

MATH 251: Practice 8

June 8, 2015

Name: Solutions

1. Evaluate the following limits.

$$\lim_{(x,y) \rightarrow (1,1)} e^{xy} + \frac{x}{y} \quad \lim_{(x,y) \rightarrow (2,1)} \frac{(x^2-4)(y^2-1)}{(x-2)(y-1)} = (x+2)(y+1).$$

Exponential is continuous,  $y \neq 0$  so

$$\lim_{(x,y) \rightarrow (1,1)} e^{xy} + \frac{x}{y} = e^1 + \frac{1}{1} = \boxed{e+1}$$

$$(x+2)(y+1) \text{ is continuous} \Rightarrow \lim_{(x,y) \rightarrow (2,1)} (x+2)(y+1) = 4 \cdot 2 = \boxed{18}$$

2. Show that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 + y^2}$$

does not exist. Hint: Consider paths of the form  $y = mx^2$  for varied  $m$ .

$$y = mx^2 \quad \lim_{x \rightarrow 0} \frac{x^2(mx^2)}{x^4 + (mx^2)^2} = \lim_{x \rightarrow 0} \frac{x^4 m}{x^4(1+m^2)} = \frac{m}{1+m^2}$$

$$m=0 \rightarrow \text{limit} = 0$$

$$m=1 \rightarrow \text{limit} = \frac{1}{2}$$

$\Rightarrow$  limit does not exist.