MATH 251: Quiz 1

May 28, 2015

Name: Solutions

1. Let $\vec{v} = \langle 2, -1, 2 \rangle$ and $\vec{w} = \langle -1, 3, 2 \rangle$. Compute

(a) $3\vec{v} + 2\vec{w}$.

(b) $\vec{v} \cdot \vec{w}$.

(c) $\vec{v} \times \vec{w}$.

(a) $3\langle 2,-1,2\rangle + 2\langle -1,3,2\rangle$

$= \langle 6,-3,6\rangle + \langle -2,6,4\rangle = \boxed{\langle 4,3,10\rangle}$

(b) $\vec{v}\cdot\vec{w} = 2\cdot(-1) + (-1)\cdot 3 + 2\cdot 2 = -2-3+4 = \boxed{-1}$

(c) $\vec{v}\times\vec{w} = \langle (-1)(2)-(2)(3), (2)(-1)-(2)(2), (2)(3)-(-1)(-1)\rangle$

$= \langle -2-6, -2-4, 6-1\rangle = \boxed{\langle -8,-6,5\rangle}$

2. Find the angle between the vectors $\langle 1, 1, -1 \rangle$ ($\vec{u}$) and $\langle 0, 1, 2 \rangle$ ($\vec{v}$) as an inverse cosine. Is this angle acute or obtuse?

$\vec{u}\cdot\vec{v} = 1\cdot 0 + 1\cdot 1 + (-1)(2) = -1$

$\|\vec{u}\| = \sqrt{3} \qquad \|\vec{v}\| = \sqrt{5}$

So $\cos\theta = \dfrac{-1}{\sqrt{15}} = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$

So $\theta = \cos^{-1}\left(\dfrac{-1}{\sqrt{15}}\right)$

$\cos\theta < 0$

$\Rightarrow$ angle is obtuse.

3. Let $\vec{u} = \langle 2, 3, -2\rangle$ and $\vec{v} = \langle 1, 1, 1\rangle$. Decompose $\vec{u}$ into $u_{//} + u_{\perp}$ with respect to $\vec{v}$.

$$\vec{u} \cdot \vec{v} = 2+3-2 = 3$$

$$\|v\|^2 = 1+1+1 = 3.$$

$$\vec{u}_{//} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} = \frac{3}{3} \vec{v} = \vec{v} = \langle 1, 1, 1 \rangle.$$

$$\vec{u}_{\perp} = \vec{u} - \vec{u}_{//} = \langle 2, 3, -2 \rangle - \langle 1, 1, 1 \rangle = \langle 1, 2, -3 \rangle.$$

So $\boxed{\vec{u} = \langle 1, 1, 1 \rangle + \langle 1, 2, -3 \rangle.}$

4. Find a vector $\vec{v}$ that is perpendicular to both $\langle 2, 1, 4\rangle$ and $\langle -2, 6, 1\rangle$.

One such vector is the cross product.

$$\vec{v} = \langle 2, 1, 4 \rangle \times \langle -2, 6, 1 \rangle$$

$$= \langle 1 \cdot 1 - 4 \cdot 6,\ 4 \cdot (-2) - 2 \cdot 1,\ 2 \cdot 6 - (-2)(1) \rangle$$

$$= \langle 1 - 24,\ -8 - 2,\ 12 + 2 \rangle$$

$$\boxed{\vec{v} = \langle -23, -10, 14 \rangle}$$