure out what angle θ has $\sec(\theta) = 2$ and $\tan\theta = \sqrt{3}$.

6. We can apply the standard formula $\int_a^b y(t)x'(t)\ dt$ to find this area.

Hint #2

1. Solving out for t in the x equation gives that $t = \frac{1}{x} - 1$. This can then be plugged into the y equation.

2. Take the derivative of the $y(t)$ equation, set to zero, and then plug this back in to find the maximum.

3. Set $x = t$, and then write y in terms of t using that.

4. To do this, we want to use sine and cosine, so one possible parametrization could be $c(t) = (2\cos(t), 2\sin(t))$.

5. We find that $\theta = \frac{\pi}{3}$. Then, we can use our standard formula for derivatives to figure out the slope of the tangent line.

6. We use $y(t) = 2 - t$ and $x'(t) = \frac{1}{t}$ to solve this problem.

Hint #3

1. This gives $\left(\frac{1}{x} - 1\right) e^{\left(\frac{1}{x} - 1\right)}$ for the function.

2. What does it mean for the particle to hit the ground? It means that $y(t) = 0$.

4. However, for this parametrization $c(0) = (2, 0)$, not the value we need. How can we modify our parametrization to fit this desired condition? The easiest way is by shifting the argument t.

5. Since $\dfrac{dy}{dx} = \dfrac{y'(\theta)}{x'(\theta)}$, we compute that $\frac{dy}{dx} = \frac{\sec\theta\tan\theta}{\sec^2\theta} = \frac{\tan\theta}{\sec\theta} = \sin\theta$.

6. We need to integrate $\displaystyle\int_1^2 (2-t)\frac{1}{t}\ dt = \int_1^2 \frac{2}{t} - 1\ dt$.

Hint #4

1. For the values at certain times, we can plug those into the parametric equations.
2. This gives the value of $t = 5$, since that is the positive solution to $y(t) = 0$, and then that can be plugged into the x equation.
4. We know that $c\left(\frac{2\pi}{3}\right) = \left(-1, \sqrt{3}\right)$ because of the values of sine and cosine. How can we use this fact to give us a new parametrization that works and has this value at 0?
5. Thus, the slope of the tangent line should be $\sin(\pi/3) = \frac{\sqrt{3}}{2}$. This will let us write the equation of the line.
6. The antiderivative here is $2\ln t - t$ and then we can plug in 1 and 2.

Answers

1. $y = \left(\frac{1}{x} - 1\right) e^{\left(\frac{1}{x}-1\right)}$. At $t = 0$, the curve is at the point $(1, 0)$. At $t = 6$, the curve is at $(1/7, 6e^6)$.

2. The maximum height is at $t = 3/2$, and is $49/4$. The particle hits the ground at $t = 5$, at position 25.

3. The parametrization we get here is $x = t$, $y = \frac{7t^2 - 2t}{3}$.

4. $c(t) = \left(2\cos\left(t + \frac{2\pi}{3}\right), 2\sin\left(t + \frac{2\pi}{3}\right)\right)$

5. $y - \sqrt{3} = \frac{\sqrt{3}}{2}(x - 2)$

6. $2\ln 2 - 2 - (2\ln 1 - 1) = 2\ln 2 - 1$