Math 152 - Worksheet 17

Section 10.5 - Ratio and Root Test

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Use the Ratio Test to determine if the series $\displaystyle\sum_{n=2}^{\infty} \frac{n^{40}}{n!}$ converges or diverges.
2. Use the Root Test to determine if the series $\displaystyle\sum_{n=3}^{\infty} \left(2+\frac{1}{n}\right)^{-n}$ converges or diverges.
3. Determine whether the series $\displaystyle\sum_{n=4}^{\infty} (0.8)^{-n} n^{-0.8}$ converges or diverges by any method covered so far.
4. Determine whether the series $\displaystyle\sum_{n=3}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right)$ converges or diverges by any method covered so far.
5. Determine whether the series $\displaystyle\sum_{n=3}^{\infty} \frac{n^2+5n}{7n^4+9}$ converges or diverges by any method covered so far.
6. Determine whether the series $\displaystyle\sum_{n=5}^{\infty} \frac{1}{n(\ln(n))^3}$ converges or diverges by any method covered so far.

Submission Problems

1. Determine whether the series $\displaystyle\sum_{n=2}^{\infty} \frac{(n!)^3}{(3n)!}$ converges or diverges by any method covered so far.
2. Determine whether the series $\displaystyle\sum_{n=5}^{\infty} \frac{\sin n}{n^2}$ converges or diverges by any method covered so far.

Hint #1

1. Take the ratio of consecutive terms of this series. How does this ratio simplify?
2. We need to take the nth root of the terms of the series, and then take the limit.
3. We see numbers raised to the nth power. Which test should be applied here?
4. The $(-1)^n$ suggests that this is an alternating series. Does the Alternating Series test apply?
5. Look at the dominant terms on the top and bottom of the terms of this series. What does this suggest?
6. The presence of a logarithm indicates that integral test may be the way to go here, since other comparison tests will not work.

Hint #2

1. The limit we need to take for the ratio test is $\lim_{n\to\infty} \dfrac{(n+1)^{40}}{n^{40}} \dfrac{1}{n+1}$.
2. If we do that, the limit t we are looking for is $\lim_{n\to\infty} \left(2 + \dfrac{1}{n}\right)^{-1}$.
3. We should apply the ratio test.
4. For this, the sequence b_n would be $\cos\left(\frac{1}{n}\right)$. Is this sequence decreasing? Does it go to zero?
5. This suggests that the series converges and that we could prove it by the limit comparison test.
6. The integral needed is $\int \dfrac{1}{x(\ln(x))^3}\, dx$. Try the substitution $u = \ln(x)$.

Hint #3

1. Since this limit is zero, what does this say about the series?
2. With this, we have that $t = \frac{1}{2}$. What does that say about the series?
3. The limit here you need to compute for this test is $\lim_{n\to\infty} \frac{1}{0.8}\frac{n^{0.8}}{(n+1)^{0.8}}$ What happens as $n \to \infty$?
4. This sequence does not go to zero. So the alternating series test does not apply. What else can we do?
5. Do the limit comparison test with the series $b_n = \frac{1}{n^2}$.
6. After this substitution, the integral is $\int_5^\infty \frac{1}{u^3}\,du$. Does this converge or diverge?

Hint #4

3. The limit goes to $\frac{5}{4}$. What does this mean for the series?
4. We already saw that the terms don't go to zero. What does that tell us?
5. The limit $L = \frac{1}{7}$, so what does that mean about the series?

Answers

1. This series converges by the Ratio test.
2. This series converges by the Root test.
3. This series diverges by the ratio test.
4. This series diverges by the nth term divergence test.
5. This series converges by the limit comparison test with $b_n = \frac{1}{n^2}$.
6. This series converges by the integral test.