MATH 251: Quiz 5

July 9, 2015

Name: ____

1. Determine if the following vector fields are conservative. If so, find a potential function.

(a) $\vec{F} = \langle ye^{xy} + y^3, xe^{xy} + 3xy^2, z^4 \rangle$.

(b) $\vec{G} = \langle x^2 + 3xy, y^4 - 3xyz, e^{xz} + 3xy \rangle$.

2. Let $\vec{c}(t) = \langle t^2, t^3 \rangle$ for $0 \leq t \leq 3$.

(a) Compute $\displaystyle\int_{\mathcal{C}} x \; ds$.

(b) Compute $\displaystyle\int_{\mathcal{C}} \langle y^2, x + y \rangle \cdot d\vec{s}$.

3. Let the surface $\mathcal{S}$ be the half of the sphere $x^2 + y^2 + z^2 = 1$ with $z \geq 0$. This surface can be parametrized by

$$G(u, v) = \langle \cos(v)\sin(u), \sin(v)\sin(u), \cos(u) \rangle$$

for $0 \leq v \leq 2\pi$ and $0 \leq u \leq \frac{\pi}{2}$.

(a) Find the upward normal vector $\vec{n}$ to this surface (as a function of u and v).

(b) Integrate $f(x, y, z) = z$ over the surface.

(c) Find the flux of the vector field $\vec{F}(x, y, z) = \langle -y, x, z \rangle$ through this sphere, with the upward normal vector.

Formulas

Let $f(x, y, z)$ be a scalar function and $\vec{F}(x, y, z)$ be a vector field. If $\mathcal{C} = \vec{c}(t)$ is a curve for $a \leq t \leq b$ and $\mathcal{S} = G(u, v)$ is a surface for u, v in some region $\mathcal{D}$, then

Scalar line integral: $\displaystyle\int_{\mathcal{C}} f(x, y, z)\ ds = \int_a^b f(\vec{c}(t))||\vec{c'}(t)||\ dt.$

Vector line integral: $\displaystyle\int_{\mathcal{C}} \vec{F}(x, y, z) \cdot d\vec{s} = \int_a^b \vec{F}(\vec{c}(t)) \cdot \vec{c'}(t)\ dt.$

Scalar surface integral: $\displaystyle\iint_{\mathcal{S}} f(x, y, z)\ dS = \iint_{\mathcal{D}} f(G(u, v))||\vec{n}(u, v)||\ dA.$

Vector surface integral: $\displaystyle\iint_{\mathcal{S}} \vec{F}(x, y, z) \cdot d\vec{S} = \iint_{\mathcal{D}} \vec{F}(G(u, v)) \cdot \vec{n}(u, v)\ dA.$

Integral Formulas:

$$\int \sin^2(x)\ dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C$$

$$\int \cos^2(x)\ dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C$$

Cross Partials Conditions

$$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x} \qquad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x} \qquad \frac{\partial F_2}{\partial z} = \frac{\partial F_3}{\partial y}$$