

MATH 251: Practice 3

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Name: Solutions

Let $u = \langle 1, 2, 1 \rangle$ and $v = \langle 2, -1, 3 \rangle$.

- (a) Compute $u \times v$.
 (b) Find the area of the parallelogram spanned by \vec{u} and \vec{v} .
 (c) Find the volume of the parallelepiped spanned by \vec{u} , \vec{v} , and $\langle -1, 1, 1 \rangle$.

$$\begin{aligned} \vec{u} \times \vec{v} &= \langle 2 \cdot 3 - (1)(-1), 1 \cdot 2 - 1 \cdot 3, 1(-1) - (2)(2) \rangle \\ &= \langle 6+1, 2-3, -1-4 \rangle = \boxed{\langle 7, -1, -5 \rangle} \end{aligned}$$

$$\begin{aligned} \text{(b) Area} &= \|\vec{u} \times \vec{v}\| \\ &= \|\langle 7, -1, -5 \rangle\| \\ &= \sqrt{49+1+25} = \sqrt{75} = \boxed{5\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \text{(c) Volume} &= |\langle -1, 1, 1 \rangle \cdot \vec{u} \times \vec{v}| \\ &= |\langle -1, 1, 1 \rangle \cdot \langle 7, -1, -5 \rangle| \\ &= |-7-1-5| = \boxed{13} \end{aligned}$$