MATH 251: Practice 6

June 3, 2015

Name: Solutions.

Consider the curve $\vec{r}(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle$, a modified "twisted cubic."

- Find the length of the curve between $t = 0$ and $t = 2$.
- Compute the curvature of $\vec{r}$ at $t = 1$.

$$r'(t) = \langle 1, 2t, 2t^2 \rangle$$

$$\|r'(t)\| = \sqrt{1+4t^2+4t^4} = \sqrt{(2t^2+1)^2} = 2t^2+1.$$

$$\int_0^2 \|r'(t)\|\,dt = \int_0^2 2t^2+1\,dt = \frac{2}{3}t^3+t \Big|_0^2 = \frac{16}{3}+2$$

$$= \boxed{22/3}$$

$$r'(t) = \langle 1, 2t, 2t^2 \rangle$$

$$r''(t) = \langle 0, 2, 4t \rangle.$$

$$r'(t) \times r''(t) = \langle 4t^2, -4t, 2 \rangle.$$

$$\|r'(t) \times r''(t)\| = \sqrt{16t^4+16t^2+4} = 2\sqrt{4t^4+4t^2+1} = 2(2t^2+1).$$

So $$\kappa(t) = \frac{\|r'(t)\times r''(t)\|}{\|r'(t)\|^3} = \frac{2(2t^2+1)}{(2t^2+1)^3} = \frac{2}{(2t^2+1)^2}.$$

$$\kappa(1) = \frac{2}{(2+1)^2} = 2/9.$$