MATH 251: Quiz 3

June 18, 2015

Name: ____

1. Find the direction of steepest ascent for the function $f(x, y) = x^3 + 2xy + y^2$ at the point $(-1, 1)$.

2. If $T(x, y, z) = x^2 + 3y - 2z^2$ and $\vec{c}(t) = \langle \sin(t), t, \cos(t) \rangle$, compute

$$\frac{d}{dt} T(\vec{c}(t)) \qquad \text{at } t = \frac{\pi}{4}.$$

3. Find $\frac{\partial f}{\partial t}$ at $s = 1$, $t = 1$, for $f(x, y, z) = 2x^3 + 3y^2 + 5z$ if

$$x = s^2 + t^2 \qquad y = s + t \qquad z = s^2 - t^2$$

4. If z is given implicitly as a function of x and y by

$$x^2 + 2y^2 - 4z^2 + 3x - 2y + z = 8$$

find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

5. Find and characterize the critical points of the function

$$f(x, y) = y^3 - 3xy - x^2 + 2x$$

6. Find the maximum value of $3x + 4y - z$ on the unit sphere $x^2 + y^2 + z^2 = 1$.

Possibly Helpful Formulas:
Gradient of a Function $f(x, y, z)$:

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \frac{\partial f}{\partial z} \right\rangle$$

Chain Rule for Paths: If we have $F(x, y, z)$ and a curve $\vec{c}(t)$, then

$$\frac{d}{dt} F(\vec{c}(t)) = \nabla F(\vec{c}(t)) \cdot \vec{c}'(t)$$

Directional Derivative of the function f in the direction of $\vec{v}$:

$$D_{\vec{v}} f = \frac{1}{||\vec{v}||} \nabla f \cdot \vec{v}$$

General Chain Rule: If we have $f(x, y, z)$ with $x = x(s, t)$, $y = y(s, t)$ and $z = z(s, t)$, then

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial s}$$

Implicit Differentiation: If $F(x, y, z) = 0$ and we want to write z as a function of x and y, we have that

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Second Derivative Test: If (a, b) is a critical point of f, then for

$$D = f_{xx}(a, b) f_{yy}(a, b) - f_{xy}(a, b)^2$$

- If $D > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a minimum.
- If $D > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a maximum.
- If $D < 0$, then (a, b) is a saddle point.
- If $D = 0$, the test is inconclusive.