MATH 251: Practice 2

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Name: Solutions

Let $u = \langle 1, 2, 1 \rangle$ and $v = \langle 2, -1, 3 \rangle$.

(a) Compute $u \cdot v$.

(b) Find the angle between u and v. You may leave this as an inverse cosine.

(c) Find the decomposition $u = u_{//} + u_{\perp}$ of u with respect to v.

(a) $\vec{u} \cdot \vec{v} = 1 \cdot 2 + 2 \cdot (-1) + 1 \cdot 3 = \boxed{3}$

(b) $\cos \theta = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|}$

$\|\vec{u}\| = \sqrt{1+4+1} = \sqrt{6} \qquad \|v\| = \sqrt{4+1+9} = \sqrt{14}$

$\cos \theta = \dfrac{3}{\sqrt{6}\sqrt{14}} = \sqrt{\dfrac{3}{28}}$

So $\boxed{\theta = \cos^{-1}\left(\sqrt{3/28}\right)}$

(c) $u_{//} = \left(\dfrac{u \cdot v}{v \cdot v}\right) \vec{v} = \dfrac{3}{14} \langle 2, -1, 3 \rangle = \boxed{\left\langle \dfrac{6}{14}, \dfrac{-3}{14}, \dfrac{9}{14} \right\rangle}$

$u_{\perp} = u - u_{//} = \langle 1, 2, 1 \rangle - \left\langle \dfrac{6}{14}, \dfrac{-3}{14}, \dfrac{9}{14} \right\rangle$

$\boxed{= \langle 8/14, 31/14, 5/14 \rangle}$